ANALYSIS OF PRESSURE-STRAIN CORRELATIONS IN A SUPERSONIC PIPE, NOZZLE AND DIFFUSER USING GREEN'S FUNCTIONS

Somnath Ghosh
Department of Aerospace Engineering,
Indian Institute of Technology, Kharagpur, India.
sghosh@aero.iitkgp.ernet.in

Rainer Friedrich
Institute of Aerodynamics and Fluid mechanics,
Technische Universitaet Munich,
Boltzmannstr 15, Garching, Germany.
r.friedrich@lrz.tum.de

ABSTRACT

Pressure-strain correlations along with the turbulent dissipation rate are important terms that need to be modelled in second-order turbulence closures. In this paper, we provide insights into the pressure-strain correlations in a supersonic pipe, nozzle and diffuser by performing Green's function analyses based on DNS and LES data. The relative importance of the rapid and slow parts of pressure-strain correlations for the axial, azimuthal and radial pressure-strain correlations is presented and it is demonstrated that properly performed LES replicates the trends found in DNS and may be used to develop models for these correlations.

INTRODUCTION

DNS studies of supersonic channel flows with isothermal walls (Coleman *et al.*, 1995; Foysi *et al.*, 2004) have revealed that compressibility effects manifest themselves as mean density and temperature variations in the near-wall region. This leads to a reduction of pressure-strain correlations at supersonic Mach numbers and in turn to an increase in Reynolds stress anisotropy (Foysi *et al.*, 2004). These observations were also made in DNS of supersonic pipe flow with isothermal wall (Ghosh *et al.*, 2010). Effects of mean dilatation and extra rates of strain add further complications to supersonic flows and lead to changes in the turbulence structure which cannot be explained only by mean property variations. Such effects were described by Bradshaw (1974) and observed in LES and DNS of canonical supersonic nozzle and diffuser flows where fully developed supersonic pipe flow serves as inflow (Ghosh *et al.*, 2008; Ghosh & Friedrich, 2014). It was observed that the Reynolds stresses decrease dramatically in the nozzle and increase in the diffuser. The pressure-strain correlations were found to play a pivotal role in changing the Reynolds stresses in these flows. Hence, it is important to gain insight into the behaviour of pressure-strain correlations in these flows and an elegant way of doing this is a Green's function analysis based on DNS data. Foysi *et al.* (2004) used Green's function to analyse pressure-strain correlations using supersonic channel flow DNS data and found the contribution of the slow terms to be greater than that of the rapid terms. Ghosh *et al.* (2010) carried out a similar study in cylindrical coordinates with DNS data of a supersonic pipe flow with isothermal wall. Recently Ghosh & Friedrich (2014) extended the Green's function analysis to a supersonic nozzle and diffuser with isothermal walls using DNS data. In this paper we analyse LES data of supersonic pipe, nozzle and diffuser flow and compare the results with those obtained with DNS. Such a Green's function analysis with LES data will enable us to easily gain insight into flows for which only LES is possible.

MATHEMATICAL AND COMPUTATIONAL DETAILS

We use modified Bessel functions to construct the Green's functions in cylindrical coordinates. The effect of axial non-periodicity in the nozzle and diffuser is taken care of by using a series expansion involving cosine functions. The procedure is detailed in Ghosh *et al.* (2010); Ghosh & Friedrich (2014) and is not repeated here due to lack of space. It is used here to analyse DNS and LES data of supersonic pipe, nozzle and diffuser flows. 5th order low-dissipation compact upwind schemes have been used in the DNS for the convective terms and 6th order central schemes for the molecular transport terms. The LES uses 6th order compact central schemes for all terms (Ghosh *et al.*, 2008). The pipe flow has a centerline Mach number of 1.5 and a friction Reynolds number of 245. $256 \times 128 \times 91$ points have been used for the DNS in the axial, spanwise and radial directions respectively where the domain size is $10R \times 2\pi R \times R$. This pipe flow also acts as inflow to the nozzle simulation which is also similarly discretized and has a domain length of $10R$ and a ratio of nozzle to pipe radius of 1.58 at the exit. The incoming pipe flow for the diffuser simulation has a friction Reynolds number of 300 and a centerline Mach number of 1.8. The diffuser domain of length $10R$ and a ratio of diffuser to pipe radius of 0.98 at the exit is discretized with $384 \times 256 \times 140$ points in the DNS. The LES uses $64 \times 64 \times 50$ points for pipe, nozzle and diffuser domains. Isothermal walls are used in all cases. Partially non-reflecting outflow conditions (Poinsot & Lele, 1992) are used for the nozzle and diffuser simulations. The modified Bessel functions are evaluated using GNU Scientific Library (GSL) software.

RESULTS

At first we present results of the Green's function analysis for the supersonic pipe flow with isothermal wall. The Green's function provides an accurate prediction of pressure-strain correlations as evidenced by the following comparison with DNS data. Figure 1 (left) shows the comparison for the axial pressure-strain correlation in the pipe flow. We find that the Green's function solution follows the DNS profile very closely. Also, we see that the slow source terms are the major contributors in this case. The contribution of the rapid terms is smaller. In figure 1 (right), we observe an accurate estimate of the azimuthal component, as well. Interestingly, for this component, the rapid terms are the major contributors as opposed to the slow terms for the axial component. For the radial component (not shown), the slow terms are again the major contributors. In fig 2 (left), we present a comparison of the axial pressure-strain correlation obtained from DNS and LES of supersonic pipe flow and the corresponding Green's function solutions. We note that although the LES underpredicts the pressure-strain correlations, the Green's function solution computed with LES data follows the LES profile very closely as in the DNS. In fig. 2 (right), we show the contributions of rapid and slow terms of the Poisson's equation to the pressure-strain correlations using DNS and LES data and note that the difference between the DNS and LES profiles for the complete correlation is clearly due to the under-prediction of the slow terms.

We now present results for the supersonic nozzle and diffuser flows and begin by looking at the mean quantities, normalized with their values at the inflow plane. Figure 3 (left) shows the area variation in the nozzle and also the decrease in bulk density and increase in bulk velocity as a result of acceleration. The corresponding profiles for the diffuser are shown in fig. 3 (right) where we see an increase in bulk density and a decrease in bulk velocity almost along the entire length of the diffuser except near the outflow as also noted in previous studies (Ghosh *et al.*, 2008; Ghosh & Friedrich, 2014). The mean density and temperature profiles in the radial direction, normalized with their wall values, in the nozzle and diffuser are shown in fig. 4. The decrease in mean temperature in the nozzle in the axial direction is associated with an axial increase in the density ratio, since mean pressure remains approximately constant in the radial direction. Opposite effects i.e. increase in mean temperature and decrease in mean density ratio are observed in the diffuser. Attenuation and amplification of turbulence intensities in the nozzle and diffuser, respectively are demonstrated in figures 5 and 6, where the axial and radial Reynolds stresses are plotted at different axial locations. As discussed in Ghosh *et al.* (2008); Ghosh & Friedrich (2014), these large changes in the Reynolds stresses can be linked to the pressure-strain correlations which are amplified in the diffuser and attenuated in the nozzle. The decrease in the axial pressure-strain correlation in the nozzle is demonstrated in figure 7 (left), which also shows the good agreement of the Green's function solutions with the LES results at both the axial locations. In 7 (right), we see that the slow terms contribute more to the axial pressure-strain correlations in the nozzle than the rapid terms, a result which has been obtained from analysis of DNS data of supersonic channel and pipe flows. The contribution of the rapid terms to the azimuthal pressure-strain correlations is larger than that of the slow terms (fig. 8, left). For the radial pressure-strain correlations, however, the slow terms are the major contributors (fig. 8, right). These results are in qualitative agreement with those obtained from DNS of nozzle and diffuser flows (Ghosh & Friedrich, 2014) and this shows the usefulness of the present analysis using LES data. We finally present results of the Green's function analysis for the diffuser flow. Figure 9 (left) shows the increase in axial pressure-strain correlations along the diffuser and also the good agreement of the Green's function result with the LES result. Similar behaviour of the slow and rapid contributions to the axial, radial and azimuthal pressure-strain correlations as in the nozzle flow is also found in the diffuser, namely the dominance of the slow terms for the axial and radial components and of the rapid terms for the azimuthal component (see fig. 10).

Conclusions

We have presented results from a Green's function analysis of supersonic pipe, nozzle and diffuser flows using mostly LES data, but also DNS data for comparisons. The analysis reveals very good agreement with the present LES data and the results follow the trends found with similar analysis of DNS data. The pressure-strain correlations constructed using the resolved scales in the LES of pipe flow lie below the DNS profile, as expected and the Green's function analysis of course shows that the LES underpredicts the contribution of the slow source terms of the Poisson equation to the pressure-strain correlation compared to the DNS. This is due to the fact that the slow terms depend on products of velocity fluctuations, their correlations and gradients of both, which are not accurately represented in an LES. As was recently observed from analysis with DNS data (Ghosh & Friedrich, 2014), and is also predicted using LES data, the slow terms have a larger contribution in the axial and radial pressure-strain correlation than the rapid terms and a smaller contribution in the azimuthal component. Thus, when carried out using adequate resolution, an LES can provide useful guidelines for modeling pressure-strain correlations.

The first author (S.G.) thanks his undergraduate students A. Dubey, A. Thagela and his doctoral student S. Mahapatra at IIT, Kharagpur for their contributions to this work.

REFERENCES

Bradshaw, P. 1974 The effect of mean compression or dilatation on the turbulence structure of supersonic boundary layers. *Journal of Fluid Mechanics* **63**, 449–464.

Coleman, G. N., Kim, J. & Moser, R. D. 1995 A numerical study of turbulent supersonic isothermal-wall channel flow. *Journal of Fluid Mechanics* **305**, 159–183.

Foysi, H., Sarkar, S. & Friedrich, R. 2004 Compressibility effects and turbulence scalings in supersonic channel flow. *Journal of Fluid Mechanics* **509**, 207–216.

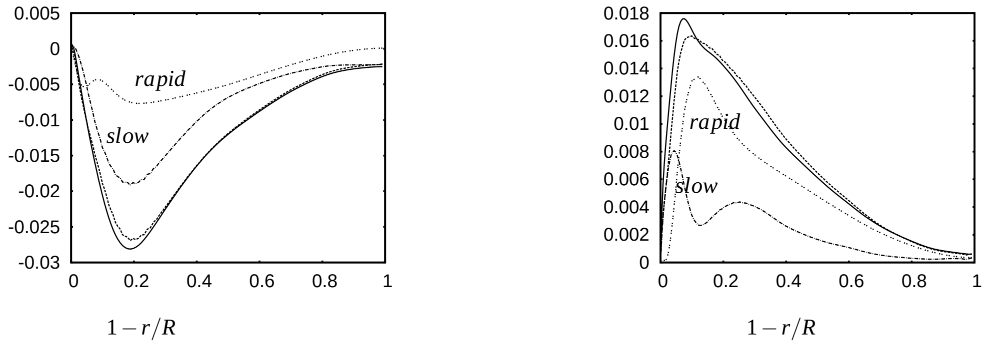

Figure 1. Comparison of Green's function solution (- - -) with DNS data (—) for axial (left) and azimuthal (right) pressure-strain correlations in pipe flow. The contributions of the slow (-.-.-) and rapid (.. .. ..) source terms are shown. All terms are normalized with $\tau_w^2/\bar{\mu}$.

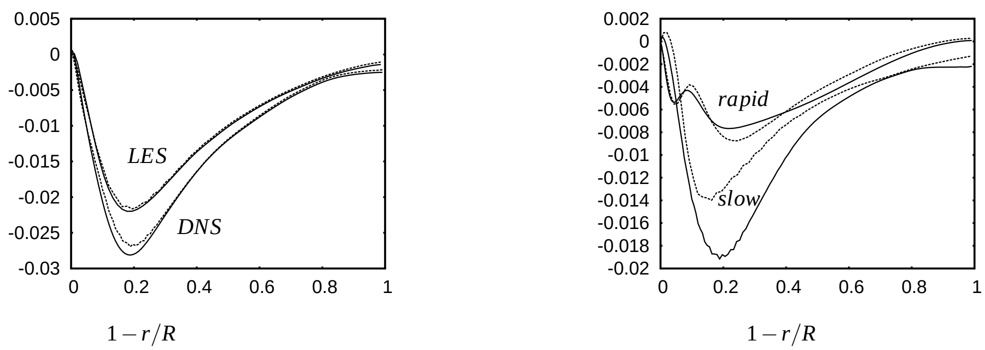

Figure 2. (left) Comparison of axial pressure-strain correlation in supersonic pipe flow from DNS and LES data (solid lines). The corresponding Green's function solutions are shown by dashed lines. (right) Slow and rapid parts of axial pressure-strain correlations obtained using Green's function from DNS (solid line) and LES (dashed line).

Ghosh, S., Foysi, H. & Friedrich, R. 2010 Compressible turbulent channel and pipe flow: similarities and differences. *Journal of Fluid Mechanics* **648**, 155–181.

Ghosh, Somnath & Friedrich, Rainer 2014 Effects of distributed pressure gradients on the pressure-strain correlations in a supersonic nozzle and diffuser. *Journal of Fluid Mechanics* **742**, 466–494.

Ghosh, S., Sesterhenn, J. & Friedrich, R. 2008 Large-eddy simulation of supersonic turbulent flow in axisymmetric nozzles and diffusers. *International J. of Heat and Fluid Flow* **29**, 579–590.

Poinsot, T. J. & Lele, S. K. 1992 Boundary conditions for direct simulations of compressible viscous flows. *Journal of Computational Physics* **101**, 104–129.

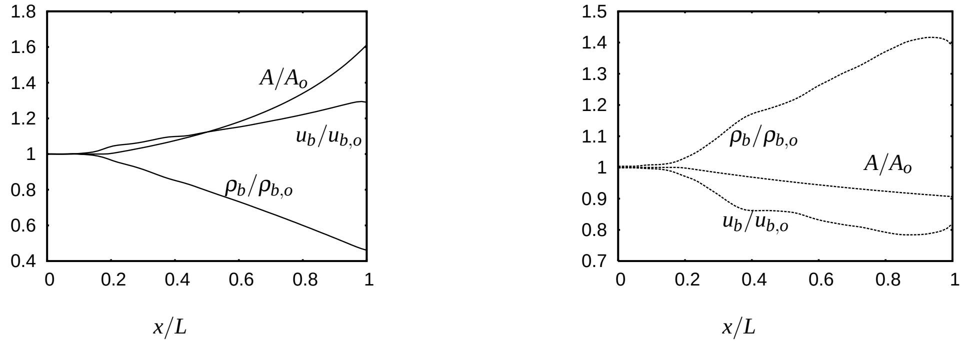

Figure 3. Area distribution, bulk velocity and bulk density in the nozzle (left) and diffuser (right), normalized with values at the inlet.

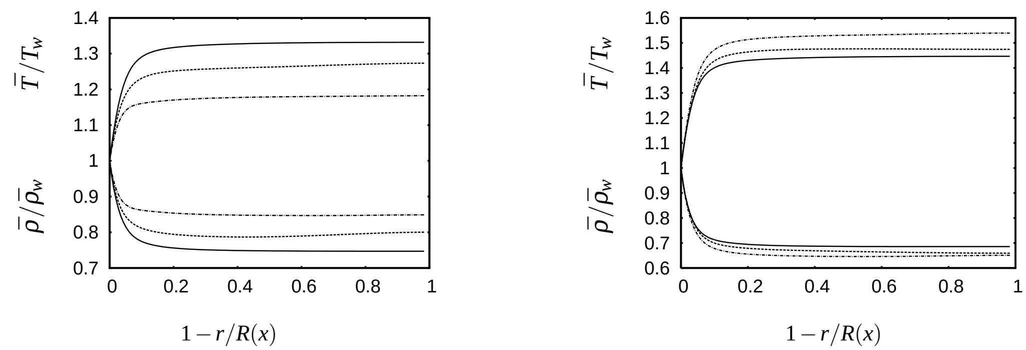

Figure 4. Mean density and temperature profiles in the nozzle (left) at — $x/L = 0.01$, ... $x/L = 0.37$, -.-.- $x/L = 0.6$ and in the diffuser at — $x/L = 0.01$, ... $x/L = 0.25$, -.-.- $x/L = 0.4$ (right), normalized with their wall values.

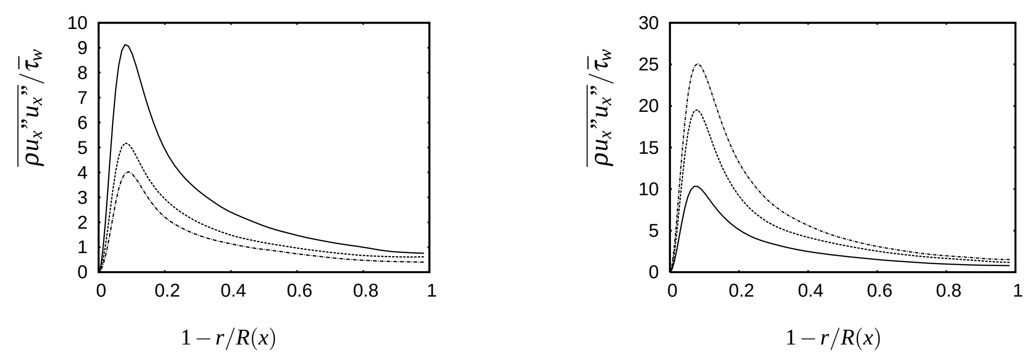

Figure 5. Axial Reynolds stress profiles in the nozzle (left) at — $x/L = 0.01$, ... $x/L = 0.37$, -.-.- $x/L = 0.6$ and in the diffuser at — $x/L = 0.01$, ... $x/L = 0.25$, -.-.- $x/L = 0.4$ (right)

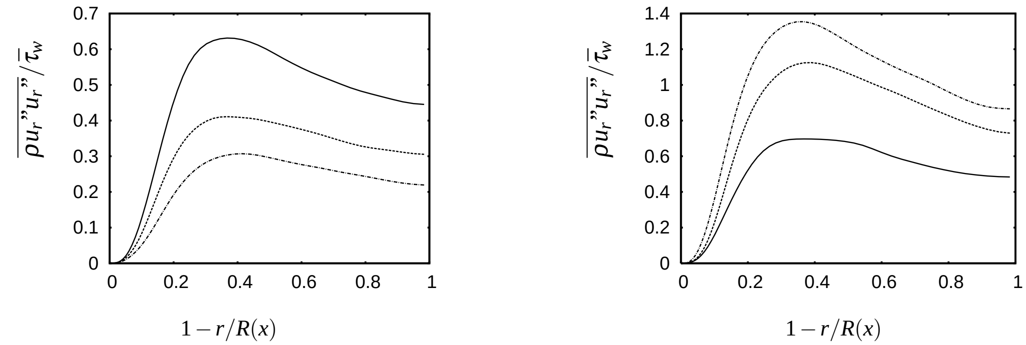

Figure 6. Radial Reynolds stress profiles in the nozzle (left) at — $x/L = 0.01$, ... $x/L = 0.37$, -.-.- $x/L = 0.6$ and in the diffuser at — $x/L = 0.01$, ... $x/L = 0.25$, -.-.- $x/L = 0.4$ (right)

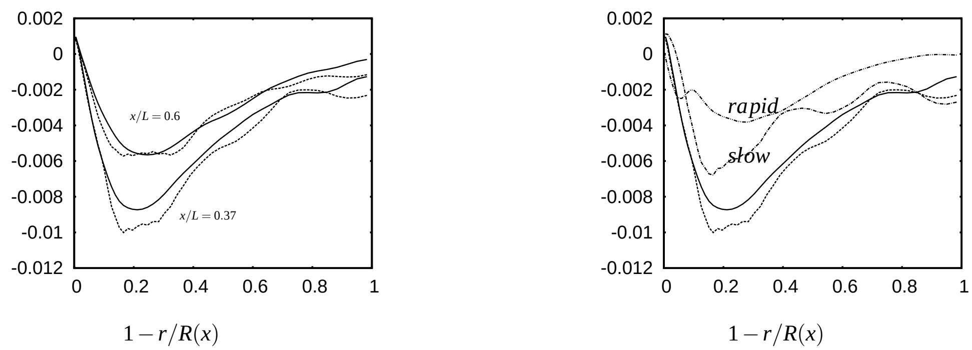

Figure 7. Comparison of Green's function solution (- - -) with LES data (—) for axial pressure-strain correlations in nozzle flow at $x/L = 0.37, 0.6$ (left). Slow and rapid parts of axial pressure-strain correlations in nozzle flow at $x/L = 0.37$. All terms are normalized with $\tau_w^2/\bar{\mu}$.

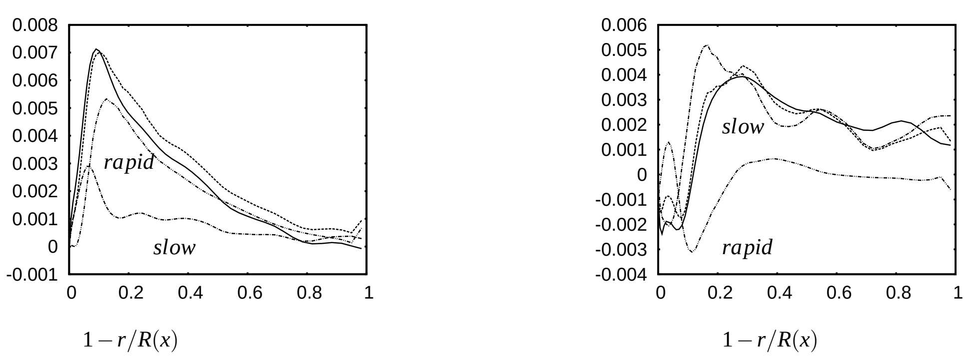

Figure 8. Comparison of LES data (—) and Green's function solutions (- - -) for azimuthal (left) and radial (right) pressure-strain correlations in the nozzle for nozzle flow at 0.37. Slow and rapid parts are also shown

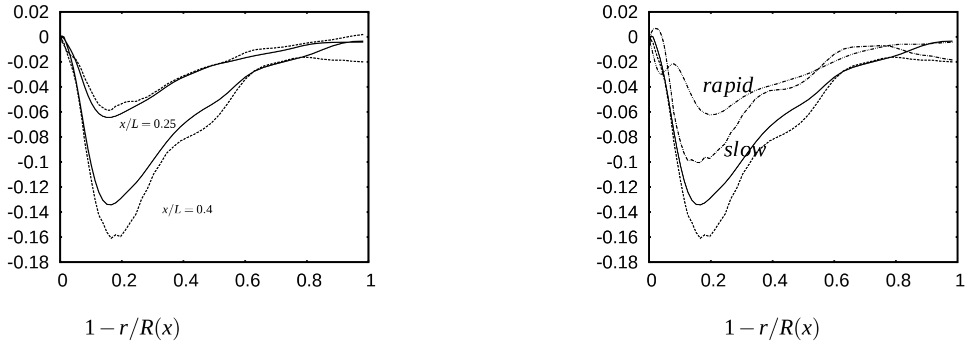

Figure 9. Comparison of LES data (—) and Green's function solutions (- - -) for axial pressure-strain correlations in the diffuser flow at $x/L = 0.25, 0.4$ (left). Slow and rapid parts of axial pressure-strain correlations in the diffuser flow at $x/L = 0.4$ (right).

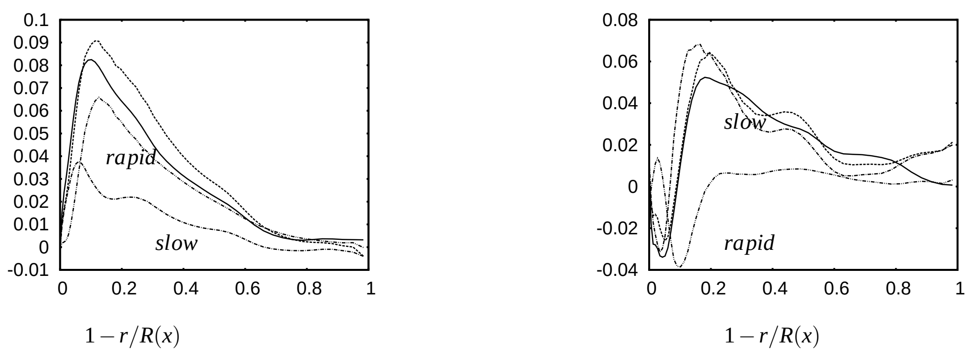

Figure 10. Comparison of LES data and Green's function solutions for azimuthal (left) and radial (right) pressure-strain correlations in the diffuser at $x/L = 0.4$. — LES, ... Green's function solution, -.-.- slow part, ... rapid part