A FAST AND DIRECT METHOD FOR CHARACTERIZING HYDRAULIC ROUGHNESS

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ABSTRACT

We describe a fast direct numerical simulation (DNS) method that promises to directly characterize the hydraulic resistance of any given rough surface, from the hydraulically smooth to the fully rough regime. The method circumvents the unfavorable computational cost associated with simulating high-Reynolds-number flows by employing minimal-span channels (Jiménez & Moin, 1991). Proof-of-concept simulations, employing the parametric-forcing roughness model (Busse & Sandham, 2012), demonstrate that flows simulated in minimal-span channels are sufficient for capturing the downward velocity shift predicted by flows in full-span channels. Owing to the minimal cost, we are able to perform direct DNS with increasing roughness Reynolds numbers while maintaining a fixed roughness height that is 40 times smaller than the half-channel height. When coupled to an unstructured-grid code, the present method promises a practical, fast and accurate tool for characterizing hydraulic resistance directly from profilometry data of rough surfaces.

INTRODUCTION

Scientists have, for years, been cataloging the relationship between surface roughness and hydraulic resistance, the former pertaining to geometry while the latter to fluid dynamics (see reviews by Jiménez, 2004; Flack & Schultz, 2010). The cataloging never ends because each rough surface is unique. In order to make predictions in full-span channels. Owing to the minimal cost, we are able to conduct parametric DNS with increasing roughness Reynolds numbers while maintaining a fixed roughness height that is 40 times smaller than the half-channel height. When coupled to an unstructured-grid code, the present method promises a practical, fast and accurate tool for characterizing hydraulic resistance directly from profilometry data of rough surfaces.

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following.

Normally, a straightforward and direct computation of roughness drag using DNS employing full-span channels is extremely expensive as it entails simultaneously capturing both the bulk flow, which scales with the half-channel height, $h$, and the near-wall flow around the roughness elements, which scales with the characteristic roughness height, $k$. Given a rough surface of fixed blockage ratio $k_s/h \lesssim 1/40$ (Jiménez, 2004), a complete characterization of hydraulic resistance requires parametric simulations that sweep through the roughness Reynolds numbers, $k^u_s \equiv kU_s/\nu \approx 5$ to 100, corresponding to the hydraulically smooth and the fully rough regimes, respectively. For the blockage ratio, $h/k_s = 40$, this means performing parametric simulations at the friction Reynolds numbers, $R_e \equiv hU_f/\nu = k^u_s(h/k_s) \approx 200$ to 4000, which are currently unfeasible for routine engineering computations. Recall that the cost of DNS, counting the number of spatial and temporal degrees of freedom, scales unfavorably as $R_e^2$ (Pope, 2000, §9.1.2). For grids conforming to the surface of the roughness elements, this cost is further exacerbated by the need for increased mesh density, and reduced time steps.

However, the extreme cost associated with conventional DNS employing full-span channels seems unnecessary. The quantity of interest from an engineering point of view is the retardation in the mean flow over roughness relative to the smooth-wall flow. This relative flow retardation or downward velocity shift, $\Delta U$, occurs mostly in the vicinity of the roughness layer, but holds constant above a few roughness heights, well into the log layer (if it exists) and the wake region, cf. Townsend’s outer-layer similarity hypothesis (Townsend, 1976). This suggests that a simulation of only the near-wall region and its interaction with the roughness geometry is required in order to extract $\Delta U \equiv \Delta U/U_c$, which is known as the (Hama) roughness function. This distillation of the problem is consistent with the observation that $\Delta U$ does not depend on the bulk flow but only on $k^u$ and other details of the roughness geometry.

A framework for simulating only the near-wall dynamics is the minimal channel as first described by Jiménez & Moin (1991) and is currently receiving renewed attention in various contexts of understanding wall-bounded turbulence (Flores & Jiménez, 2010; Hwang, 2013; Lozano-Durán & Jiménez, 2014). Presently, we exploit this framework for measuring $\Delta U$ by fully resolving the near-wall Navier–Stokes dynamics and its interaction with the (modelled) roughness geometry. The prohibitive cost of conventional DNS is alleviated by use of these minimal-span channels, which are designed to preclude the bulk flow that scales with $h$. Without the bulk flow, the cost of DNS with roughness now only scales as $k^u_s^{3/4}$, which is quite feasible for the engineering task at $k^u_s \approx 5$ to 100. In general, the computational cost is $(h/k_s)^{3/4}$ times less than that of a conventional DNS in a full-span channel.

In the remainder, we show results from simulations using the parametric-forcing roughness model of Busse & Sandham (2012) that demonstrate the veracity of the present method.

### SIMULATIONS

We solve the following Navier–Stokes equations of motion between two no-slip, impermeable walls at $z = 0$ and $2h$:

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j^2} + f(t) \delta_{i1},$$

$$\frac{\partial u_i}{\partial x_j} = 0,$$

where $u_i$ is the velocity; $t$ is time; $x_j$ is the spatial coordinate; $\rho$ is the density; $p$ is the pressure; $f(t)$ is the spatially uniform, time-varying, pressure gradient that drives the flow at constant mass flux; and the last term on the right-hand side of (1) represents the body force due to roughness. Here $(x_1, x_2, x_3)$ or $(x, y, z)$ are taken as the (streamwise, spanwise, wall-normal) coordinates. The effect of any roughness geometry, which includes both pressure and viscous drag, can always be formally written as a forcing term on the right-hand side of the Navier–Stokes equation but, for the present purposes, the form adopted here is meant to represent a generic roughness in the spirit of the parametric-forcing model of Busse & Sandham (2012). The roughness forcing is active only in the streamwise direction and opposes the flow, $a_i = a \delta_{i1}$, and $F(x_3, k)$ is the shape function that depends on the characteristic roughness height $k$, chosen here to be the step function

$$F(x_3, k) = \begin{cases} 1 & x_3 < k \\ 2h - x_3 < k, & \text{otherwise.} \end{cases}$$

In general, the shape function depends on the geometry of the roughness being modeled. For example, MacDonald et al. (2014) show that an exponential shape function closely models a three-dimensional single-mode sinusoidal roughness. The roughness factor, $a$, is thought to scale with the roughness density, that is, the frontal area per unit volume (Nikora et al., 2007; Busse & Sandham, 2012), measured in inverse-length (area per unit volume) units. Presently, $a = 1/(40k)$ and $k = h/40$ for all the simulations. This simple roughness model has the advantage of appearing completely homogeneous to the flow over it. This idealized roughness, which retains only information about the roughness height $k$, will be used to show that $k$ imposes further restrictions on the minimal span of the computational domain. Periodic boundary conditions are imposed in the streamwise and spanwise directions with respective domain sizes, $L_x$ and $L_z$. The parameters for the 20 separate simulation cases for this study are documented in table 1. The streamwise and spanwise grids are uniform, the wall-normal grid is stretched with the cosine mapping and the chosen resolutions are comparable to other DNSs (Moser et al., 1999; Bernardini et al., 2014). The streamwise domain sizes are large enough to accommodate the near-wall streaks, which are 1000 wall units long.

### RESULTS

#### Mean velocity profiles at fixed friction Reynolds number

The mean velocity profiles, plotted in inner wall units, $U^+ \equiv U/U_f$ and $z^+ \equiv zU_f/\nu$, from all the simulations are shown in figure 1. We first focus on figure 1(c), which
Table 1. Simulation cases listed with nominal Reτ. Each of these 10 cases is run using smooth and rough walls, that is, a total of 20 separate simulations. In the smooth cases, α = 0, while in the rough cases α = 1/(40k) and h/k = 40. Δz_ε is the wall-normal resolution at the channel centre, where it is coarsest.

<table>
<thead>
<tr>
<th>Reτ</th>
<th>L_3^+</th>
<th>L_5^+</th>
<th>N_k</th>
<th>N_c</th>
<th>Δx^+</th>
<th>Δy^+</th>
<th>Δz_ε</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimal</td>
<td>180</td>
<td>3707</td>
<td>116</td>
<td>384</td>
<td>24</td>
<td>192</td>
<td>9.7</td>
</tr>
<tr>
<td>Minimal</td>
<td>395</td>
<td>3707</td>
<td>116</td>
<td>384</td>
<td>24</td>
<td>256</td>
<td>9.7</td>
</tr>
<tr>
<td>Full</td>
<td>590</td>
<td>3707</td>
<td>1854</td>
<td>384</td>
<td>384</td>
<td>256</td>
<td>9.7</td>
</tr>
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<td>Minimal</td>
<td>590</td>
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<td>768</td>
<td>9.7</td>
</tr>
<tr>
<td>Minimal</td>
<td>2000</td>
<td>3707</td>
<td>232</td>
<td>384</td>
<td>48</td>
<td>768</td>
<td>9.7</td>
</tr>
<tr>
<td>Minimal</td>
<td>2000</td>
<td>3707</td>
<td>463</td>
<td>384</td>
<td>96</td>
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</tr>
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<td>Minimal</td>
<td>4000</td>
<td>3707</td>
<td>463</td>
<td>384</td>
<td>96</td>
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<td>9.7</td>
</tr>
</tbody>
</table>

shows four mean velocity profiles, all at Reτ ≈ 590, the two in gray from minimal-span channels with L_3^+ = 116 and the two in black from full-span channels with L_3^+/h ≈ 3π. The two solid lines, one full-span and one minimal-span, correspond to the smooth-wall channels and the two dashed lines, one full-span and one minimal-span, correspond to the (modeled) rough-wall channels. Other parameters, including the grid resolutions and streamwise domain lengths, are held fixed. It is evident that the full-span and minimal-span simulations agree in the near-wall region, z^+ ≤ 46. This has been previously shown for the smooth-wall case by Flores & Jiménez (2010); Hwang (2013). With the top of the roughness-forcing region located by the dashed vertical line at k^+ = Reτ/(h/k) ≈ 590/40 ≈ 15, it can be seen that the full- and minimal-span profiles also agree for the rough-wall case in the vicinity of the roughness, suggesting that the minimal flow accurately captures the essential physics of the near-wall roughness-affected region. Consistent with Flores & Jiménez (2010); Hwang (2013), the minimal-channel profiles exhibit an exaggerated wave. Further, the minimal-channel profiles diverge from the full-channel profiles at precisely the same location, z^+ ≈ 46, regardless of whether the wall is rough or smooth. The agreement in the profiles below z_c suggest that the minimal flow in this region is unconstrained by the minimal span, behaving as if it were in a full-span channel. Above z_c, the minimal flow is constrained and, in the absence of eddies up to size z that typically characterize this location in full-span channels, the mixing of momentum is significantly reduced, leading to a profile with a much sharper increase in mean velocity compared to the usual log behavior in full-span channels. Interestingly, the profiles in the region above z_c are both constrained in exactly the same way. This is further supported by plotting the downward velocity shifts relative to the smooth-wall case, U^+ - U^+_s, shown in the inset of figure 1(c). Above the roughness-forcing region, z^+ ≈ k^+ ≈ 15, the shift is parallel and impervious to the minimal-span constraint, suggesting a kind of generalized outer-layer similarity in the sense of Townsend (1976). Typically, for profiles at unmatched Reτ, the Hama roughness function Δμ^+ is obtained by the shift in the log region where it is well defined. However, this is unnecessary in the present simulations at matched Reτ since the shift U^+ - U^+_s is constant everywhere above the roughness and is therefore well defined, so that we can unambiguously set Δμ^+ = [U^+_s - U^+]/z_k. The behavior of U^+ - U^+_s, where it increases in where z < k and remains constant where z > k, is consistent with the expectation that it is the dynamics of the near-wall roughness-affected flow alone that sets Δμ^+. And further, the result that U^+ - U^+_s is virtually identical in full- and minimal-span simulations supports the idea that the minimal flow faithfully captures the relevant flow dynamics that sets Δμ^+. This demonstrates the efficacy of the present method.

Effect of span

Many of the aforementioned behaviors in figure 1(c) at Reτ ≈ 590, k^+ ≈ 15 extend to other Reynolds numbers, e.g. Reτ ≈ 950, k^+ ≈ 24 (figure 1d). In particular, the roughness-affected region is faithfully predicted by the minimal-span simulations, and Δμ^+ can be unambiguously determined by evaluating U^+_s - U^+_s wherever z > k for both full- and minimal-span simulations. The minimal- and full-span profiles still diverge at z^+ ≈ 46 for Reτ ≈ 950, as with Reτ ≈ 590 (compare figures 1c, d), because the minimal span is held fixed at L_3^+ ≈ 116, consistent with previous studies (Flores & Jiménez, 2010; Hwang, 2013) that show z_c ∝ L_3^+ ∝ 0.3-0.4L_3. With L_3 held fixed, and with increasing Reτ, there would be a point where k > z_c. This suggests a criterion in order for the minimal-channel method to work as intended, namely, that k needs to be smaller than z_c. This criterion makes physical sense. When the roughness is no longer immersed in the natural unconfined flow below z_c, it follows that Δμ^+ can no longer be accurate because Δμ^+ is a measure of the interaction between the roughness and the natural unconfined flow. The simulations at Reτ ≈ 2000, 4000, respectively in figures 1(e, f), with various L_3, quan-
Figure 1. Mean velocity profiles of simulated turbulent channel flow at (a) $Re \tau \approx 180$, (b) $Re \tau \approx 395$, (c) $Re \tau \approx 590$, (d) $Re \tau \approx 950$, (e) $Re \tau \approx 2000$ and (f) $Re \tau \approx 4000$ through smooth-wall (solid) and $h/k = 40$ rough-wall (dashed), minimal-span (gray) and full-span (black) channels. The vertical dashed line marks the top of the roughness-forcing region, $z < k$. The inset shows that the velocity shift, $U^+_s - U^+_r$, stays the same for both minimal- and full-span channels above the roughness-forcing region. The smooth-wall full-channel $Re \tau \approx 180, 395$ profiles in (a, b) are from Moser et al. (1999); the smooth-wall full-channel $Re \tau \approx 2000$ profile in (e) is from Hoyas & Jiménez (2006) and the smooth-wall full-channel $Re \tau \approx 4000$ profile in (f) is from Bernardini et al. (2014).

As $L^+_y$ increases with values 116, 232 and 463, the location where the minimal- and full-span simulations diverge, $z^+_c$, increases with values 46, 93, 185, that is, $z^+_c \approx 0.4L^+_y$. It is reassuring that the minimal-span smooth-wall profiles capture more and more of the full-channel profiles of Hoyas & Jiménez (2006) and Bernardini et al. (2014) as $L^+_y$ increases (solid lines in figures 1e, f). For $Re \tau \approx 2000$ (figure 1e), the thinnest minimal-span is $L^+_y \approx 116$, corresponding to $z^+_c \approx 0.4(116) \approx 46$, which is only slightly below the top of the roughness-forcing region at $k^+ \approx 50$. The criterion, $k < z^+_c \approx 0.4L^+_y$, is only slightly violated, leading to undetectable discrepancies in $U^+_s - U^+_r$, as shown in the inset. However, for $Re \tau \approx 4000$ (figure 1f), the top of the roughness forcing region $k^+ \approx 100$ is unequivocally larger than $z^+_c \approx 46$ of the thinnest minimal channel, and clear discrepancies result in $U^+_s - U^+_r$, as shown in the inset. Once the spans are widened to $L^+_y \approx 232, 463$, such that $z^+_c \approx 93, 185 > k^+ \approx 100$, the criterion $k < z^+_c$ is more or less satisfied, and $U^+_s - U^+_r$ collapses, yielding a well-defined $\Delta U^+$. Although the criterion, $k < z^+_c$, is developed here with the present modeled homogeneous roughness, where $k$ measures the distance between the top of the roughness-forcing region and the hydraulic origin, we expect that, in practice, a conservative criterion would be $k_t < z^+_c$, where $k_t$ is the maximum peak-to-valley roughness height.
Hama roughness function and skin-friction coefficient

A sweep in roughness Reynolds numbers, from $k^+ \approx 5$ to 100, corresponding to $Re_k \approx 180$ to 4000 (table 1) is needed in order to fully characterize the roughness transition, from the hydraulically smooth to the fully rough regime. The Hama roughness function, $\Delta U^+$, for such a sweep is shown in figure 2(a). Only $\Delta U^+$ from simulations satisfying $k < z_c$ are shown. Here, even the $Re_k \approx 4000$ case is feasible owing to the significant saving in computational cost associated with the minimal-channel technique. As a comparison, the smooth-wall full-channel profile in figure 1(f) requires a simulation with $8192 \times 4096 \times 1024$ grid points (Bernardini et al., 2014) while the minimal-channel case at the same $Re_k = 463$ only requires $384 \times 96 \times 1024$ grid points, amounting to a factor of 910 saving in number of grid points. The horizontal axis is $k_x = 1.6k$, where the constant 1.6 relates this particular (modeled) roughness height $k$ with parameter $\alpha = 1/(40k)$ to its dynamic behavior measured against the equivalent sand-grain roughness $k_s$. The constant 1.6 can be determined owing to the availability of data in the fully rough regime that can be matched to the fully rough asymptote of uniform sand grains, $\log(k_s^+)/ \kappa + A - 8.5$, where $\kappa \approx 0.4$, the von Kármán constant and $A \approx 5.2$, the smooth-wall log-law intercept. It is interesting to note that the top-hat version of the parametric-forcing model (Busse & Sandham, 2012) that is employed in the present study closely models the uniform-sand-grain roughness behavior in the transitionally rough regime, $4 \lesssim k_s^+ \lesssim 70$. It is often thought that the pressure drag of the roughness elements dominates the viscous drag in the fully rough regime, $k_s^+ \gtrsim 70$–100. This idea is quantified using the present roughness model. Figure 2(b) shows the modeled drag fraction $F_{mod}/(F_{mod} + F_{visc})$ versus $\Delta U^+$, where $F_{mod} \equiv \int_0^1 (\mu_{eff} \mu_{eff} / \mu_0) \alpha u \nu |u| \; dx \; dy \; dz$ and $F_{visc} \equiv v \Delta U^+ / dz_{00}$. The model $F_{mod}$ can be interpreted as a pressure drag because it scales as the square of velocity. As expected, the drag partition undergoes a transition from a viscous-drag-dominated regime to a modeled-drag-dominated regime as $\Delta U^+$ increases. However, even at $\Delta U^+ \approx 9$ corresponding to $k_s^+ \approx 160$, we observe that $F_{mod}/(F_{mod} + F_{visc}) \approx 0.72$, indicating that a residual influence from viscous drag remains. However, figure 2(c) shows that the skin-friction coefficient, $C_f$, appears to already reach a constant value, independent of $Re_k$, a standard method used to show that a flow is in the fully rough regime. The bulk velocity, $U_{b+}$, for a full-span rough channel flow is used in $C_f$ and $Re_b$, $U_{b+}$ is readily estimated from $U_{b+} = U_{b+} - (U_{b+} - U_{b+})$, where all profiles are at matched $Re_k$, which are available in this case.

CONCLUSIONS

We have presented a novel, fast and direct method for characterizing the hydraulic resistance of any given surface roughness. The way in which a particular roughness transition from the hydraulically smooth to the fully rough regime is, to the first approximation, described by how the
roughness geometry interacts with the near-wall flow. The method presented herein shows that this interaction, so far as the mean drag is concerned, is captured in the absence of the bulk flow using the idea of minimal-span channels.

The dynamic drag characterization of a rough surface is encapsulated in the $\Delta U^+$, which we show can be accurately determined using minimal-span channels. The fact that $\Delta U^+$ is plotted against $k^+$ (or $l_0^+$) in the literature and not against $Re_T = h^+$ acknowledges that $\Delta U^+$, to a large extent, depends only on the roughness-affected near-wall flow. The minimal-span channel is a method for simulating this near-wall flow of thickness $O(k)$ without resolving the outer scale $h \gg k$, thereby breaking the curse of the (outer) Reynolds number. The savings in computational cost for the present method are possible because capturing only the near-wall flow requires far less grid points than capturing the full flow. The present method can be used to obtain the drag characteristics of many surfaces very quickly.

We propose that all the following criteria must be simultaneously satisfied when choosing the minimal span, $L_y$:

(i) $L_y > 100 U_\tau / U_{14}$, the minimal span must be wide enough to accommodate the near-wall streaks;

(ii) $L_y > k / 0.4$, the minimal span must be wide enough to immerse the roughness in unconfined wall turbulence; and

(iii) $L_y > \lambda_\text{c}$, the minimal span must be wide enough to capture the widest features of the roughness, $\lambda_\text{c}$.

Criterion (i) is fairly well established (e.g. Jiménez & Moin, 1991; Hwang, 2013), necessary to capture the interaction, and perhaps the destruction (Jiménez, 2004), of the near-wall streaks by the roughness elements as $k^+$ increases. Criterion (ii) is rigorously demonstrated in the present study, and stems from the distance-from-the-wall ($z$) scaling in the log region where eddies have span or sizes, here set by the domain span $L_y$, that scale with $z$. Although not explored in this study, criterion (iii) is presumably necessary if $\Delta U^+$ is to capture the whole effect, from the widest scales, $\lambda_\text{c}$, to the thinnest scales, of the roughness geometry.

In many ways, the present idea is not new. Large-eddy simulation (LES) directly simulates or resolves the large-scale flow that are deemed to be dependent on large-scale geometry while modeling the small-scale flow that are deemed to be universal. Here, in the case of the minimal-channel method, the idea of LES is used in reverse, and can be interpreted as small-eddy simulation (SES), a term first coined, and used in the same context, by Jiménez (2003). The minimal-channel method directly simulates or resolves the small-scale flow that are deemed to be dependent on the small-scale roughness geometry while modeling the large-scale flow that are deemed to be universal. In LES, the small scales are understood through the phenomenological theory of Kolmogorov, while in SES (minimal-channel method), the large scales are understood through the phenomenological theory of Townsend. In other words, we are directly simulating the non-universal parts of wall roughness while leaving the universal parts to be described by Townsend’s outer-layer similarity hypothesis.

In Chung et al. (2015), the efficacy of the present method is rigorously demonstrated by comparison with DNS of explicitly gridded roughness simulated using a finite-volume code.

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