

Energy balance in turbulent gas-solid channel flow

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ABSTRACT

The aim of the present study is to examine the implications of particle additives on the transfer, conversion and dissipation of mechanical energy in a turbulent gas-solid channel flow. To achieve this goal we have performed two-way coupled direct numerical simulations (DNSs) of gas-solid channel flow. Equations for fluid mean flow kinetic energy (KE) and fluid turbulent kinetic energy (TKE) are used in the results analysis. To highlight the influence of particles, the KE budgets were compared with the results of un-laden channel flow, i.e. without any additives. It was found that in the un-laden flow, 57.2% of the energy input was directly dissipated in the mean flow, whereas 40.2% was converted to turbulence through mean shear production before being dissipated by viscous action at small scales. By contrast, in the particle-laden flow, the interaction of the particles and fluid appears in the energy budgets. In the mean-flow energy balance, the mean dissipation accounted for 59.1% of the energy supply. This is comparable with the un-laden flow. However, the energy loss from the mean flow reduced from 40.2% to 13.7%, but was partly compensated by the new sink term (24.7%) which represents negative work done by the particles. The results also suggested that the mean flow loses kinetic energy to particles in the centre region of the channel, whereas it gains energy from the particles in the near-wall region. In the TKE budget, the particles released kinetic energy to the turbulence and this energy is likely obtained from the mean flow. This extra energy supply compensates partially for the substantial reduction of the mean shear production to about $2/3^{\text{rd}}$ of the production in the un-laden channel. Ultimately TKE is dissipated by deformation work due to the fluctuating viscous stresses. We concluded that the particles play an intermediary role in the energy transfer and conversion from the mean flow to the turbulence.

INTRODUCTION

Particle-laden flow is one of the most common two-phase flows, which is found both in nature and industry, such as air transport of pollutants, fluidized bed in chemical processes, and dispersion of volcanic ash in the atmosphere. The complexity of the turbulent fluid motion leads to fascinating dynamics of particle suspensions as well as complicated particle-fluid interactions, such as kinetic energy transfer between the solid and gas phases (see Zhao et al. 2013). It is known that the addition of tiny particles can modulate the fluid motion and either augmentation or attenuation of the turbulence has been observed (Balachandar & Eaton 2010, Squires & Eaton 1990). Such phenomena have been widely explored by means of experiments and numerical simulations, such as Squires & Eaton (1990), Pan & Banerjee (1996), Dritselis and Vlachos (2008), Zhao et al. (2010).

Kinetic energy budgets are one of the primary tools to examine the turbulent flow. Andersson & Barri (2008) investigated KE transport and conversion in an unladen turbulent plane Couette flow. Mansour et al. (1988) have shown and analysed the Reynolds stress budgets and the turbulent dissipation in a turbulent channel flow. In flows of gas-solid mixtures the suspended solid particles interact with the fluid and make the energy exchange processes more complex than in an unladen channel flow (Balachandar & Eaton 2010). In the presence of particles, a reaction force from each and every particle affects the fluid motion through an extra force term in the Navier-Stokes equations. The presence of this particle force may give rise to significant modifications of the flow field, both in isotropic turbulence (Squires & Eaton 1990) and in channel flows (Li et al. 2010). In this work the two-way coupled Eulerian-Lagrangian approach is employed to investigate particle suspensions in turbulent channel flow and we mainly focus on the influence of tiny inertial particles on the KE and TKE balance of the particle-laden flow.

GOVERNING EQUATIONS

Turbulent channel flow is computed by means of DNS in an Eulerian frame of reference. The motion of the incompressible and isothermal Newtonian fluid is governed by the mass and momentum conservation equations:

$$\nabla \cdot \vec{u} = 0; \quad \rho \left[\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla) \vec{u} \right] = -\nabla p + \nabla \cdot \vec{\tau} + \vec{f}^p \quad (1)$$

The flow is driven in the streamwise x -direction by a constant mean pressure gradient $\nabla \bar{p}$. In two-way coupled simulations, the last term \vec{f}^p in the momentum equation represents the feedback force per unit volume from the particle phase on the fluid.

The spherical particles in the flow are treated as point-particles in a Lagrangian frame; see e.g. Li et al. (2001) or Zhao et al. (2010). Each individual particle is tracked at every time step and the translational motion of the individual particles is only affected by particle inertia through the Schiller-Naumann-corrected Stokes drag, while other forces, such as gravity, lift, and virtual-mass forces, are neglected in order to isolate the interaction between turbulence and particles. The size of the particles is smaller than the Kolmogorov length scale in the flow field and the force on a particle can therefore be treated as a point force. The position of a particle and its translational velocity can be obtained from:

$$\frac{d\vec{x}}{dt} = \vec{v}_p; \quad \frac{d\vec{v}}{dt} = \frac{C_{N-S}}{\tau_p} \left[\vec{u}(\vec{x}_p, t) - \vec{v}_p \right] \quad (2)$$

Here C_{N-S} is the correction coefficient (Schiller and Naumann, 1933) and response time is $\tau_p = 2Da^2/9\nu$, where D is density ratio between particles and the fluid, a is particle radius and ν is fluid kinematic viscosity.

According to Newton's third law, each and every particle acts back onto the local fluid with a point force $-\vec{F}_m$ where the subscript m refers to the particle ID number. The feedback force on the fluid from n_p particles within a given cell volume:

$$\vec{F}_m = m_p \frac{C_{N-S}}{\tau_p} \left[\vec{u}(\vec{x}_p, t) - \vec{v}_p \right]; \quad \vec{f}^p = -\frac{1}{V_{cell}} \sum_m^n \vec{F}_m \quad (3)$$

This two-way coupled scheme is essentially the same as that employed by Squires and Eaton (1990) in the simulation of isotropic turbulence and later by Dritselis and Vlachos (2008) and Zhao et al. (2013) in simulations of wall turbulence.

The flow in the channel is driven by a constant pressure gradient, which provides a continuous power supply:

$$\langle E \rangle = \int_{-h}^h -U_x \nabla \bar{p} dz \quad (4)$$

Here U_x is mean velocity in the streamwise x -direction and the integration is from wall to wall in the wall-normal z -direction. This input of mechanical energy will ultimately be converted into internal thermal energy.

The overall kinetic energy associated with the mean flow U_x is given by integrating all terms in the balance equation for the mean flow kinetic energy KE in the wall-normal direction:

$$0 = \underbrace{\int_{-h}^h -U_x \nabla \bar{p} dz}_{\langle E \rangle} + \underbrace{\int_{-h}^h \overline{\rho u_x u_z} \frac{\partial U_x}{\partial z} dz}_{\langle S \rangle} - \underbrace{\int_{-h}^h \mu \left(\frac{\partial U_x}{\partial z} \right)^2 dz}_{\langle \mathcal{E}_{mean} \rangle} + \underbrace{\int_{-h}^h \overline{f_x^p U_x} dz}_{\langle W_{f,mean} \rangle} \quad (5)$$

Here, the first term is the mechanical energy input by the driving pressure gradient (4), whereas the other terms are the integrated mean flow-turbulence interaction term $\langle S \rangle$, the viscous dissipation term $\langle \mathcal{E}_{mean} \rangle$ and the particle-fluid interaction term $\langle W_{f,mean} \rangle$, respectively. While the first term is consistently positive and the second and third terms are negative, the role of the last term will be shown in the results section.

Similarly the balance of the overall turbulent kinetic energy TKE is given as:

$$0 = - \underbrace{\int_{-h}^h \overline{\rho u_x u_z} \frac{\partial U_x}{\partial z} dz}_{\langle P \rangle} - \underbrace{\int_{-h}^h \mu \left(\frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_i} \right) dz}_{\langle \mathcal{E} \rangle} + \underbrace{\int_{-h}^h \overline{f_i^p u_i} dz}_{\langle W_{f,fluctuation} \rangle} \quad (6)$$

The production term in equation (6) is balanced by the turbulent dissipation and by particle-fluid fluctuation interactions. Please notice that the production term $\langle P \rangle = -\langle S \rangle$.

Based on the analysis above a set of energy balance equations in a particle-unladen channel flow becomes:

$$\begin{aligned} 0 &= \langle E \rangle - \langle P \rangle - \langle \mathcal{E}_{mean} \rangle \\ 0 &= \langle P \rangle - \langle \mathcal{E} \rangle \\ 0 &= \langle E \rangle - \langle \mathcal{E} \rangle - \langle \mathcal{E}_{mean} \rangle. \end{aligned} \quad (7)$$

The analogous equations for particle-laden channel flow with consideration of fluid-particle interactions are:

$$\begin{aligned} 0 &= \langle E \rangle + \langle P \rangle - \langle \mathcal{E}_{mean} \rangle + \langle W_{f,mean} \rangle \\ 0 &= -\langle P \rangle - \langle \mathcal{E} \rangle + \langle W_{f,fluctuation} \rangle \\ 0 &= \langle E \rangle - \langle \mathcal{E} \rangle - \langle \mathcal{E}_{mean} \rangle + \langle W_{f,mean} \rangle + \langle W_{f,fluctuation} \rangle. \end{aligned} \quad (8)$$

COMPUTATIONAL DETAILS

The channel flow is computed by means of DNS at a friction Reynolds number $Re_\tau = 180$, which is based on the distance $2h$ between the two parallel walls. The size of the computational domain is $12h$ and $6h$ in the streamwise x -direction and the spanwise y -direction, respectively. The Navier-Stokes equation (1) for particle-laden flows is discretized on 192^3 grid-nodes. Periodic boundary conditions are imposed in the two homogeneous directions and no-slip and impermeability boundary conditions are enforced at the solid channel walls at $z = -h$ and $z = h$. The DNS-solver is the same as that used by Zhao et al. (2010). A pseudo-spectral method using Fourier series in the two homogeneous directions and a second-order finite-difference scheme in the wall-normal direction is employed for the spatial derivatives on a staggered grid system. The time advancement is carried out with a second-order explicit Adams-Bashforth scheme.

One set of 4 million particles with response time $\tau_p^+ = 30$ was released randomly in an already fully-developed turbulent channel flow. The superscript '+' indicates a quantity non-dimensionalized by using the fluid viscosity ν and frictional velocity u_τ . The particle radius is $a^+ = 0.36$ and the volume fraction is around 9×10^{-4} . Particle-wall collisions are perfectly elastic. The statistics are obtained as averages in time and in homogeneous x - y planes after a sufficient level of steadiness is achieved at $t^+ = 10800$. The time window for the sampling is $\Delta t^+ = 18000$.

RESULTS AND DISCUSSIONS

Statistics of mean KE and TKE balances in both unladen and particle-laden channel flows are shown in Figure 1 and Figure 2, respectively. The mechanical energy input E in the unladen channel flow is balanced by the mean flow-turbulence interaction term, i.e. the transfer of energy from the mean flow to the turbulence, and mean ϵ , which is the viscous dissipation of the kinetic energy of the mean motion. The negative peak of mean flow-turbulence interaction term S can be observed around $z^+ \approx 12$ where the turbulence intensity is maximum (Kim et al. 1987) and the maximum ϵ_{mean} is found at the wall where viscous effects are largest. With particle additives the extra term $W_{f,\text{mean}}$, the so-called fluid-particle interaction term, is imposed into the balance equation (5). This term acts as a sink in the central region of channel from $z^+ = 30$ to 330 but as a source term in the near-wall region. In the presence of particles, the magnitude of the mean flow-turbulence interactions S is dramatically damped whereas the profiles of mean ϵ and E remain almost the same as in the un-laden flow.

It is well known that the $S = -P$ plays an important role in transferring kinetic energy from mean flow (KE) into turbulence (TKE) by means of mean shear. Finally the TKE is transformed into thermal energy by means of turbulence dissipation ϵ . In other words, the overall P in the flow system should be equal to the overall turbulence dissipation ϵ , i.e. $\langle P \rangle = \langle \epsilon \rangle$. In the particle-laden flow, however, a contribution from the fluctuating part of the particle-fluid interactions should be considered in the TKE budgets, i.e. $W_{f,\text{fluctuation}}$ (equation 6). By comparing the un-laden and particle-laden flows in Figure 2, we can observe an attenuation of P and turbulent dissipation ϵ as well as a non-negligible contribution from fluid-particle interactions in the TKE-balance. Consistent with the finding reported by Zhao et al. (2013), $W_{f,\text{fluctuation}}$ is a source term all across the channel. This suggests that the particles tend to transfer kinetic energy to the fluid turbulence.

To get a comprehensive impression of the overall energy input, transport, conversion, and dissipation in the turbulent channel flow, it is instructive to integrate the individual terms across the channel, i.e. from the lower wall to upper wall. For both un-laden and laden channel flow, Table 1 and Table 2 summarize the total KE and TKE balance according to equations (7) and (8), respectively. Similar as the observations discussed above, the integrated energy input E and the mean viscous dissipation in Table 1 remain almost the same as in the un-laden flow, whereas the mean flow-turbulence interaction term S is attenuated. The mean particle-fluid interaction term plays the role as a sink term, which means that the particles extract kinetic energy from fluid mean flow.

In Table 2, a modest imbalance between the production P and the turbulence dissipation ϵ is firstly observed in the un-laden flow. The present dissipation terms have been compared with those from the clean channel flow of Hoyas and Jiménez (2006) and our results are slightly underpredicted (not shown). This could possibly be caused by the limited numerical accuracy of the 2nd-order central-difference scheme in the wall-normal direction. This may explain the slight imbalance of

the integrated TKE budgets in Table 2. Here, we can see that the fluctuating fluid-particle interaction term is acting as a source term. This shows that the particles contribute to turbulent kinetic energy of the fluid phase. It is likely that parts of this energy stems from the loss of mean flow kinetic energy. The turbulent energy dissipation is anyhow reduced in the particle-laden flow.

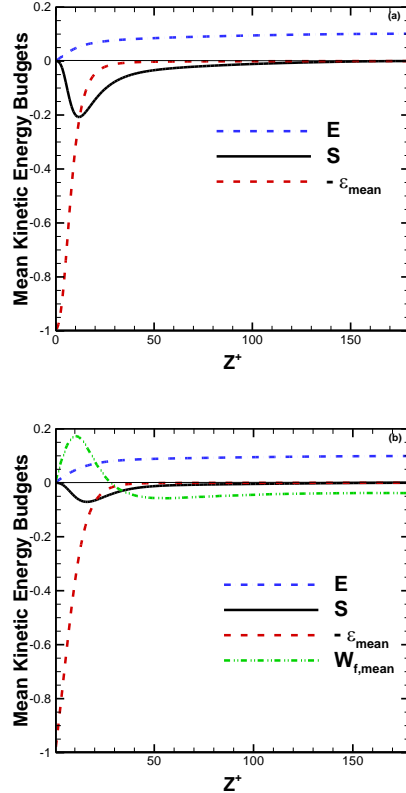


Figure 1: Profiles of source and sink terms of KE: unladen flow (a) and particle-laden flow (b). E is the kinetic energy input by the mean pressure gradient; ϵ_{mean} is dissipation induced by the mean velocity gradient; S is meanflow-turbulence interaction term; $W_{f,\text{mean}}$ is the fluid-particle interaction term.

Table 1: Integrated source and sink terms of the mean flow KE. Brackets ' $\langle \rangle$ ' indicate the integration of the terms from bottom wall to upper wall.

Case	$\langle E \rangle$	$\langle -\epsilon_{\text{mean}} \rangle$	$\langle S \rangle$	$\langle W_{f,\text{mean}} \rangle$
Unladen	31.3	-17.9	-12.6	0.0
Percentage	100%	57.2%	40.2%	0.0%
Laden	31.7	-18.7	-4.4	-7.8
Percentage	100%	59.1%	13.7%	24.7%

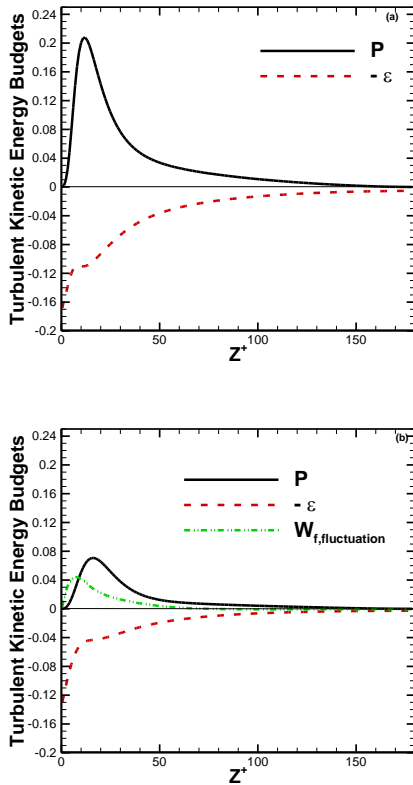


Figure 2: Profiles of source and sink terms of TKE: unladen flow (a) and particle-laden flow (b). ϵ is turbulent dissipation; W_f is the fluid-particle interaction term.

Table 2: Integrated source and sink terms of the TKE.

Case	$\langle P \rangle$	$-\langle \epsilon \rangle$	$\langle W_{f,fluctuation} \rangle$
Unladen	12.6	-11.6	0.0
Percentage	100%	92.3%	0.0%
Laden	4.4	-5.7	2.0
Percentage	68.5%	89.1%	31.5%

CONCLUDING REMARKS

In the present work we have examined the implications of particle additives on the conversion, transfer, and dissipation of mechanical energy in a particle-laden channel flow by means of two-way coupled Eulerian-Lagrangian DNSs. All results are compared with the unladen channel flow to examine the role of inertial particles on the turbulence modulation.

We first looked into the budgets of KE and TKE in Figures 1 and 2 obtained from DNS data. All budget terms were next integrated from the lower wall to the upper wall to investigate the overall kinetic energy transport and conversion shown in Tables 1 and 2. We found that the mean flow loses energy due to the particles in the channel central region, whereas the flow gains energy from the particles in the near-wall region. Overall, it is likely that the particles gained energy from the mean flow, will thereafter release the energy to the turbulence. In spite of this energy supply from the particles to the turbulence, the reduced mean-shear production P is not fully compensated. According to the statistical results presented, we conclude that the extra term which accounts for particle-fluid interactions plays an important role in KE and TKE transfer.

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