Direct Numerical Simulation of Rapidly Distorting Compressible Homogeneous Turbulent Flow

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Abstract

The gas-kinetic scheme is applied into the direct numerical simulation of rapidly distorting turbulence, to investigate the effects of mean flow compressibility on turbulence fluctuations. Different distortion Mach numbers are considered and the results show that as 'implicit effect' (Simone et al., 1997), the redistribution relaxation process in the initial stage has great influences on flow field evolution. Compared with shear flows, mean flow compressibility has distinct 'explicit effect' on turbulence energy in strain flows. The priori estimate of three typical turbulence models for redistribution term shows that high order turbulence model is required for pressure-deviatoric term, and the compressibility corrections of pressure-dilatation part should also be carried out upon high order models.

Introduction

In turbulence model studies, the redistribution term has always been with great difficulty due to the modelling of fluctuating pressure, especially in compressible flows. Meanwhile, most published works focus on shear flows (Simone et al., 1997; Sarkar et al., 1991; Kumar et al., 2013). Flows dominated by strain have received much less attention and require more fundamental researches (Blaisdell et al., 1996, Cambon et al., 1993). In the studies of redistribution modelling, Rapid Distortion Theory (RDT) is commonly adopted (Pope, 2000). RDT is valid for the strain rate $Sk/\varepsilon >> 1$, so that the influence of the fluctuation velocity (nonlinear term) and dissipation term can be neglected. Thus only the rapid pressure-strain part is considered. The slow pressure-strain term related with the small scale turbulent fluctuations can be modelled by Rotta model.

It is worth to be mentioned that RDT flow condition is not only an ideal model but also commonly exists in actual turbulent flows. Lee et al. (1990) has observed that Sk/ε can reach a maximum value of 15-20 in incompressible channel flow. The value is even larger as 25 in our *Ma*=0.3 compressible simulation, which reflects the specific effect on turbulent statistics of mean flow compressibility.

Considering the limitations of the linearization in RDT, the direct numerical simulation (DNS) is adopted in this paper to analyze the compressibility effect on the redistribution term in plane stain flows, starting from homogeneous isotropic turbulent flows.

Numerical Scheme

The multidimensional gas-kinetic scheme (GKS) (Li & Fu, 2006; Xu, 2001) is adopted in the present study, which is based on the BGK equation,

$$f_t + \mathbf{u}\cdot\nabla_x f = \frac{g-f}{\tau} \qquad (1),$$

Here the distribution function f is the only physical quantity need to be solved, g is the Maxwellian distribution. τ is the particle collision time which can be determined by $\tau = \mu / p$. f has the analytical solution in form,

$$\begin{aligned} & f\left(\mathbf{x},t,\mathbf{u},\xi\right) \\ & = \frac{1}{\tau}\int_0^t g\left(\mathbf{x}',t',\mathbf{u},\xi\right)\mathrm{e}^{-(t-t')/\tau}\mathrm{d}t' \\ & + \mathrm{e}^{-t/\tau} f_0\left(\mathbf{x}-\mathbf{u}t,\mathbf{u},\xi\right) \end{aligned} \qquad (2),$$

where $\mathbf{x}' = \mathbf{x} - \mathbf{u}(t-t')$ is the particle trajectory and f_0 is the initial distribution function at the beginning of each time step. The relevant macroscopic quantities including fluxes can be obtained by taking moments of f. As f is the function of time, by integral GKS can achieve high order accuracy both in space and time. In this paper, the 2nd order scheme is used. GKS can be directly applied in the DNS of turbulence (Li et al., 2003, Kumar et al. 2013). Combined with turbulence models, GKS can simulate high-Reynolds-number turbulent flows on RANS scale (Li et al., 2010, Tan et al., 2011).

To simplify the boundary conditions to a periodic one, the moving mesh method is used with the coordinate transformation (Jin et al., 2010), $t=\tilde{t}, \mathbf{x}=\tilde{\mathbf{x}}+\mathbf{U}t$, and here $\mathbf{U}$ is the given mean flow velocity,

$$\mathbf{S}=\begin{bmatrix}1.0 & & \\ & -1.0 & \\ & & 0\end{bmatrix}S \qquad (3).$$

$$U_i(t)=\frac{\partial x_i}{\partial t}=\sum_j S_{ij}x_j(0)e^{S_{ij}t}$$

The moving mesh leads to the requirement of large acceleration, and will result in significant loss of numerical accuracy if the acceleration is not considered in the evolution of *f*. In the present study, the GKS is improved, based on BGK equation with acceleration (Tian et al., 2007),

$$f_t+\mathbf{u}\cdot\nabla_x f+\mathbf{a}\cdot\nabla_u f=\frac{g-f}{\tau} \qquad (4).$$

The microscopic characteristic relation of gas molecule for Eq.(4) is replaced by,

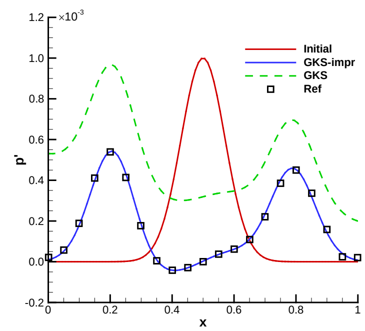
$$\mathbf{x}'=\mathbf{x}-\mathbf{u}(t-t')+\frac{1}{2}\mathbf{a}(t-t')^2 \qquad (5).$$

Thus the acceleration can be explicitly included in distribution function *f* and the fluxes at computational cell interface automatically contain the correction.

The improved GKS is tested with the propagation of pressure perturbation in gravitational field. The initial pressure is perturbed as (Tian et al., 2007),

$$p(x,0)=p_0(x)+\eta e^{\alpha(x-x_0)^2} \qquad (6).$$

η=0.001 is the amplitude of the perturbation. Figure 1 shows the results of original GKS and modified GKS (marked as GKS-impr). It can be easily observed that the improved GKS with source term effect considered in fluxes can accurately catch the influence of external force on small pressure perturbation. The accuracy of the modified scheme is further validated, and turns out to keep 2nd order.



(b)

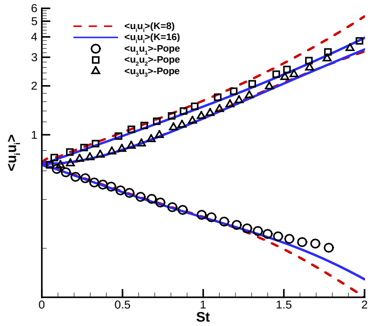
Figure 1 Propagation of the pressure perturbation in gravitational field.

During the rapidly distorting of the flow, the flow field energy keeps growing due to the acceleration. Thus the time step is reduced until the accumulative error can be neglected. In this paper, the plane strain distortion is focused on and the strain rate is set as $S=Ma_g\cdot a$, where a is the initial sound speed and keep constant in different cases. For simplicity, the flow field is initialized with homogeneous isotropic turbulence with energy spectrum as (Blaisdell et al., 1996),

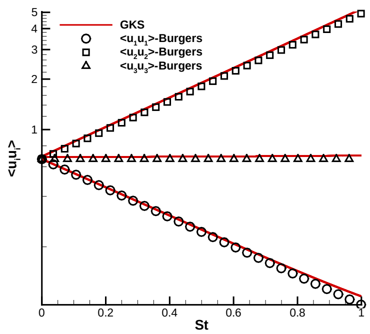
$$E(k)=Ak^4 e^{-2k^2/k_0^2} \qquad (7).$$

As the mean flow compression effect is mainly considered, the initial turbulence Mach number is set as 0.05 to reduce the turbulence compressibility. So the ‘pre-computation’ (Simone et al., 1997.) of thermal parameters in compressible flows is not necessary. The Re_λ (defined on Taylor length) of initial field is 72 and the computational domain is $2\pi^3$ divided by 128^3 grids.

Two cases are computed firstly for validation, one is near incompressible (Ma_g=0.2) and the other is with very high Mach number (Ma_g=100). Here Ma_g is the gradient Mach number, defined by mean flow strain rate, unit length and sound speed $Ma_g=S\Delta/a$ (simply called as Mach number unless otherwise specified). Figure 3 is the Reynolds stress results compared with spectrum method (Pope, 2000) and the asymptotic solution of Burger equation. Besides, the single Fourier mode in rapidly shear distorting flow (Kumar et al., 2013) has also been adopted to validate the scheme, which is not shown here.



(a)



(b)

Figure 2 The evolution of the Reynolds stress for plane strain rapid distortion. (a): Ma_g=0.2; (b): Ma_g=100.)

From Eq.(7), the prime timescale for initial pressure fluctuation is $(ak_0)^{-1}$. Two cases in Figure 2 represent solenoidal flow and pressure-release flow, respectively, which show different flow mechanisms. At long distortion time, due to the exponentially accelerated eddy stretching in the expansion direction, the lower wave number fluctuations are generated, and affect the other directions by nonlinear process. The turbulence flow fields develop to pressure-release with respect to the mean flow distortion. Thus DNS simulation will diverge from RDT results at large distortion time. To extend the solenoidal flow stage analyzed in this paper, greater wave number of fluctuation (in refined grid) can be adopted, as the comparison between $k_0 = 8$ and $k_0 = 16$ in Figure 2. Considering the computational cost, $k_0 = 8$ is still chosen, with a little shorter distortion time for analysis.

Results and Analysis

In compressible RDT, there exist three characteristic time scales, which are mean flow field time scale τ_d, dissipation time scale τ_t and compressibility time scale τ_a. According to these scales, two Mach numbers can be defined, which is distortion Mach number $M_d = \tau_a / \tau_d$ ($M_d = Ma_g / 2$ in this paper), reflecting the mean flow compression effect on turbulent fluctuations and turbulence Mach number $M_t = \tau_a / \tau_t$, the compressibility of turbulence. In this paper, the mean flow compressibility on solenoidal turbulent flows are studied, so the gradient Mach numbers are chosen from 0.2 to 1 with the same initial flow conditions.

For compressibility ‘implicit effect’, the evolutions of anisotropy with different Mach number are shown in Figure 3. With the augmentation of compressibility, the anisotropic tensors gradually develop from two-component to one-component.

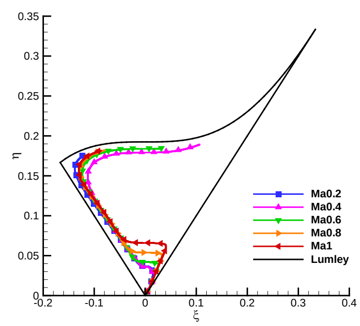


Figure 3 Flow fields anisotropy evolution with different gradient Mach number.

The differences in anisotropy evolution mainly derive from the anisotropy distortion of initial homogeneous flow fields, so that the redistribution terms growing from zero. After enough distortion time, the redistribution terms will get balance with the flow anisotropy and the pressure-deviatoric ratio R_1 (defined in following) will enter the plateau stage as in Figure 4. In the present study, this initial adjustment is simply called the redistribution relaxation. As the initial sound speed is constant and the strain rates is related with Mach number in the computations, the timescale ratio of mean flow field and turbulence varies with Mach number. When the dimensionless distortion times are transformed to the same strain rate, the redistribution relaxation process can be normalized and the plateau stage can be easily chosen with different Mach number in the following data analysis. Besides, the results show that the mean flow compressibility inhibits the redistribution growth in the redistribution relaxation stage.

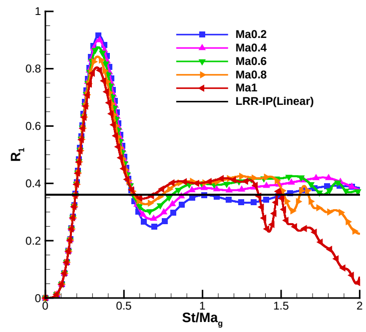


Figure 4 R_1 with different Mach number.

For further analysis, the redistribution tensor Φ_{ij} is decomposed into pressure-dilatation and pressure-deviatoric parts as (Gatski et al., 2013),

$$\bar{\rho}\Phi_{ij} = \frac{2}{3}\bar{\rho}\Phi\delta_{ij} + \bar{\rho}\Phi_{ij}^{d} \qquad (8).$$

As pressure-deviatoric part no longer contains explicit compression effect, modelling of this part is expected to be similar with incompressible case. The ratio of pressure-deviatoric tensor to production tensor is defined as,

$$R_1 = \frac{\Phi_{ij}^{d}\Phi_{ij}^{d}}{P_{ij}P_{ij}} \qquad (9).$$

In linear model (LRR, SSG etc.), the ratio is a constant 0.36. In compressible flows, variation of the ratio against St/Ma_g is as Figure 4. Deducting the section influenced by redistribution relaxation and lower wavenumber fluctuations produced by nonlinear mechanism, the averaged value of R_1 between $0.9 \le St/Ma_g \le 1.3$ for different Mach number is shown in Figure 5. An interesting increase with Mach number is found, from about 3% for Ma_g = 0.2 to 10% for Ma_g = 1.

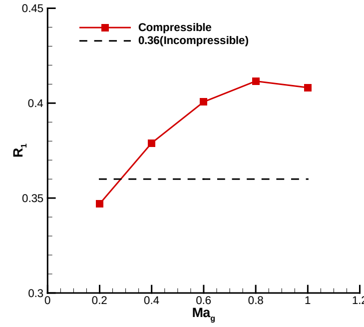


Figure 5 Averaged value of R_1 with Mach number.

Considering the different anisotropy at the beginning of plateau stage with different Mach number due to the redistribution relaxation, the deviation of R_1 from linear models may come from the nonlinear terms. This is clarified with the help of a cubic quasi-isotropic turbulence model for pressure-deviatoric term, FLT (Fu et al., 1987), which was further modified by Huang et al. (2008) to simulate plane strain flows. An a priori estimate is carried out based on the present DNS data. The difference is measured by R_1^E, which is defined as,

$$R_1^E = \frac{\left(\Phi_{ij}^{d} - \Phi_{ij}^{FLT*}\right)\left(\Phi_{ij}^{d} - \Phi_{ij}^{FLT*}\right)}{P_{ij}P_{ij}} \qquad (10).$$

The averaged value against Mach number is shown in Figure 6. The minor value of R_1^E means that compressibility 'implicit effect' can be neglected with FLT model for the present moderate Mach numbers. The high order incompressible redistribution model can be used to simulate pressure-deviatoric part directly.

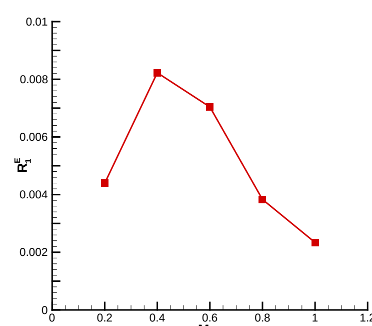


Figure 6 Averaged value of R_1^E with Mach number.

For the pressure-dilatation part called

compressibility 'explicit effect', the evolution equation of the turbulent kinetic energy is,

$$\bar{\rho}\frac{DK}{Dt} = \bar{\rho}P + \bar{\rho}\Phi \quad (11).$$

In Eq.(11), pressure-dilatation part plays as dissipation role for turbulent kinetic energy. The ratio of pressure-dilatation term to production term is defined as,

$$K_{\Phi P} = \frac{\Phi}{P}, \quad (12)$$

which is used to measure the effect of pressure-dilatation under different Mach number, as shown in Figure 7. The averaged value of $K_{\Phi P}$ in the plateau stage with Mach number is presented in Figure 8. In shear flow studies, pressure-dilatation part is associated with turbulence Mach number, which represents small scale turbulent compressibility (Sarkar, 1992). The result in Figure 8 reflects that pressure-dilatation part in strain flow shows distinct explicit compression effect related with mean flow compressibility, which is approximately linear with mean flow Mach number.

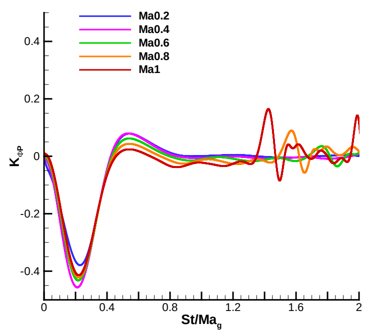


Figure 7 $K_{\Phi P}$ with different Mach number.

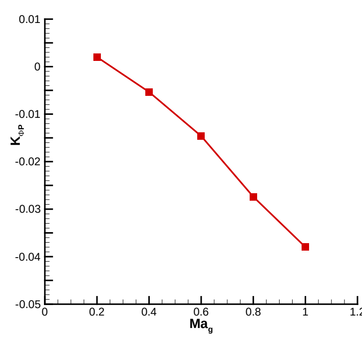


Figure 8 Averaged value of $K_{\Phi P}$ against Mach number.

Based on the near incompressible plane strain distortion case (Ma_g=0.2), three typical redistribution models are estimated: the linear LRR-IP, SSG model and modified cubic quasi-isotropic FLT model. Figure 9 is the result of Reynolds-stress invariants from different models. Recalling Figure 5 and Figure 6, despite that linear model can well predict the contraction of redistribution tensor in near incompressible cases, the component prediction need high order model. In compressible cases, the higher order terms get dominant in the model, and cannot be neglected even in the contraction of redistribution tensor. So the compressible model must be developed upon high order model.

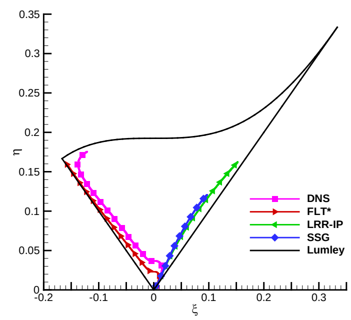


Figure 9 Reynolds-stress invariants of turbulence model and DNS.

The preliminary studies on high Mach number rapidly distorting turbulence have also been conducted. The results show great differences with current moderate Mach number results. The mean flows compressibility has stronger effects on turbulence. Further studies will be carried out.

Conclusions

Based on DNS of rapid plane strain distortion flows under different Mach numbers, this paper mainly focuses on the compressibility effect on the redistribution term. In strain dominated flows, high order turbulence model is required for pressure-deviatoric term, and suppression effects of the initial redistribution relaxation process cannot be ignored. The correction of pressure-dilatation part should be related with mean flow Mach number.

Acknowledgement

This work is supported by the National Natural Science Foundation of China (Project No.11172154, 10932005) and National Key Basic Research and Development Program (2014CB744100).

References

Blaisdell, G. A., Coleman, G. N., & Mansour, N. N. (1996). Rapid distortion theory for compressible homogeneous turbulence under isotropic mean strain. *Physics of Fluids (1994-present)*, 8(10), 2692-2705.

Cambon, C., Coleman, G. N., & Mansour, N. N. (1993). Rapid distortion analysis and direct simulation of compressible homogeneous turbulence at finite Mach number. *Journal of Fluid Mechanics*, 257, 641-665.

Fu, S., Launder, B. E., & Tselepidakis, D. P. (1987). Accommodating the effects of high strain rates in modelling the pressure-strain correlation. *Mechanical Engineering Department Report TFD/87/5, [Ph D Thesis]*, University of Manchester Institute of Science and Technology.

Gatski, T. B., & Bonnet, J.-P.. Compressibility, turbulence and high speed flow. *Academic Press*, 2013.

Huang, S., & Fu, S. (2008). Research of the rapid pressure-strain correlation model in the rapid distortion limit. *Science in China Series G: Physics, Mechanics and Astronomy*, 51(9), 1380-1389.

Jin, C., Xu, K., & Chen, S. (2010). A three dimensional gas-kinetic scheme with moving mesh for low-speed viscous flow computations. *Adv. Appl. Math. Mech*, 2(6), 746-762.

Kumar, G., Girimaji, S. S., & Kerimo, J. (2013). WENO-enhanced gas-kinetic scheme for direct simulations of compressible transition and turbulence. *Journal of Computational Physics*, 234, 499-523.

Lee, M. J., Kim, J., & Moin, P. (1990). Structure of turbulence at high shear rate. *Journal of Fluid Mechanics*, 216, 561-583.

Li, Q. B., & Fu, S. (2006). On the multidimensional gas-kinetic BGK scheme. *Journal of Computational Physics*, 220(1), 532-548.

Li, Q. B., & Fu, S. (2003). Numerical simulation of high-speed planar mixing layer. *Computers & Fluids*, 32(10), 1357-1377.

Li, Q.B., Tan, S., Fu S. & Xu, K. (2010). Numerical simulation of compressible turbulence with gas-kinetic BGK scheme, *13th Asian Congress of Fluid Mechanics*, Dec. 17-22, 2010, Dhaka, Bangladesh.

Pope, S. B. Turbulent flows. *Cambridge university press*, 2000.

Sarkar, S., Erlebacher, G., & Hussaini, M. Y. (1991). Direct simulation of compressible turbulence in a shear flow. *Theoretical and Computational Fluid Dynamics*, 2(5-6), 291-305.

Sarkar, S. (1992). The pressure–dilatation correlation in compressible flows. *Physics of Fluids A: Fluid Dynamics (1989-1993)*, 4(12), 2674-2682.

Simone, A., Coleman, G. N., & Cambon, C. (1997). The effect of compressibility on turbulent shear flow: a rapid-distortion-theory and direct-numerical-simulation study. *Journal of Fluid Mechanics*, 330, 307-338.

Tan, S., Li, Q.B., Fu, S., Zeng, S., “Engineering Simulation of Turbulence with Gas-Kinetic BGK Scheme,” *Proceeding of the Sixth International Conference on Fluid Mechanics*, Vol. 1376, No. 1, 2011, pp. 78-80.Tian, C. T., Xu, K., Chan, K. L., & Deng, L. C. (2007). A three-dimensional multidimensional gas-kinetic scheme for the Navier–Stokes equations under gravitational fields. *Journal of Computational Physics*, 226(2), 2003-2027.

Xu, K. (2001). A gas-kinetic BGK scheme for the Navier–Stokes equations and its connection with artificial dissipation and Godunov method. *Journal of Computational Physics*, 171(1), 289-335.