Investigation of particle-fluid interaction at high Reynolds-number using PIV/PTV techniques

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ABSTRACT

Particulate turbulent flows are highly complicated due to the interaction between the fluid turbulence and the dispersed particles. An experimental investigation of the turbulent motion of solid particles in liquid phase at a high Reynolds number (*Re*) of 320,000 is conducted in a vertically upward section of a closed slurry flow loop. The solid particles are 0.5, 1, and 2 mm glass beads with volumetric concentration of 0.1, 0.4 and 0.8 percent, respectively. Measurements of the glass beads turbulence is conducted using particle tracking velocimetry (PTV) while the surrounding liquid phase is characterized by particle image velocimetry (PIV) of 18 micron hollow glass tracers. The average velocity profiles show that the large glass beads lag behind the liquid flow at the central region of the pipe while they move faster than the fluid phase near the wall. Investigation of Reynolds stress components of the two phases shows that the turbulent intensity of the solid particles is significantly larger than the liquid phase in both streamwise and radial directions. Concentration profiles show that 0.5 and 1 mm particles tend to accumulate in the core of the flow while 2 mm particles create a center-peaked distribution. It has also been observed that the presence of the solid particles has negligible effects on the turbulence intensity of the liquid phase at the investigated *Re* number.

INTRODUCTION

The presence of small solid particles affects the transport of mass, momentum, and heat in turbulent pipe flows that are widely encountered in a variety of engineering systems. The motion of solid particles and their interactions with the fluid turbulence introduce a complicated system with additional parameters including the *Re* number and Stokes number of the particle, particle/fluid density ratio, flow direction, and solid phase concentration. The complexity of particle-laden turbulent flows has limited the success of models and numerical simulations in accurate prediction of transport mechanism in particular at high *Re* numbers.

The limitations are also extended to experimental investigations at high *Re* particle-laden flows. Balachandar and Eaton (2010) have mentioned lack of experimental measurement in turbulent particle-laden flows at *Re* larger than 30,000. This limitation to low *Re* is partially due to the focus of previous experiments on particle-laden flows with air as the carrier phase of the solid particles. Kulic et al. (1994) and Varaksin et al. (2000) conducted measurement of turbulent statistics in air flows with *Re* smaller than 15,000. Their results showed attenuation of carrier phase turbulence due to the presence of the solid particles smaller than 0.1 mm. Tsuji et al. (1984) investigated larger particles with diameter of 0.2 to 3 mm in an upward gas flow pipe at a *Re* of about 20,000. They observed that larger particles augmented fluid turbulence while the smaller ones attenuated it. The same conclusion was obtained by Kussin and Sommerfeld (2002) who performed phase Doppler anemometry in a horizontal air channel with glass beads of 0.1 to 0.625 mm diameters and *Re* around 50,000. Hosokawa and Tomiyama (2004) and Kameyama et al. (2014) conducted experiments in solid-liquid turbulent flows with particles of 0.625 to 4 mm in diameter at Reynolds numbers of 15,000 and 19,500, respectively. Zisselmar and Molerus (1979) investigated solid-liquid flows in horizontal pipe at a Reynolds number of 100,000 and observed attenuation of liquid phase turbulence upon addition of 53 μm glass beads.

An overview of experimental investigations on particle-laden flows presented in Table 1 highlights lack of any experiment at *Re* number higher than 100,000 relevant slurry flow applications in petroleum engineering that consist of solid particles in oil/water flows. Moreover, it is difficult to extrapolate the conclusions on particle turbulent motion in gas flows to liquid flows at high Reynolds due to difference in density ratios (ρ_P / ρ_f) and stokes numbers of the solid particles.

Table 1. Summary of experimental investigations on particle-laden flows

REF.	Carrier Phases	Flow direction	Particle size (mm)	Re	ρ_P / ρ_f	Mass fraction	Volume fraction
Varaksin *et al.* (2000)	Gas	Down	0.05	15,300	2100	0.04 to 0.55	2.e-5 to 5.8e-4
Lee and Durst (1982)	Gas	Up	0.1-0.8	8,000	2100	0.55 to 0.71	0.58 e-3 to 1.2 e-3
Kulic *et al.* (1994)	Gas	Down	0.05-0.09	13,800	2100,7300	0.02 to 0.44	0 to 2e-4
Kussin and Sommerfeld (2002)	Gas	Horizontal	0.06-0.625	30,705 to 57,284	2100	0.09 to 0.5	5e-3 to 5e-4
Hosokawa and Tomiyama (2004)	Liquid	Up	1-4	15,000	3.2	2.2e-3 to 5.7e-3	7e-3 to 1.8e-2
Kameyama *et al.* (2014)	Liquid	Up/down	0.625	19,500	2.5	2.4e-3	0.6
Zisselmar and Molerus (1979)	Liquid	Horizontal	0.053	100,000	2.5	7e-3 to 2.4e-2	1.7e-2 to 5.6e-2

In addition to the fluid's turbulence, turbulent motion of particles in two-phase flows is also crucial for developing a thorough understanding of particulate systems. The interaction between the solid and liquid phases is through several forces such as viscous drag, pressure gradient due to local acceleration, added mass, Basset history integral in addition to gravity and electrostatic forces (Melling, 1997). Based on the review of the works on the particle fluctuations in the literature; one can conclude that the streamwise fluctuations of particles are equal to or higher than the liquid phase (Varaksin et al., 2000, Kussin and Somerfeld, 2002, Kameyama et al., 2014). However, there is not such an agreement on the lateral (or radial) fluctuations of particles. While the majority of experimental works (Caraman et al., 2003, Boree et al., 2005, Kussin and Somerfeld, 2002, Kameyama et al., 2014) suggest that lateral fluctuations of particles are equal or higher than the fluid, Kulic et al. (1994) and Varaksin et al. (2000) obtained lateral fluctuations smaller than the fluid. Vreman (2007) associated these discrepancies to the parameters such as wall roughness and electrostatics which have not been characterized in the experimental investigations. Finally one should note that Varaksin et al. (2000) and Kulic et al. (1994) speculated that their result might have been tampered by insufficient pipe length and electrostatic charges on particles, respectively. In the current experiments, using long developing section, long smooth acrylic test section, and water as carrier phase helped to diminish those unwanted effects.

These investigations show that there is a large group of parameters affecting the turbulence intensity of the solid and liquid phases. Development of models requires a larger dataset and systematic investigation of each parameter. A key parameter in the bi-directional relation between the dispersed and the carrier phase is the volume fraction (φ_v) or mass fraction (φ_m) (Balachandar and Eaton, 2010). At particle concentration (φ_v) higher than 10^{-6} significant modulation of the fluid turbulence statistics is observed (Elghobashi, 1994). Gore and Crowe (1991) suggested the effect of particles on fluid turbulence is a function of some non-dimensional parameters as

$$M\% = f(\mathrm{Re}, \mathrm{Re}_p, u/U_s, \rho_p/\rho_f, \varphi_v) \quad (1)$$

where M is the turbulence modulation, Re_p is the particle Reynolds number, u is velocity fluctuations, U_s is the slip velocity between fluid and particle. Tanaka and Eaton (2008) introduced a new dimensionless parameter, Pa_{st} (particle moment number) to classify attenuation and augmentation of fluid turbulence by solid particles as following

$$Pa_{st} = St\,\mathrm{Re}^2\left(\frac{\eta}{L}\right)^3 \quad (2)$$

where St is stokes number, η is Kolmogorov length scale and L is the flow geometry dimension. As shown in Eq.1 and Eq.2, Reynolds number has a direct impact on the particle and fluid turbulence interaction which promotes extension of the experimental investigations to higher *Re.*

The current experimental investigation provides detailed characterization of turbulent motion of solid particles dispersed in water in a 2 inch upward flow at *Re* of 320,000. Glass beads were used as the particulate phase with diameters of 0.5, 1 and 2 mm and volumetric concentrations (φ_v) of 0.1, 0.2, and 0.8 percent. A combined PIV/PTV technique is applied for simultaneous measurement of turbulent statistics of both phases as detailed in the subsequent sections.

EXPERIMENTS

Fig.1 illustrates a schematic of the close-circuit slurry loop operated using a centrifugal pump controlled by a Variable Frequency Transformer. Flow rates are measured by a magnetic flow meter (FoxBoro IM T25) and temperature is held constant at 25 ºC throughout the whole experiments by a double pipe heat exchanger. Water and solid particles are loaded through the feeding tank. Once the loop is loaded with the mixture, the tank will be bypassed and flow circulates through the closed loop. Measurements are conducted in the upward section with a pipe diameter of D = 50.6 mm. A transparent test section made from acrylic is located 90D after the bend providing sufficient straight pipe for fully developed regime. In order to minimize image distortion due to the curvature of the pipe wall, a rectangular box was placed around the test section. Glass beads with diameter of 0.5, 1 and 2 mm were used as particulate phase with volumetric concentrations (φ_v) of 0.1, 0.4 and 0.8 percent respectively. The density of glass beads is 2500 kg/m3 resulting in $\rho_p / \rho_f = 2.5$. Average bulk velocity is 5.74 m/s which correspond to Re of 320,000.

A planar Particle Image/Tracking Velocimetry (PIV/PTV) technique is employed to capture the motion of both liquid and particulate phases. Flow is seeded by 18 µm hollow glass tracers with density of 600 kg/m3 (60P18 Potters Industries). PIV Images are captured by a CCD camera (Imager Intense, Lavision) with 1376×1040 pixel resolution which has a physical pixel size of 6.45×6.45 µm. The required PIV illumination is provided by an Nd:YAG laser (Solo III-15, New Wave Research) which creates a sheet with a thickness of about 1 mm. For each set of experiments, 10,000 pairs of double-frame images are acquired using Davis 8.2 (LaVision). Magnification and spatial resolution of the imaging system is set at 0.27 and 42.6 pixel/mm, respectively.

The images capture both the large glass beads and the PIV tracers. The large glass beads are detected using “imfindcircle” function in MATLAB-R2013a which is based on Hough transform for detection of circular objects. The script requires the range of acceptable particle radius (set to ±%20 of average particle radius) and also a gradient-based threshold for edge detection as input parameters. The latter is based on the high intensity gradient at the sharp boundary of in-focus particles while the out-of-focus particles have a smooth gradient. Two different low and high gradient-based thresholds are considered for edge-detection. The low threshold is applied to detect and mask out all the in-focus and out-of-focus particles from both frames for PIV analysis of the liquid phase. However a higher threshold is applied to only detect the in-focus particles for the PTV process. By having the centroid location and radius of the in focus particles; displacement (i.e. velocity) of the detected glass beads are measured in the PTV algorithm developed in MATLAB.

In the PIV process, the images with the detected particles are imported to Davis 8.2 software to calculate the velocity of the liquid phase. First the detected particles will be masked out and then two nonlinear filters including subtract sliding background and particle intensity normalization filters are applied on the images. Those filters helped minimising the noise and stability in results. Then cross-correlation method with 32×32 pix^2 window size and 75% window overlap is applied to obtain the instantaneous velocity field of liquid phase. The interrogation windows which have more than 1% overlap with the masked areas are rejected ensuring no bias in the measurement of the liquid phase.

U, u, and v are average velocity in flow direction, fluctuating velocity in flow direction and fluctuating velocity in radial direction, respectively. The radial direction is indicated by r starting from the centre of the pipe (r/R=0) toward the pipe wall located at r = 25.3 mm (r/R=1).

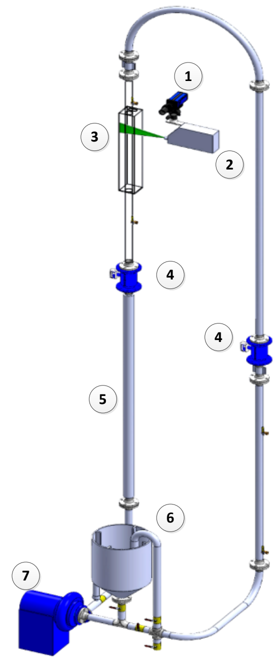

Figure 1. Schematic of the test rig consisting of (1)- laser, (2)-camera, (3)-acrylic pipe and viewing box, (4)-magnetic flow meters, (5)-double pipe heat exchanger, (6)-Feeding tank, (7)-pump

RESULTS AND DISCUSSION

The average velocity profiles for both liquid and solid phases are shown in Fig.2. The liquid velocity profiles are almost identical to the single phase (Unladen flow) profile indicating that the presence of particles at this low concentration has negligible effect on the velocity profile of the liquid phase. However particles lag behind the liquid phase in the core of the flow due to the gravity and the lag is larger for larger particles. The results show that slip velocities of the particles at the pipe centreline are close to the corresponding terminal (settling) velocities of each particle which is in agreement with results of Sato and Hishida (1996). By going towards the wall, the velocity difference between the particles and the liquid phase becomes smaller until at r/R=0.85, all the particles and liquid have almost the same velocity. After this cross-over point, the velocity of particles is larger than the liquid velocity. Also among the particles, the bigger ones take the lead and have larger velocity in this region. Therefore, the slip velocity between the particles and fluid at the near wall region is opposite of that in the core of the flow.

The abovementioned phenomenon was reported by others (Lee and Durst, 1982, Tsuji et al., 1984, Kameyama et al., 2014) and is attributed to the “slip boundary condition” of the particles at the wall. Particles after colliding with the wall bounce off to the core of the flow while preserving most of their streamwise momentum. Moreover high velocity particles are transported from core of the flow to the wall region by their lateral fluctuations. Since these particles have high response (or relaxation) time (τ_p), they don’t quickly respond to the surrounding liquid gradient field and do not effectively decelerate in this region. Particles with larger size have even longer response time; therefore, larger particles acquire higher velocity in near wall region.

The streamwise turbulent fluctuations $<u^2>$ of the liquid phase show small variations by the addition of glass beads compared to the unladen flow as observed in Fig.3. The highest modulation in the liquid turbulence intensity is observed near the wall. The liquid phase $<u^2>$ shows negligible variation with the addition of the 1 and 2mm glass beads. However, a slight attenuation of $<u^2>$ is observed by adding the 0.5 mm particles. Fig.4 shows that the particles also introduce small changes to the radial velocity fluctuations $<v^2>$ of the liquid phase. The graph shows that the 2 mm particles slightly augmented and 0.5 mm particles attenuated radial fluctuation of the fluid while 1mm have no detectable effect on $<v^2>$ of the liquid phase. It is worthy of note that the average modulation of turbulence (the magnitude of change in $<u^2>^{0.5}$ and $<v^2>^{0.5}$) in both radial and streamwise directions does not go beyond 5%. These changes are also very small in comparison with other works. Tsuji et al. (1984) reported turbulence augmentation of 100% and 150% at the centreline for 1 and 3mm polystyrene particles with around 0.4 vol% concentrations in an upward air flow at *Re* number around 23,000. Also Hosokawa and Tomiyama (2004) showed that 1, 2.5mm ceramic particles at concentration around 0.8 volumetric percent in an upward water flow with *Re*=15,000 augmented the turbulent intensity by more than 100% at the pipe centreline. This shows that increasing the *Re* number has a dampening effect on the extent of turbulence modulation.

Hosokawa and Tomiyama (2004) showed that the extent of modulation increases by increasing U_s/u. Since fluctuating velocity (u) is a function of the bulk velocity (U_b), the functionality can be rewritten as U_s/U_b. As discussed above, the slip velocity (U_s) at the pipe centre equals the terminal settling velocity of the particle (V_t) hence we can see that the turbulence modulation can also be considered as a function of V_t/U_b. Since the terminal settling velocity is almost constant; by increasing the Reynolds number (or U_b) this ratio goes to zero therefore the modulation becomes small, at least at this low concentration. In other words, when the velocity ratio of the two phases approaches one, the impact of particles on the fluid turbulence at low mass concentration can be small.

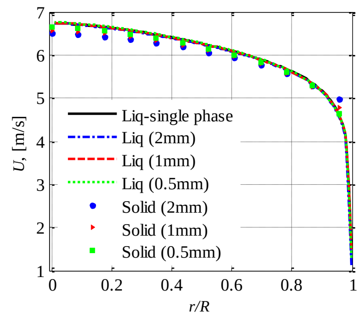

Figure 2. Average velocity profiles of liquid and solid phase along the radial direction.

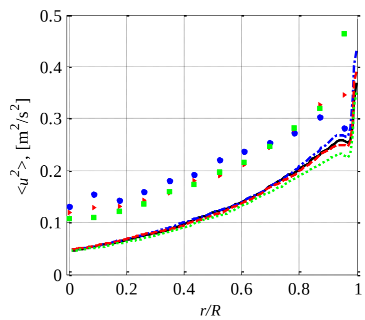

Figure 3. Streamwise turbulent fluctuations of liquid and solid phases. Legends are the same as Fig.2.

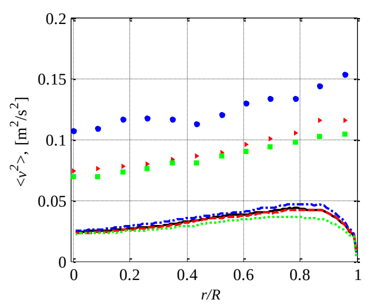

Figure 4. Fluctuating velocity profiles of liquid and solid phase in the radial direction. Legends are the same as Fig.2

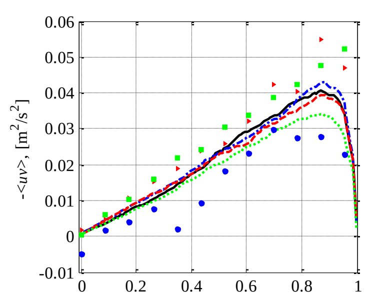

Figure 5. Reynolds stresses $<uv>$ of liquid and solid phase along the radial direction. Legends are the same as Fig.2.

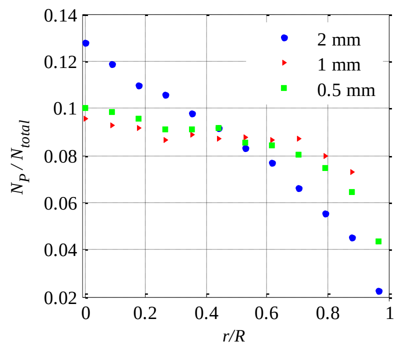

Figure 6: Number density distribution of particles. Legends are the same as Fig.2.

Fig.3 illustrates that particles have a larger <u^2> than the carrier phase almost everywhere across the pipe radius which is in agreement with other works (Kulic et al., 1994, Varaksin et al., 2000, Caraman et al., 2003). The streamwise turbulent intensity is larger for the larger particles at the pipe centreline while the smaller ones have a higher intensity near the wall which to the authors' knowledge has not been scrutinized in the literature. Boree et al. (2005) provided a graph of <u^2> for 60 and 90 µm particles showing that smaller particles have a larger <u^2> in the near wall region.

These particles are partially responsive to the fluid turbulence in the core of the flow but the fluid turbulence does not play a role as a source of production for particle fluctuations in the near wall region because of high Stokes number of particles in this region. The higher streamwise fluctuation of the particles is credited for their higher inertia. Since particles have higher inertia than the fluid element they can move further in the flow field while keeping their initial inertia. This leads to particles with larger axial fluctuations than the liquid phase. On the same ground, larger turbulence production of 2mm particles is expected due to their higher inertia. However, production of axial turbulence for 0.5 mm particles is larger near the wall which can be attributed to the larger mean velocity gradient of 0.5 mm particles in this region.

The radial fluctuations of particles are also observed to be higher than the fluid (fig.4). The 2mm glass beads also demonstrated much larger radial fluctuation than the rest. The results are in agreement with other works such as Kameyama et al. (2014) and Boree et al. (2005). In order to investigate the sources for particle fluctuations in the radial direction, the cross section of the pipe is divided in to two sections: core region and near wall region. In the core region, in addition to the fluid turbulence, particles are subject to the lift force which acts towards the pipe centre and particle-particle interactions which is a dispersive force and scatters the particles towards the pipe wall. The interaction of the 2 mm particles with the fluid turbulence is lower than the other smaller particles because of its higher response time. However, the other two sources work stronger in favour of 2mm particles thus they have much higher radial turbulence in the core of the flow. In the near wall region, fluid turbulence and particle-particle interaction are negligible. In this region, the lift force is reversed due to higher velocity of particles relative to the liquid phase and pushes the particles towards the wall (Lee and Durst, 1982). Therefore, the reverse lift force and the particle-wall collisions in near wall region are regarded as the main sources for the particles radial fluctuations which again are stronger for 2 mm particles.

The profiles of Reynolds stress (-<*uv*>) for liquid phase and the glass beads are illustrated in Fig.5. Reynolds stress (-<*uv*>) for 0.5 and 1 mm particles are slightly higher than that of single phase while the Reynolds stress of the 2 mm particles is lower than that of the unladen flow. Boree and Caraman (2005) and Caraman et al. (2003) showed that -<*uv*> for 60 and 90µm glass beads in air flow is slightly larger than the fluid. In general, the normal Reynolds stress of the solid particles was observed to increase with the particle size in Fig.5. However, the 0.5 and 1 mm particles have a much larger Reynolds shear stress than the 2 mm particles which can be attributed to the fact that 2 mm particles are less involved with the fluid turbulence because of larger inertia.

Concentration profiles of the glass beads (fig.6) are obtained from PTV data using the number of particles (N_p) at any radial position, normalized by the total number of detected particles (N_{total}) across the radius. Concentration profiles for 0.5 and 1 mm particles are almost constant in core of the flow (*r/R*<0.7) and then it rapidly declines in the near wall region. This trend was observed by Kameyama et al. (2014) for 625 µm glass beads in the upward liquid flow. However the concentration profile for 2 mm particles is linearly declining from pipe centreline towards the pipe wall.

These trends show that there is a lift force that pushes the particles away from the wall and it is more effective in case of 2mm particles. It is clear that particles are subject to the liquid velocity gradient from pipe centre (*r/R*=0) to crossing point (*r/R*=0.85) which causes particles to spin and creates a lift force towards the pipe centreline. At the core region of the flow (below *r/R*=0.7) the velocity gradient becomes smaller and the magnitude of the lift force declines. This explains the flat concentration profiles for 0.5 and 1 mm in the core of the flow. However Because of the large size of 2mm particles, this force is still effective even below *r/R*=0.7 and keeps pushing the particles towards the centreline which creates a centre-peaked profile for 2mm particles.

CONCLUSION

In this investigation, the turbulent motions of solid particles have been measured in upward solid/liquid flows at a high unprecedented *Re* of 320,000. Results show that the 0.5, 1 and 2 mm glass beads lag behind the fluid in the core of the flow (*r/R*<0.85) because of the gravity and the lag almost equal to the particle terminal velocity. However, solid particles move faster than the liquid phase in the near wall region (*r/R*>0.85) because of their ~~lack of fast~~slower response to the surrounding liquid and slip boundary condition at the wall. Particles didn't have any significant effect on the turbulence of the liquid phase while literature shows substantial augmentations for much lower *Re*. This was attributed to the low ratio of terminal velocity to the bulk velocity (V_t/U_b).

Higher fluctuating velocities of particles in streamwise direction are due to their higher inertial which causes the particles to have longer radial movements. Because of higher radial transport of larger particles in the core, therefore, 2mm particles have larger axial fluctuations in the core of the flow. However in near wall region, curvature of velocity profile (hence mean shear rate) for 0.5 mm particles is largest which leads to create larger production source of axial fluctuations. Production sources for radial fluctuations are fluid turbulence, particle-particle interactions and lift force towards the centre in the core of the flow. However the production sources near the wall are reversal lift force and particle-wall collisions. Since these sources stronger for bigger particles they end up having larger radial fluctuations. Concentration distributions are mainly influenced by the lift force which has the particles accumulating in the core region. Because of more vigorous lift in case of 2mm, the concentration distribution appears to be a centre-peaked profile.

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