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A FAMILY OF HIGH ORDER TARGETED ENO SCHEME FOR COMPRESSIBLE FLUID SIMULATIONS

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ABSTRACT

Though classical WENO schemes achieve great success and are widely accepted, they show several shortcomings, such as too dissipative for reproducing turbulence and lack of numerical robustness with very high-order, in practical complex simulations. In this paper, we propose a family of high-order targeted ENO schemes for highly compressible fluid simulations involving wide range of sales. In order to increase the robustness of the very high-order classical WENO schemes, the reconstruction is built up by dynamically assembling a set of low-order upwind biased candidate stencils with incrementally increasing widths. Strong scale-separation formulations are proposed to extract the discontinuities from high wave-number physical fluctuations effectively. Coupled with a sharp cut-off technique, one candidate stencil is abandoned only on the condition that it is crossed by strong discontinuities, otherwise always applied with optimal standard weight. The background linear scheme is optimized with the constrain of preserving the approximate dispersion-dissipation condition. While such optimization leads to one-order degeneration, it provides favorable spectral property for intermediate and high wave-number region. By means of quasi-linear analyses and practical numerical experiments, a set of generally case-independent parameters are determined. The formulations of arbitrary high-order schemes are presented in a straightforward way. Five-point and six-point stencil schemes are further designed and analyzed in details. A variety of benchmark-test problems, including broadband waves, strong shock and contact discontinuities are studied. Compared to well established classical WENO schemes, present schemes suggest much improved property of robustness, low numerical dissipation in transition region and sharp discontinuity-capturing capacity, therefore are promising for DNS and LES of shock-turbulence interactions.

1 Introduction

We propose a family of high-order targeted ENO (TENO) scheme, preserving low dissipation for low to intermediate wave-number range, sharp shock-capturing capacity and good robustness with one set of predefined parameter. Several new ideas are introduced to resolve the aforementioned problems suffered by classical WENO schemes: (i) Different from the baseline WENO idea, a set of low-order (higher than third-order) approximate candidate polynomials with incremental stencil widths are dynamically assembled to build up the high-order reconstruction. Each candidate stencil has at least one-point upwind, eliminating the possible unstableness induced by the completely downwind stencil. In this way, the scheme could gradually degenerate to third order according to the local flow features, thus avoid the multiple discontinuities and recover the robustness of classical fifth-order WENO scheme; (ii) Inspired by the work of Hu and Admas Hu & Adams (2011), a much stronger scale-separation formulation is designed to segregate the discontinuities from the high wave-number physical fluctuations; (iii) Unlike WENO's idea, in which the weighting strategy prefers to assign a relatively larger weight to the smoother stencil for the sake of numerical stability, the certain candidate stencil is abandoned only when containing a genuine discontinuity with predefined strength, otherwise it is always applied with standard weight. With ADR analyses and practical numerical simulations, the predefined strength is determined by a sharp cut-off procedure. This technique depresses the dissipation to the limit of the background linear scheme for wave-number region of interest while preserving the robustness of shock capturing. For specific problems, e.g. turbulence simulations, this predefined strength could be further optimized to tolerate more unstable physical disturbances; (iv) As analysed above, the background linear scheme is one main source of numerical dissipation in transition region. The optimization algorithm mentioned by Hu and Admas Hu *et al.* (In press) is adopted, but with a relax constrain of preserving the dispersion and dissipation condition. A K-th order scheme can be optimized to satisfy this constrain by adjusting the combination of two $(K-1)$-th order stencils, leading to a $(K-1)$-th order scheme with favourable spectral property. It is straightforward and fairly simple to construct arbitrary high-order TENO scheme. As an canonical example, we further focus on the fifth-order and six-order TENO schemes and analyse their performances in broadband-test problems. The numerical results demonstrate that new proposed schemes are robust enough with one set of generic parameters and perform better than several well established WENO schemes for all the test problems.

2 Numerical results

2.1 Double Mach reflection of a strong shock

The two-dimension case is taken from Woodward and Colella Woodward (1984) and has been intensively adopted in literatures for the validation of high-resolution shock capturing schemes as it contains both the strong shock and complex solution features. The initial condition is as follows:

$$(\rho,u,v,p)=\begin{cases}(1.4,0,0,1) & y<1.732(x-0.1667)\\ (8,7.145,-4.125,116.8333) & otherwise\end{cases} \tag{1}$$

The computational domain is $[0,0]\times[4,1]$ and the simulation end time is $t=0.2$. Initially, a right-moving Mach 10 shock in air is positioned at x = 0.1667 with a incident angle of 60^0 to the x-axis. The post-shock condition is imposed from x = 0 to x = 0.1667 whereas a reflecting wall condition is enforced from x = 0.1667 to x = 4 at the bottom. For the top boundary condition, the fluid variables are set to exactly describe the motion of Mach 10 shock wave. General inflow and outflow condition is employed for the left and right side of the computational domain. Multi-resolution based adaptive mesh refinement method is adopt with Lmax = 4 indicating an effective resolution of 1024×256 (h = 1/256). Only domain $[2,3]\times[0,0.9]$ is presented for the facilitation of comparison. Apart from WENO-CU6-M, other schemes overcome the strong shock whereas manufacturing results which have significant difference in the "blown-up" portion around the double Mach stems. As described in Fig. 1, the TENO5-opt and TENO5 resolve the finest small structure and the strongest wall jet among all the five-point stencil schemes, and even better than fifth-order WENO-JS scheme at resolution h = 1/960 (see literature Shi *et al.* (2003)). When considering the TENO6, TENO6-opt and WENO-CU6, flagrant enhancement in resolving the fluctuation structures can be observed in Fig. 2. TENO6 reproduces richest small structures, more than ninth-order WENO-JS scheme at resolution h = 1/960 (see literature Shi *et al.* (2003)). TENO6-opt resolves a little less structures than TENO6 due to the dissipation added in the optimization procedure, however, the result is still flourishing superior than WENO-CU6. This demonstrates that the proposed targeted ENO schemes feature strong scale separation capability and excellent fluctuation resolving property.

2.2 Rayleigh-Taylor instability

As another canonical problem, which contains both discontinuities and complex flow structures, the invicid Rayleigh-Taylor instability case proposed by Xu and Shu Xu & Shu (2005) is considered. The initial condition is

$$(\rho,u,v,p)=\begin{cases}(2,0,-0.025c\cos(8\pi x),1+2y), & 0\le y<1/2,\\ (1,0,-0.025c\cos(8\pi x),y+3/2), & 1/2\le y\le 1,\end{cases} \tag{2}$$

where the sound speed is $c=\sqrt{\gamma\frac{p}{\rho}}$ with $\gamma=\frac{5}{3}$. The computational domain is $[0,0.25]\times[0,1]$. Reflective boundary conditions are imposed at the left and right side of the domain. Constant primitive variables $(\rho,u,v,p)=(2,0,0,1)$ and $(\rho,u,v,p)=(1,0,0,2.5)$ are set for the bottom and top boundaries, respectively. Initially, the interface located at y = 0.5 separates the heavy and light fluid, forming a contact discontinuity. For inviscid simulation solving Euler equations, the smaller magnitude of numerical dissipation from high-order shock-capturing scheme results in richer fine structures.

Fig. 3 shows the solutions from all 8 schemes at a resolution 128×512, i.e. grid space $\Delta x=\Delta y=1/512$. TENO5-opt and TENO5 have resolved much richer vortical structures than WENO-JS and WENO-Z. Furthermore, the low-dissipation property of TENO5-opt and TENO5 allows for breaking the symmetry of flow field which is not the nature of Rayleigh-Taylor instability. A more dissipative scheme is more likely to preserve this symmetry even when increasing the resolution significantly, see Fig.2 in Shi *et al.* (2003). For six-point stencil schemes, further improved resolution is obtained especially with TENO6, TENO6-opt and WENO-CU6-M. When the mesh resolution is doubled in both coordinate directions, the solution from WENO-CU6 also loses symmetry in Fig. 4. Another noticeable outcome is that, small structures resolved from TENO5-opt and TENO5 with resolution of 128×512 are comparable to that from WENO-JS and WENO-Z with resolution of 256×1024.

2.3 Inviscid simulation of the three-dimensional Taylor-Green vortex

The three-dimensional Taylor-Green vortex is a basic prototype for studying the transition to turbulence. Extensive studies with DNS and LES can be found in Brachet *et al.* (1983)Shu *et al.* (2005)Hickel *et al.* (2006). The initial condition with well-resolved single-mode velocity field is

$$\begin{aligned}
&u(x,y,z,0)=\sin(x)\cos(y)\cos(z),\\
&v(x,y,z,0)=-\cos(x)\sin(y)\cos(z),\\
&w(x,y,z,0)=0,\\
&\rho(x,y,z,0)=1.0,\\
&p(x,y,z,0)=100+\tfrac{1}{16}[(\cos(2x)+\cos(2y))(2+\cos(2z))-2],
\end{aligned} \tag{3}$$

where the pressure is defined to limit the Mach number less than 0.1 so that the nearly incompressible property is reproduced throughout the simulation. The computational domain is $[0,2\pi]\times[0,2\pi]\times[0,2\pi]$ with periodic boundary conditions at the six domain boundaries. The simulation is run until $t=10$. Since no physical dissipation is considered, the total kinetic energy should be conserved as the initial value π^3 throughout the entire simulation. On the other hand, the enstrophy which measures the growth of vorticity, can quantify the resolution capability on predicting the

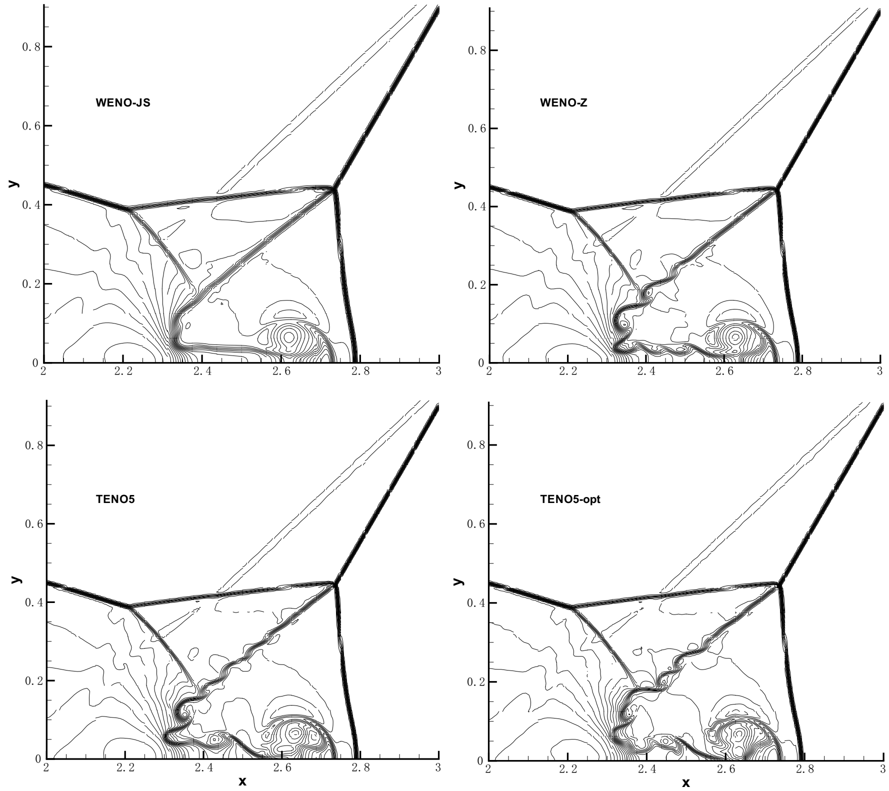

Figure 1. Double Mach reflection a strong shock: density solution from five-point stencil schemes. Resolution at h = 1/256. The Figure is drawn with 43 density contours from 1.887 to 20.9.

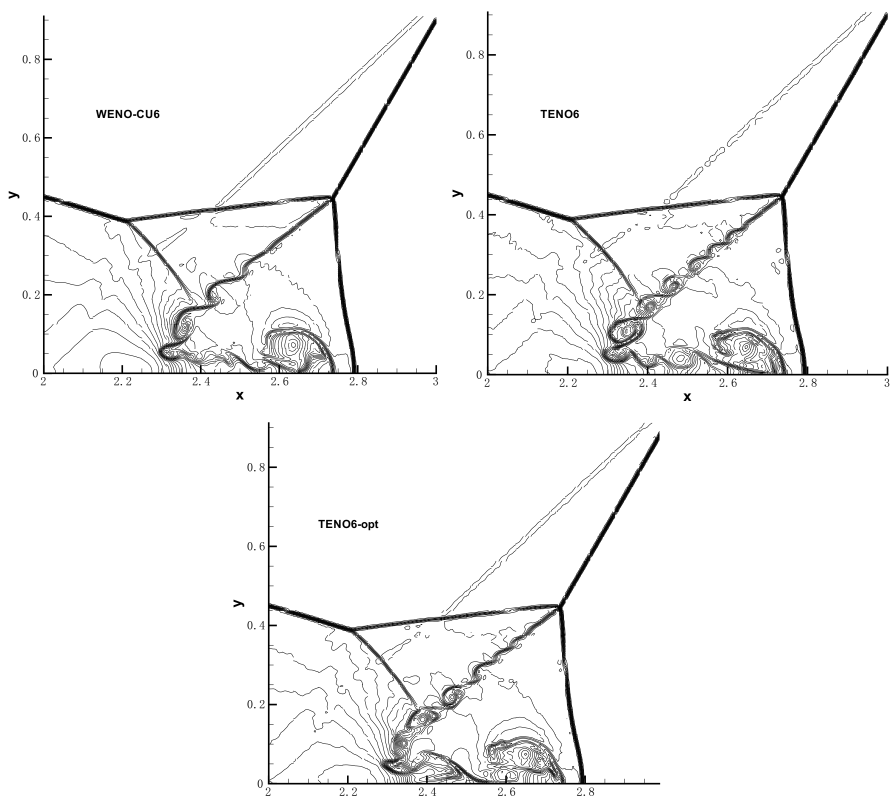

Figure 2. Double Mach reflection a strong shock: density solution from six-point stencil schemes. Resolution at h = 1/256. The Figure is drawn with 43 density contours from 1.887 to 20.9. WENO-CU6-M fails to pass current case.

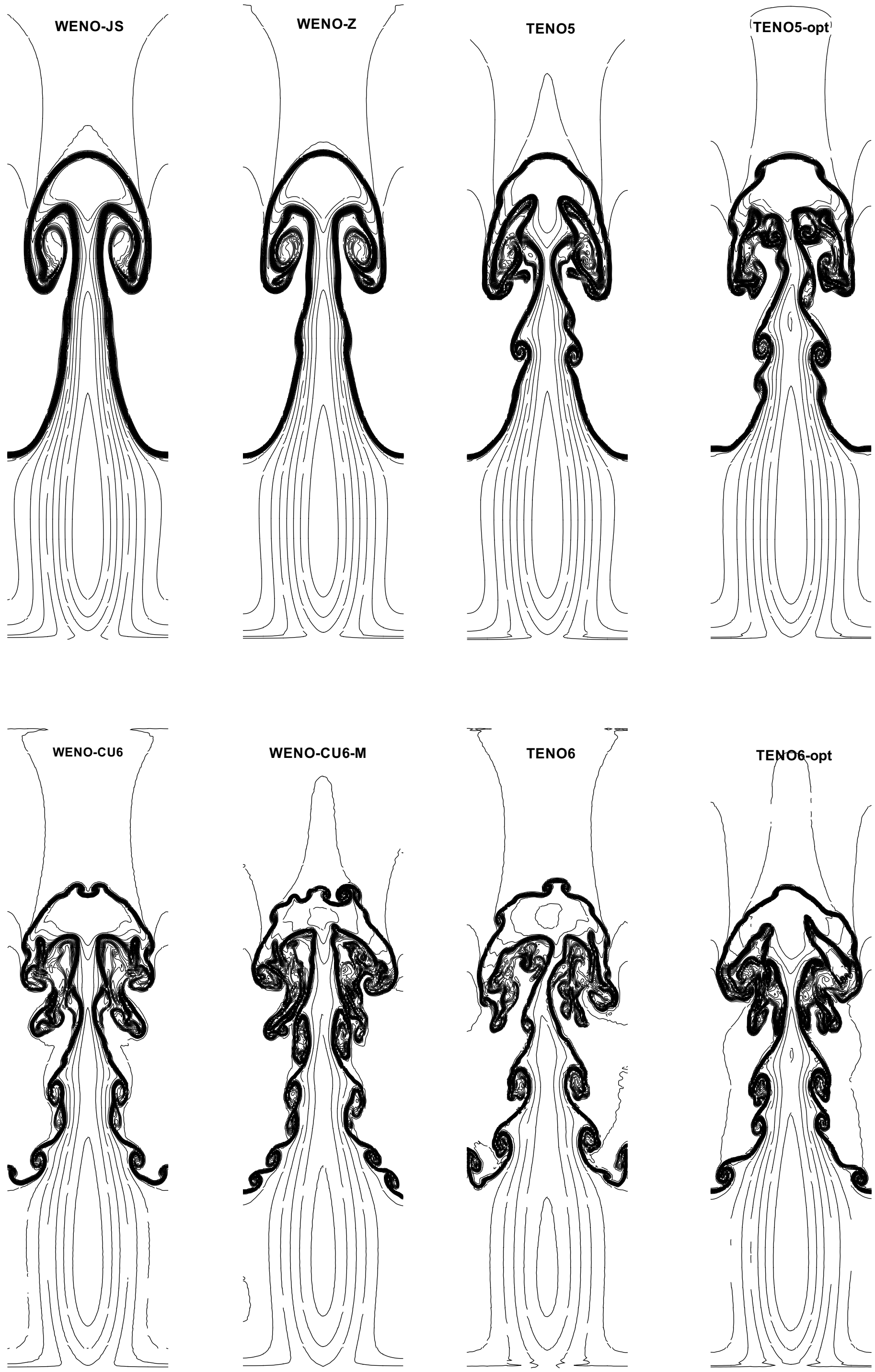

Figure 3. Rayleigh-Taylor instability: density contours. Resolution 128×512.

singularities.

Fig. 5 provides the time history of the normalized values of mean kinetic energy and mean enstrophy with resolution $128 \times 128 \times 128$. TENO5 and TENO5-opt preserve the total kinetic energy the best while WENO-JS loses it significantly. Furthermore, TENO5 and TENO5-opt resolve much more enstrophy than WENO-Z and WENO-JS scheme since $t \approx 4$ by offering less numerical dissipation in intermediate wave-number range. The predicted enstrophy from TENO5-opt is as large as two times of that from WENO-JS in the later time stage. In Fig. 6, TENO6 and TENO6-opt perform essentially better than WENO-CU6 and WENO-CU6-M with respect to both the kinetic energy conservation and the enstrophy prediction. Note that current schemes never over-predict the total kinetic energy whereas the optimized scheme WENO-BO unphysically over-predicts them due to its anti-dissipation property in the intermediate wave-number range, see Fig.9 in Arshed & Hoffmann (2013).

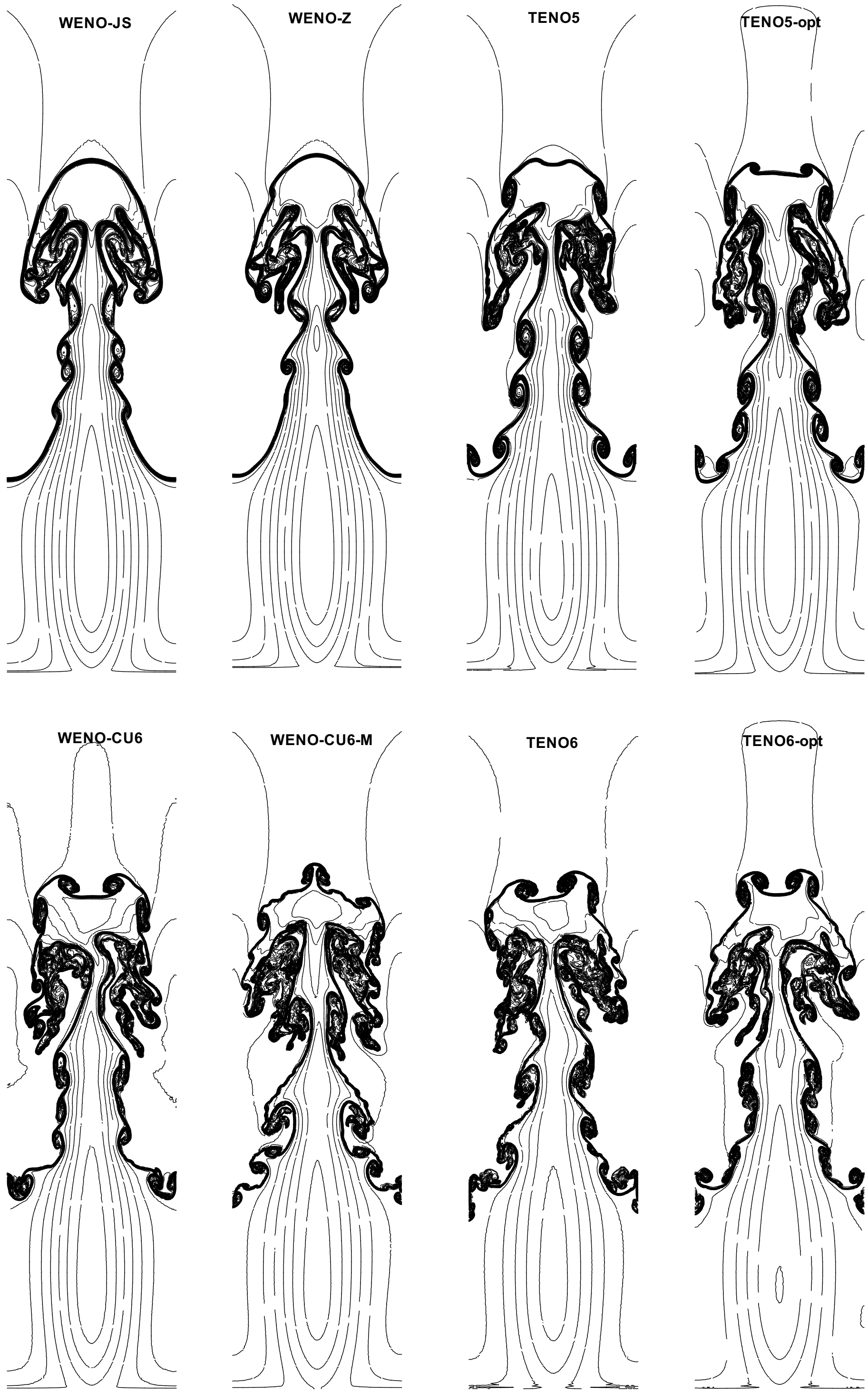

Figure 4. Rayleigh-Taylor instability: density contours. Resolution 256×1024.

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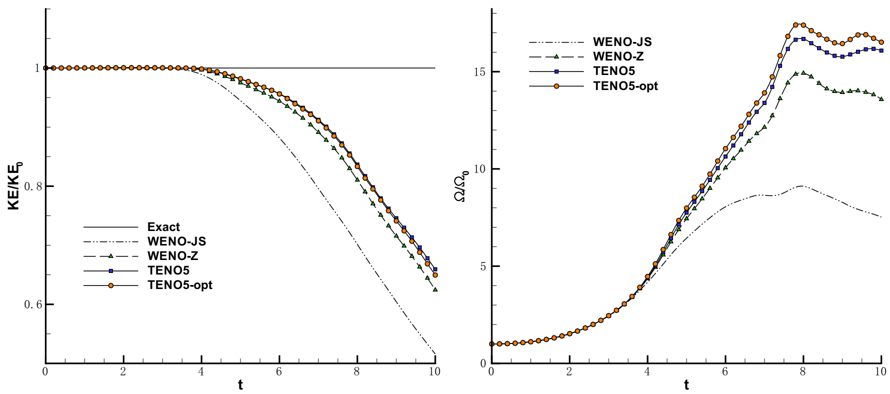

Figure 5. Inviscid simulation of the three-dimensional Taylor-Green vortex problem: solutions from five-point stencil schemes. Mean kinetic energy (left) and mean enstrophy (right). Discretization on a $128 \times 128 \times 128$ uniform grid. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

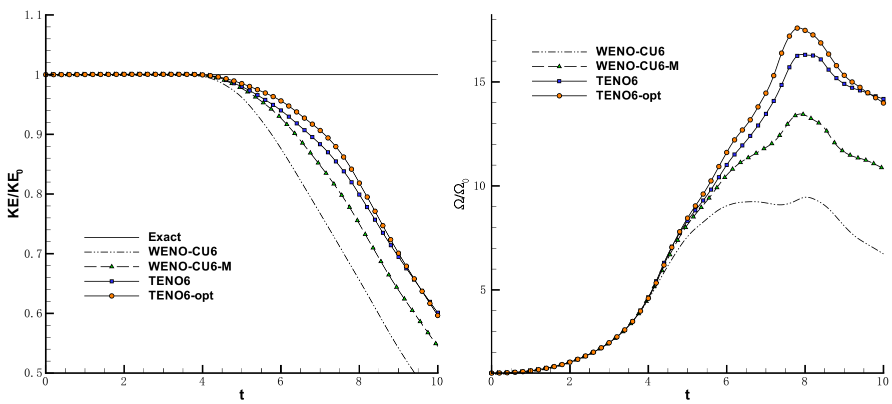

Figure 6. Inviscid simulation of the three-dimensional Taylor-Green vortex problem: solutions from six-point stencil schemes. Mean kinetic energy (left) and mean enstrophy (right). Discretization on a $128 \times 128 \times 128$ uniform grid. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

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