ON THE EXISTENCE OF NON CLASSICAL DYNAMIC REGIMES IN FRACTAL/MULTISCALE GRID TURBULENCE DECAY

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INTRODUCTION

Grid turbulence decay is among the main research topics in turbulence theory. This topic has received a large attention since the beginning of the 20th century. Most of the existing works deal with the analysis of self-similar and/or self-preserving solutions, which exhibits an algebraic decay law for turbulent kinetic energy. The identification of the parameters that govern the decay exponent of decay energy is still an active field of research.

More recently, grid turbulence decay has been renewed by considering new grid topologies, namely fractal/multiscale grids, which have been reported by some authors to lead to decay regimes that escape classical theories for isotropic turbulence decay, e.g. Hurst & Vassilicos (2007); Valente & Vassilicos (2011). These possibly new decay regimes still raise some controversies. While some authors claim to observe the exponential decay law predicted by George's theory, other authors report unusually fast algebraic decay regime. Some groups also do not observe unusual decay, only classical decay regimes. Some researchers have also hypothesized that unusual decay law originate in a breakdown of isotropy and homogeneity, and should be considered as artifacts.

The present study aims at providing new elements related to the possible occurrence of unusual decay regime due to fractal/multiscale production mechanisms. To this end, a spectral model based on EDQNM-closed Lin equation is developed, in which turbulence production in the fractal grid wake is mimicked by an isotropic forcing term. The time evolution of the forcing term is derived from self-similar wake elements, and it accounts in a Lagragian way for the downstream evolution of the grid wake. In this way, pure turbulence production effects are isolated from possible anisotropic or inhomogeneous effects, leading to an deep insight into flow physics. All details can be found in Meldi *et al.* (2014).

PHYSICAL MODEL

The present physical model is an extended Lin equation for time evolution of the 3D energy spectrum $E(k)$ with a forcing term that accounts for turbulent kinetic energy production due to the fractal wake induced shear downstream the grid :

$$\frac{\partial E(k,t)}{\partial t} + 2\nu k^2 E(k,t) = T(k,t) + \langle \mathscr{F}(k,t) \rangle \qquad (1)$$

The forcing term is based on the assumption that the fractal kinetic energy production is isotropic. Therefore, the present study enables for a clear separation between multiscale/fractal production effects and the influence of anisotropy/inhomogeneity.

The model introduced for spectral HIT DNS proposed by Mazzi & Vassilicos (2004) is used as a starting point. This empirical model represents the effects of fractal grids in HIT grid experiments and, as a consequence, can be defined as a fractal forcing:

$$\mathbf{P} \cdot \hat{\mathbf{F}}(\mathbf{k},t) = (\mathbf{k} L_\gamma)^\beta \, a_k \, f \, \mathbf{e}(\mathbf{k},t) \qquad (2)$$

where $\mathbf{P}$ is the projector on the plane normal to $\mathbf{k}$, $\hat{\mathbf{F}}(\mathbf{k},t)$ is the Fourier transform of the physical forcing term, L_γ is the largest scale forced, f is a scalar function and $\mathbf{e}(\mathbf{k},t)$ is a unit vector. The coefficient $a_k \in I = [0;1]$ is 1 for the forced modes and 0 for the other modes. The fractal dimension of the grid is related to the grid geometry. The parameter β represents a scaling exponent which is related to the pattern of the multi-scale grid. Typical values considered in Mazzi & Vassilicos (2004) are $\beta = 0.1 - 0.7$.

The resulting energy input rate spectrum using the model in equation 2 is:

$$\mathscr{F}(k) = \frac{1}{2} \sum_{S(k)} [(\mathbf{P} \cdot \hat{\mathbf{F}}) \cdot \hat{\mathbf{u}}^* + (\mathbf{P} \cdot \hat{\mathbf{F}})^* \cdot \hat{\mathbf{u}}] \qquad (3)$$

where the sum is over shells of radius k in the wavevector space and $*$ indicates the complex conjugate. Following Mazzi & Vassilicos (2004) work, the expression of $\mathscr{F}(k)$ can be manipulated observing that $\mathbf{e}(\mathbf{k},t)$ can be defined as:

$$\mathbf{e}(\mathbf{k},t) = \gamma_R \frac{\hat{\mathbf{u}}(\mathbf{k},t)}{|\hat{\mathbf{u}}(\mathbf{k},t)|} + \gamma_I \frac{\mathbf{k} \wedge \hat{\mathbf{u}}(\mathbf{k},t)}{|\mathbf{k} \| \hat{\mathbf{u}}(\mathbf{k},t)|} \qquad (4)$$

where γ_R and γ_I are a purely real coefficient and a purely imaginary coefficient, respectively. Equation 4 allows to express the forcing term $\mathbf{P} \cdot \hat{\mathbf{F}}(\mathbf{k},t)$ as a function of the velocity field only. Choosing $f\gamma_I = i f \gamma_R$ the model reads as:

$$\mathbf{P} \cdot \hat{\mathbf{F}}(\mathbf{k},t) = (k L_b)^\beta \, a_k \, f \, \gamma_R \left[\frac{\hat{\mathbf{u}}(\mathbf{k},t)}{|\hat{\mathbf{u}}(\mathbf{k},t)|} + i \frac{\mathbf{k} \wedge \hat{\mathbf{u}}(\mathbf{k},t)}{|\mathbf{k} \| \hat{\mathbf{u}}(\mathbf{k},t)|} \right] \qquad (5)$$

Equation 5 and the condition $\mathbf{e}(\mathbf{k},t) \parallel \hat{\mathbf{u}}(\mathbf{k},t)$ can be used to simplify the expression for the energy input rate spectrum $\mathscr{F}(k)$. Considering that in isotropic turbulence $\hat{\mathbf{u}}^*(\mathbf{k},t) = \hat{\mathbf{u}}(-\mathbf{k},t)$:

$$\mathscr{F}(k) = f\,\gamma_R\,(kL_b)^{\beta}\,a_k \sum_{S(k)} \frac{\hat{\mathbf{u}}(\mathbf{k},t)\cdot\hat{\mathbf{u}}(\mathbf{k},t)}{|\hat{\mathbf{u}}(\mathbf{k},t)|} \tag{6}$$

In the following, we consider the ensemble average of the energy input rate spectrum in Equation 6:

$$\langle \mathscr{F}(k) \rangle = f\,\gamma_R\,(kL_b)^{\beta}\,a_k \sum_{S(k)} \frac{\langle \hat{\mathbf{u}}(\mathbf{k},t)\cdot\hat{\mathbf{u}}(\mathbf{k},t)\rangle}{\sqrt{\langle \hat{\mathbf{u}}(\mathbf{k},t)\cdot\hat{\mathbf{u}}(\mathbf{k},t)\rangle}} \tag{7}$$

this term can be implemented in the Lin equation 1, in order to study the effects of a fractal forcing over the time evolution of the turbulence statistical quantities. The term $\langle \hat{\mathbf{u}}(\mathbf{k},t)\cdot\hat{\mathbf{u}}(\mathbf{k},t)\rangle$ can be derived by the theory of isotropic turbulence:

$$\langle \hat{u}_i(\mathbf{k}',t)\cdot\hat{u}_j(\mathbf{k},t')\rangle = \hat{U}_{ij}(\mathbf{k},t,t')\,\delta(\mathbf{k}+\mathbf{k}') \tag{8}$$

where $\hat{U}_{ij}$ is the spectral tensor. This tensor is hermitian and, in the case $t = t'$:

$$\hat{U}_{ii}(\mathbf{k},t,t) = \hat{U}(k,t) = \frac{E(k,t)}{2\pi k^2} \tag{9}$$

$\hat{U}(k,t)$ is the trace (real and positive) of the spectral tensor. Equation 9 allows for deriving an explicit expression of $\langle \mathscr{F}(k) \rangle$ as a function of the energy spectrum:

$$\langle \mathscr{F}(k) \rangle = 2\,f\,\gamma_R\,(kL_b)^{\beta}\,a_k\,k\,\sqrt{2\,\pi\,E(k,t)} \tag{10}$$

The model constant $2\,f\,\gamma_R$ is set so that the energy input rate associated to the fractal forcing is balanced by the energy dissipation rate $\varepsilon(t)$.

The model in Equation 10 allows for the study of statistically steady forced isotropic turbulence. Therefore, it must be modified to describe turbulence decay, which is associated to a decay of the turbulence production due to the spreading of the wakes and the induced decrease of their shear. This is done interpreting the present spectral description in a Lagrangian way, using a self-similar wake model to describe each bar wake and neglecting their interactions. The latter point is not a fundamental problem here, since homogeneity and isotropy are enforced thanks to the use of Lin's equation. The model is designed to recover a physically meaningful description of the decay that occurs at a distance larger than the interaction length downstream of the grid. The interaction length is the distance needed for all the wakes to have interacted in wind tunnel experiments, which is reminiscent of the classical formation region in regular grid experiments. In the present case, it is assumed that, downstream of the interaction length, the velocity field is influenced by all wakes at all locations in the wind tunnel.

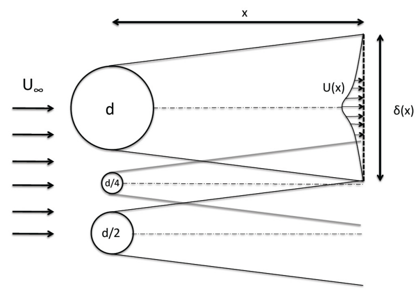

Figure 1: Simplified scheme representing the generation of multi-scale turbulence, in HIT grid experiments.

The present schematic physical model for the multi-scale/fractal grid wake is displayed in Figure 1.

Self-similarity assumption and dimensional analysis for planar wake analysis yield :

$$\delta \propto (t-\tau)^{1/2}, \qquad U \propto (t-\tau)^{-1/2}, \qquad \overline{uv} \propto (t-\tau)^{-1} \tag{11}$$

where U, δ and τ are the wake velocity deficit amplitude, the wake thickness and the time virtual origin, respectively.

The decay law of the model should be consistent with the Taylor frozen turbulence hypothesis.This hypothesis must be considered as an approximation, as it is not completely fulfilled in grid turbulence experiments. Nevertheless, it allows for a direct comparison between the EDQNM time evolution and the experiment's spatial evolution. This is done connecting the virtual time origin τ of the present model with the wake interaction length scale x introduced in a number of experimental works by the Imperial college group (Mazellier & Vassilicos, 2010; Valente & Vassilicos, 2011; Gomes-Fernandes *et al.*, 2012). Clearly, EDQNM results for times smaller than τ cannot be compared with experimental results, because of the strong anisotropy in the near grid flow field region. The grid turbulence physical dynamics through which the fractal isotropic turbulence is reached at x are completely different from the numerical steady model used to attain the EDQNM fractal converged initial state for $t < \tau$.

The turbulence production associated to a single wake is evaluated as

$$TP = \frac{dU}{dy}\,\overline{uv} \propto \frac{U}{\delta}\overline{uv} \propto (t-\tau)^{-2} \tag{12}$$

Here, the characteristic time unit t_G for t is d/U_∞, with d the bar diameter and U_∞ the upstream velocity. Therefore, at mode k, the decay law for turbulence production can be written as

$$TP(k,t) \propto (1+\alpha\,kL_\gamma(t-\tau))^{-2} \tag{13}$$

where L_γ denotes the largest scale at which forcing occurs, which is approximated as the diameter of the largest bar in the grid. The parameter α is the ratio between the characteristic grid time t_G and the initial turbulent turnover time t_0:

$$\alpha = \frac{t_G}{t_0} = \frac{d}{U_\infty}\,\frac{\varepsilon(0)}{\mathscr{K}(0)} = \frac{d}{L(0)}\,\frac{\sqrt{\mathscr{K}(0)}}{U_\infty} \qquad (14)$$

The parameter α can be thought of as the ratio between two characteristic lengths and two characteristic velocities. The ratio $\sqrt{\mathscr{K}(0)}/U_\infty$ is related to the wind tunnel features and can be easily measured and quantified. On the other hand, the ratio $d/L(0)$ governs the relationship between the turbulent flow evolution and the perturbation due to the multi-scale grid. If $d/L(0) \ll 1$, the initial turnover time will be very large with respect to the grid time, so that the turbulent flow dynamics will be almost static with respect to the multi-scale grid effects. Conversely, for progressively larger values of $d/L(0)$, the two dynamic effects will have similar evolution times. In the limit of $d/L(0) \to +\infty$, the forcing term effects will decay fast after the time τ, and a HIT free decay regime from an initial multi-scale grid spectrum will be triggered.

The resulting multi-scale/fractal forcing term, which is obtained combining Eqs. 10 and 13, is:

$$F(k,t) = \begin{cases} 2\,(kL_b)^\beta\, a_k\, k\sqrt{2\pi E(k,t)}, & t \leq \tau \\ 2\,(kL_b)^\beta\, a_k\, k\sqrt{2\pi E(k,t)}\,(1+\alpha k L_\gamma (t-\tau))^{-2}, & t \geq \tau \end{cases} \qquad (15)$$

where L_γ is the largest scale forced. The coefficient $a_k \in I = [0;1]$ is 1 for the forced modes and 0 for the other modes. The fractal dimension of the grid is related to the grid geometry. The parameter β represents a scaling exponent which is related to the pattern of the multi-scale grid. Typical values considered in Mazzi & Vassilicos (2004) are $\beta = 0.1 - 0.7$. The parameter $\alpha = \frac{d}{U_\infty}\frac{\varepsilon(0)}{\mathscr{K}(0)} = \frac{d}{L(0)}\frac{\sqrt{\mathscr{K}(0)}}{U_\infty}$ (with d the bar diameter and U_∞ the upstream uniform velocity) measures the ratio of the timescale largest grid bar d/U_∞ to the turbulent time scale $\mathscr{K}(0)/\varepsilon(0)$. It is related to the grid topology. The exponent -2 in the decay law for turbulent production directly stems from the self-similar planar wake theory, which is assumed to be relevant to describe rod wake dynamics downstream the fractal grid. The steady forcing for $t < \tau$ is used to obtain an initial solution which inherits some features from turbulence generation in the formation region in wind tunnels. Therefore, the time τ, which is interpreted as the virtual time origin for the decay, is related to the wake interaction length as defined in wind tunnels, e.g. Mazellier & Vassilicos (2010); Valente & Vassilicos (2011).

RESULTS AND MAIN CONCLUSIONS

A systematic study of the influence of all parameters has been conducted, along with a comparison with available experimental data. A good agreement is observed with the later (see figure 2). Typical results are illustrated in figure 3.

The main conclusions are (i) the key parameter that governs the nature of turbulence decay is α and (ii) several decay regimes can be observed. The 3 regimes are: Present results display some anomalous decay regimes over finite times before recovering algebraic decay laws, during which the classical behaviour predicted by the Comte-Bellot – Corrsin theory does not hold. Similar observations were reported in many experimental investigations. But an open issue is to know if these anomalous decay regimes are associated either with very fast algebraic decay laws or exponential decay laws. To investigate this problem, the computed time evolution of kinetic energy is approximated at every time step by the power-law relation $\mathscr{K}(t) \propto t^{n_\mathscr{K}}$ where the time evolution of the power-law exponent $n_\mathscr{K}(t)$ is now a function of time. A careful examination of present results yields the identification of three regimes, governed by the value of α:

- *Rapidly decaying production term ($\alpha > 10^{-2}$).* For such cases, decay exponents $n_\mathscr{K}(t) < -2$ are observed during a short transient period, which is consistent with Valente & Vassilicos (2011) and Hearst & Lavoie (2014), who reported $n_\mathscr{K} \sim -2,5$ and $n_\mathscr{K} \sim -2,79$, respectively. It is worth reminding that the lower bound compatible with the Comte-Bellot – Corrsin analysis is $n_\mathscr{K} = -10/7$. In this regime, the production term vanishes very quickly and has no significant influence on the turbulence decay. Additional runs were carried out with the same initial condition (i.e. applying the steady forcing term for $t < \tau$) but completely removing the forcing term for $t \geq \tau$, in which the same transient period with anomalous decay was observed, showing that these anomalous fast decay regimes may be governed by the unusual shape of the energy spectrum that can be produced by the fractal forcing. During these fast decay regimes, the expected features of the exponential decay are not observed, i.e. $L/\lambda \neq const$ and $C_\varepsilon \neq Re_\lambda^{-1}$.
- *Very slowly decaying production term ($\alpha < 10^{-3}$).* Here, the decay of turbulence appears to be driven by the decay of the production term, and an exponential decay in good agreement with George's prediction is observed. But it is worth noting that the local approximation of such decay regimes via local algebraic laws leads to the definition of very slow decay rate over finite time, with $0 < n_\mathscr{K} \leq -1$.
- *Intermediary decay regimes*, in which no clear trend can be identified.

The global picture can be further complexified accounting for the intensity of the production effect. Here, finite Reynolds number effects are excluded. For very high and very low α values a governing dynamic effect can be isolated. In this case, $C_\varepsilon = const$. On the other hand, for intermediate values of α the turbulence production effects are not sufficiently strong to completely drive the turbulence dynamics, but yet they are not small enough to be overwhelmed by dissipation. This results in a time evolution of C_ε, which appears to be α dependent and not far from the $C_\varepsilon \propto Re_\lambda^{-1}$ observed by Valente & Vassilicos (2011). This is a clear confirmation that the states $C_\varepsilon = const$ and $C_\varepsilon \propto Re_\lambda^{-1}$ should not be automatically associated with classical grid turbulence and fractal grid turbulence, respectively. Summarising, these modified decay laws are related to production effects related to the geometry of the grid. The characteristic evolution times of these effects are as well governed by the grid topology and they can actually be significantly longer than most physically observable turbulent flow evolution times. For infinite times, a classical turbulence decay is expected.

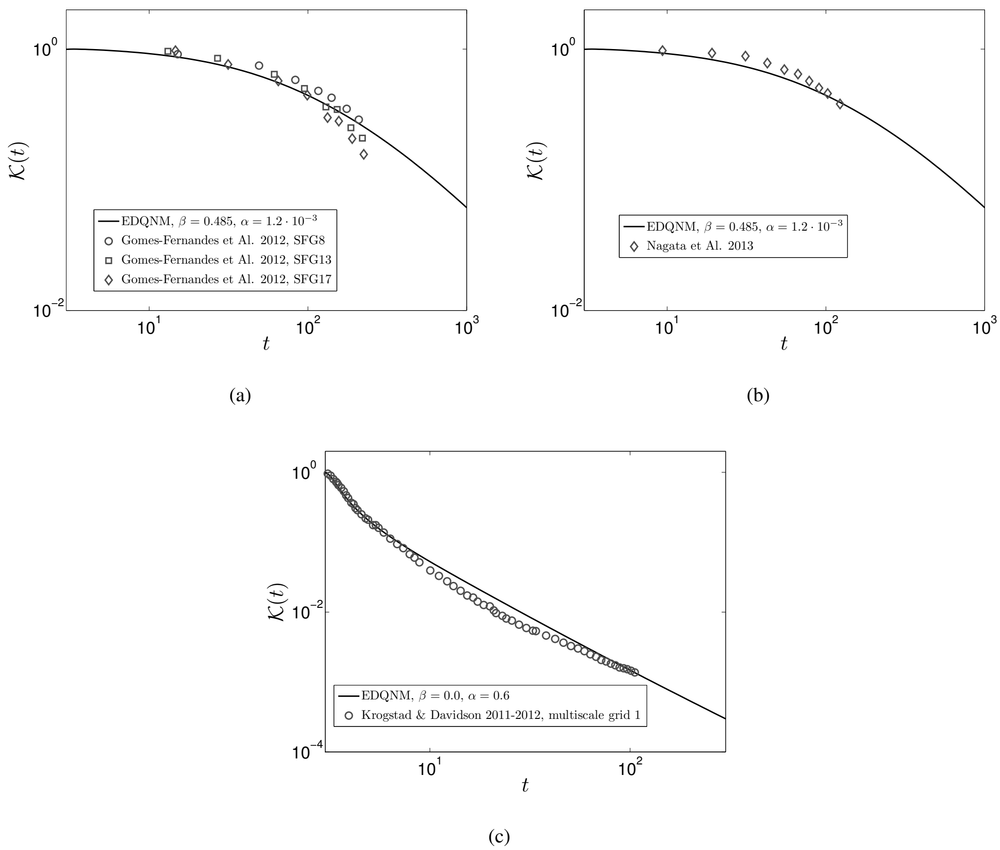

Figure 2: Validation of EDQNM results by comparison with different experimental data sets.

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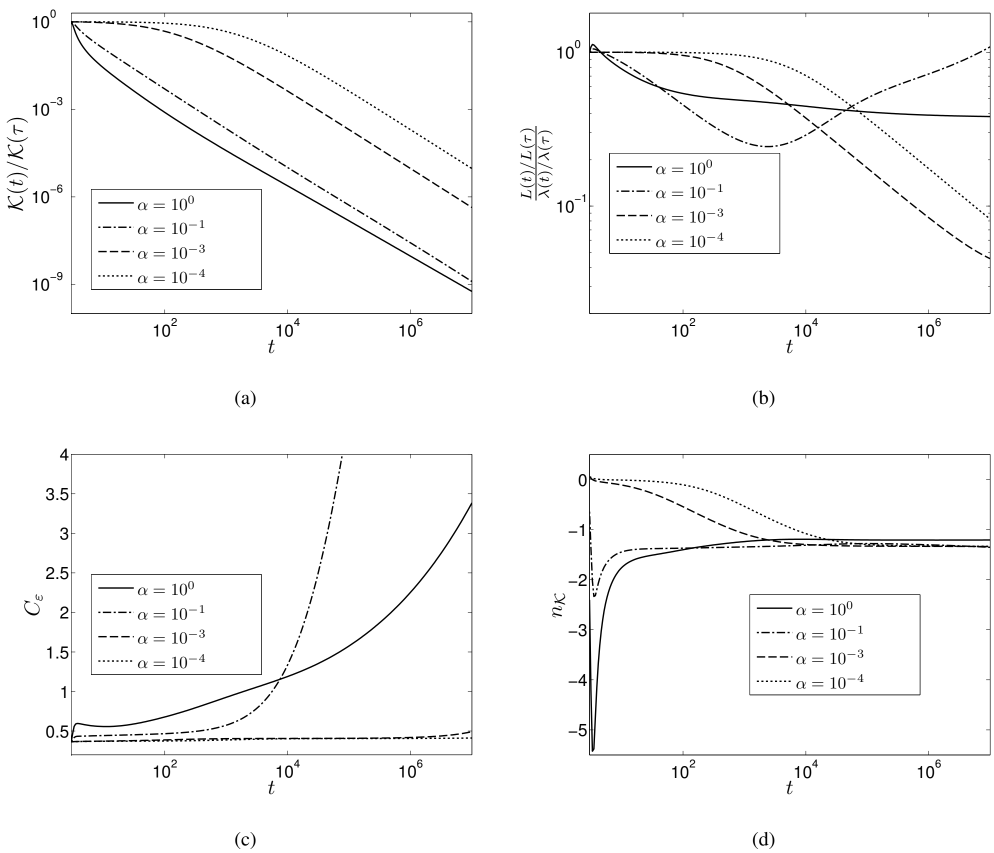

Figure 3: Analysis of non-classical decay regimes looking at time evolution of (a) turbulent kinetic energy, (b) turbulent length scale ratio L/λ, (c) dissipation parameter $C_\varepsilon = \varepsilon L/\sqrt{\mathcal{K}}$ and (d) instantaneous turbulence kinetic energy decay exponent (assuming that the decay can be approximated as an algebraic function at all times).