# A SCALE SELF-RECOGNITION MIXED SGS MODEL BASED ON THE UNIVERSAL REPRESENTATION OF KOLMOGOROV LENGTH BY GS VARIABLES

**N. Fukushima***, **Y. Naka, K. Hiraoka, M. Shimura, M. Tanahashi and T. Miyauchi**
Department of Mechanical and Aerospace Engineering
Tokyo Institute of Technology
2-12-1 Ookayama, Meguro-ku, Tokyo 152-8550, Japan
*email: nfukushim@navier.mes.titech.ac.jp

## ABSTRACT

Direct numerical simulation of homogenous isotropic turbulence (HIT) have been conducted at relatively high Reynolds numbers. By analyzing the DNS database, characteristics of GS-SGS energy transfer are investigated in detail. Especially, dependences of GS-SGS energy transfer by Leonard, cross and Reynolds terms, and the total GS-SGS energy transfer on filter-width to Kolmogorov scale, $\Delta/\eta$, are revealed. The characteristics of conventional eddy viscosity models and scale similarity model are investigated in terms of the GS-SGS energy transfer. It is found that Smagorinsky model can predict energy transfer by Reynolds term well for large $\Delta$ where Reynolds term is dominant and Bardina model has a potential to predict cross term well especially for small $\Delta$ where cross term is dominant. Based on the assumption of local equilibrium and the fact that the Smagorinsky coefficient is the function of $\Delta/\eta$, a new method to predict $\Delta/\eta$ by using only resolved scale and a new subgrid-scale (SGS) model, a scale self-recognition mixed SGS model, are proposed. Superiority of the scale self-recognition mixed SGS model has been demonstrated through static and dynamic tests in HIT and turbulent channel flow. The correlation coefficient between the total GS-SGS energy transfer obtained from filtered DNS data and statically predicted by the proposed model is very high with any size of $\Delta$ in HIT. Compared with the conventional SGS models, the present model dynamically gives the best prediction of both instantaneous and statistical characteristics of the turbulent flows.

## INTRODUCTION

With the development of computational resources and technologies, large eddy simulation (LES) is of growing importance on computer aided design system. Several subgrid-scale (SGS) models have been developed. Smagorinsky model (Smagorinsky (1963)) is a basic eddy viscosity model but the model coefficient depends on flow fields. Therefore, dynamic Smagorinsky model (Germano *et al.* (1991); Lilly (1992)) is widely used, whose model coefficient is computed dynamically under the assumption of scale similarity. However, it has much numerical cost and averaging procedure of the coefficient is required to overcome instability problem. Kobayashi (2005) developed coherent structure Smagorinsky model whose coefficient is locally determined by the second invariant of velocity gradient tensor. The demand of local determination of the coefficient increases in terms of effective use of massive-parallel computer systems. On the other hand, Bardina model (Bardina *et al.* (1980)) is based not on eddy viscosity concept but on scale similarity assumption.

Conventionally, it has been supposed that in turbulence there are eddies having various scales hierarchically and that energy transfer from large- to small-scale occurs. For a few decades, intensive direct numerical simulation (DNS) studies have been conducted to clarify the relation between turbulent coherent structures and the energy transfer. In our previous study, temporal-spatial intermittent distributions of strong forward and backward scatter are observed around turbulent coherent structures even for homogenous isotropic turbulence (HIT), and characteristics of GS-SGS energy transfer depend on the ratio of the filter width to turbulent scale (Tanahashi *et al.* (2006)). In complex turbulence in real applications, it is important to predict local GS-SGS energy transfer accurately. Further clarification of the energy transfer is necessary for evaluating and developing SGS models.

In this study, detailed investigation of the GS-SGS energy transfer and evaluation of characteristics of SGS models in terms of the energy transfer are conducted by analyzing DNS results of HIT at high Reynolds number. From the results, a new method to predict a ratio of filter width to Kolmogorov length by using only resolved scale and a new SGS model, a scale self-recognition mixed SGS model, are

Table 1. Numerical condition for DNS of homogeneous isotropic turbulence.

| $Re_\lambda$ | $Re_{l_E}$ | $N^3$ | $L/l_E$ | $S_{u'}$ | $F_{u'}$ |
|---|---|---|---|---|---|
| 175.4 | 1518.5 | $512^3$ | 6.39 | -0.536 | 6.25 |
| 222.7 | 2087.6 | $640^3$ | 6.63 | -0.534 | 6.34 |
| 256.1 | 2899.1 | $800^3$ | 6.36 | -0.538 | 6.47 |
| 287.6 | 3694.1 | $960^3$ | 6.47 | -0.564 | 6.97 |
| 344.1 | 4575.7 | $1280^3$ | 6.48 | -0.567 | 7.20 |
| 393.8 | 5779.4 | $1536^3$ | 6.19 | -0.554 | 7.52 |

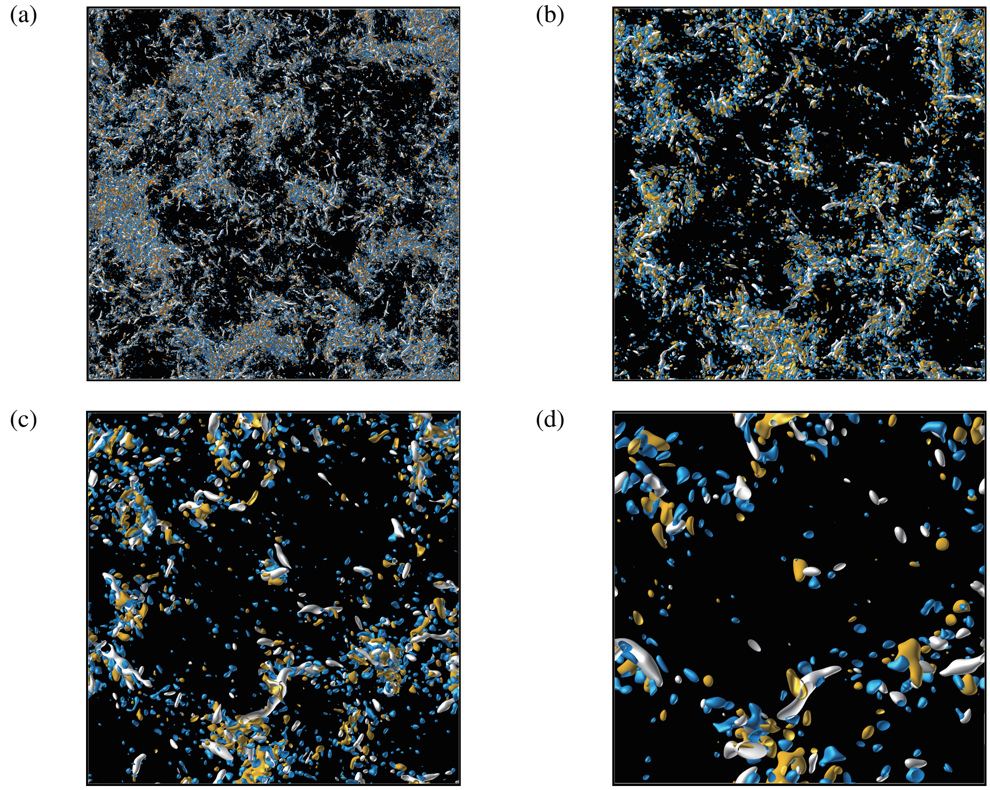


Figure 1. Instantaneous distributions of vortical structures of GS component (white), positive GS-SGS energy transfer (yellow) and negative GS-SGS energy transfer (blue) at $Re_\lambda = 344.1$: (a) $\Delta = 15\eta$, (b) $\Delta = 30\eta$, (c) $\Delta = 60\eta$ and (d) $\Delta = 120\eta$.

proposed and the superiority is demonstrated through static and dynamic tests in HIT. In addition, the applicability of the present model is demonstrated in a turbulent channel flow.

## DNS DATABASE OF HOMOGENEOUS ISOTROPIC TURBULENCE

DNS of homogeneous isotropic turbulence up to $Re_\lambda = 344.1$ have been conducted. Here, $Re_\lambda$ denotes Reynolds number based on Taylor micro scale. Spectral methods are implemented in all directions with fully de-aliasing and third order Runge-Kutta scheme is applied for time advancement. The spatial resolution is $\eta k_{max} \approx 1$ for all Reynolds number and the size of the computational domain, $L$, has been selected to be larger than $6l_E$ which is enough to resolve the largest scale motions of turbulence. Here, $\eta$, $k_{max}$, $l_E$ denote Kolmogorov scale, the maximum wave number and integral scale, respectively. The numerical conditions are summarized in Table 1, where $Re_{l_E}$, $N^3$, $S_{u'}$ and $F_{u'}$ are Reynolds number based on integral scale, number of total grid points, skewness and flatness of longitudinal velocity gradient. The details of numerical method can be referred to Tanahashi *et al.* (2002, 2006).

In this study, both sharp cut-off and Gaussian filters are applied to the DNS data in wavenumber space to obtain the resolved scale for scale separation. It should be noted that resolved scale in practical LES cannot be modeled by applying either a Gaussian filter or a sharp cut-off filter only. Results with applying both a sharp cut-off filter and a Gaussian filter are shown below.

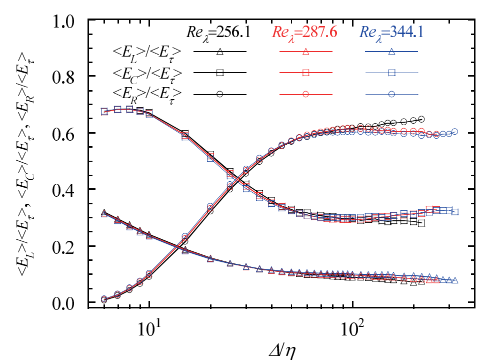


Figure 2. Dependences of contributions of Leonard, cross and Reynolds terms to total energy transfer on filter-width for $Re_\lambda = 256.1$, 287.6 and 344.1.

## FILTER WIDTH DEPENDNCY ON CHARACTERISTICS OF GS-SGS ENERGY TRANSFER

Instantaneous distributions of vortical structures of GS component, positive GS-SGS energy transfer (forward scatter) and negative GS-SGS energy transfer (backward scatter) at $Re_\lambda = 344.1$ are shown in Fig. 1. The dependency on filter-width, $\Delta$, are investigated. Regions of strong forward and backward scatter are distributed around vortical structures of GS component in any size of filter-width, while these structures change from elongated to blunt shapes with increase in filter-width. Therefore, it is supposed that there are close relationship between the vortical structures and GS-SGS energy transfer at each filter-width. It is noted that

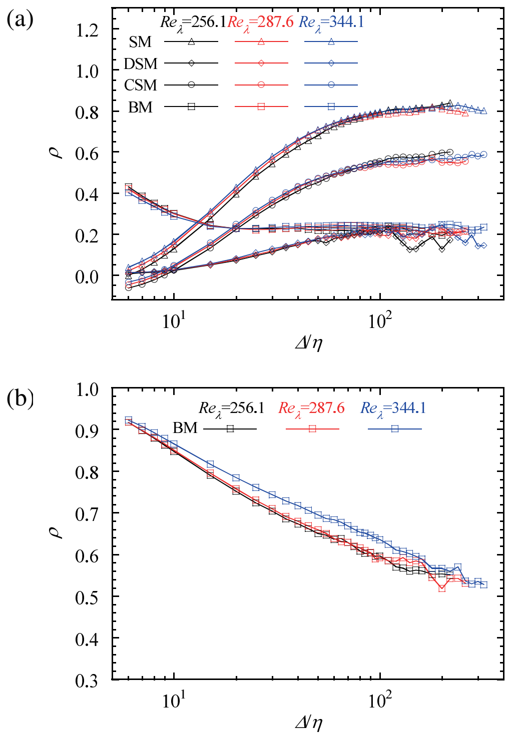


Figure 3. Dependences of correlation coefficient between GS-SGS energy transfer by Reynolds term (a) and cross term (b) obtained from DNS and predicted by SGS models on filter-width for $Re_\lambda = 256.1$, 287.6 and 344.1.

backward scatter plays an important role on GS-SGS energy transfer in relatively high Reynolds number turbulence. Therefore, it is necessary to predict distribution of GS-SGS energy transfer, including backward scatter, accurately.

To clarify characteristics of the GS-SGS energy transfer in detail, Fig. 2 shows dependences of averaged contributions of Leonard, cross and Reynolds terms to the total GS-SGS energy transfer, $< E_\tau > \equiv < -\tau_{ij}\overline{S_{ij}} >$, on $\Delta$, for $Re_\lambda = 256.1$, 287.6 and 344.1. The averaged contribution by cross term, $< E_C >$, is the largest for $\Delta < 30\eta$ and that by Reynolds term, $< E_R >$, is the largest for $\Delta > 30\eta$ while that by Leonard term, $< E_L >$, is low in any size of $\Delta$. As filter width increases, the averaged contribution ratios by Reynolds, cross and Leonard terms reach to 60%, 30% and 10%, respectively. These tendencies are independent of Reynolds number.

These results can further be explained in terms of a probability density function (PDF) of the energy transfer, representing the ratio of forward and backward scatter, by each term (not shown here). For the energy transfer by Leonard term, the PDF is almost symmetric for any size of $\Delta$. For cross term, the PDF is not symmetric for small $\Delta$ and relatively symmetric for large $\Delta$. The energy transfer by Reynolds term is mostly positive and its magnitude is relatively small. The PDF for Reynolds term is asymmetric in particular for larger $\Delta$. Consequently, the averaged contribution ratio of Leonard term is low in any size of $\Delta$ while cross term shows the largest contribution for small $\Delta$ and Reynolds term is dominant for large $\Delta$, respectively.

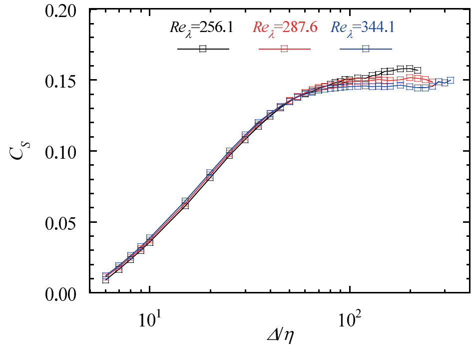


Figure 4. Dependences of the Smagorinsky coefficient ($C_S$) on filter-width for $Re_\lambda = 256.1, 287.6$ and 344.1.

## STATIC TEST OF CONVENTIONAL SGS MODELS

Characteristics of eddy viscosity models (Smagorinsky (Smagorinsky (1963)), dynamic Smagorinsky (Germano *et al.* (1991), Lilly (1992)) and coherent structure Smagorinsky models (Kobayashi (2005))) and scale similarity model (Bardina model (Bardina *et al.* (1980))) are investigated in terms of the GS-SGS energy transfer. Hereafter, in figures, they are called SM, DSM, CSM and BM, respectively. Figure 3(a) shows filter-width dependences of correlation coefficients between the GS-SGS energy transfer by Reynolds term obtained from filtered DNS data and predicted by SGS models. The energy transfer, $E_R$, predicted by Smagorinsky and coherent structure Smagorinsky models has a strong correlation with filtered DNS data for $\Delta > 80\eta$ and a weak correlation with filtered DNS data for small $\Delta$ where backward scatter occurs in high volume fraction of the domain, which cannot be predicted by these models. On the other hand, Bardina and dynamic Smagorinsky models can predict the backward scatter. However, the energy transfer, $E_R$, predicted by Bardina and dynamic Smagorinsky models correlates poorly with filtered DNS data in any size of $\Delta$. The scale similarity assumption could not be realized locally even in the high Reynolds number turbulence. In Fig. 3(b), the correlation coefficients between the energy transfer by cross term, $E_C$, by Bardina model and from filtered DNS data show a high value even for large size of $\Delta$ while the coefficients decrease slightly with increase of $\Delta$.

These tendencies of the model prediction for each term are independent of Reynolds number. Consequently, Smagorinsky model can predict energy transfer by Reynolds term well for large $\Delta$ where Reynolds term is dominant and Bardina model has a potential to predict cross term well especially for small $\Delta$ where cross term is dominant. It is well-known that one model gives a good prediction in some conditions, but it may fail to predict SGS stress in the other conditions. This is explained by the fact that the models proposed until now cannot predict the ratio of filter width to Kolmogorov length scale ($\Delta/\eta$). LES with high accuracy can be conducted by predicting $\Delta/\eta$. In addition, $\Delta/\eta$ depends on spatiotemporal intermittency in turbulence. Therefore, a new method to predict $\Delta/\eta$ by variables at resolved scale and a new SGS model based on $\Delta/\eta$ are proposed. We name the newly proposed model "a scale self-recognition mixed SGS model".

## SCALE SELF-RECOGNITION MIXED SGS MODEL

The newly developed scale self-recognition mixed SGS model (SSRM) in this study is written as follows:

$$\tau_{ij} = (\overline{\bar{u}_i\bar{u}_j} - \bar{\bar{u}}_i\bar{\bar{u}}_j) + (\bar{\bar{u}}_i\bar{u}'_j + \bar{u}'_i\bar{\bar{u}}_j - 2\bar{\bar{u}}_i\bar{\bar{u}}_j) + (-2(C_S\Delta)^2|\bar{S}|\bar{S}_{ij}) \quad (1)$$

$$C_S = C_\infty(1-\alpha e^{-\beta(\Delta/\eta)}) \quad (2)$$

$$\Delta/\eta = a\left(2\Delta^6|\bar{S}|\bar{S}_{ij}\bar{S}_{ij}/\nu^3\right)^b \quad (3)$$

Here, $\alpha$, $\beta$, $a$ and $b$ are model constants. The model formulation is similar to the conventional mixed model. For Leonard and cross terms, Bardina model with the coefficient of 1.0 is implemented to satisfy the Galilean invariance. For Reynolds term, Smagorinsky model is applied. It is noted that the Smagorinsky coefficient, $C_S$, is not a constant, but the function of $\Delta/\eta$.

Figure 4 shows filter-width dependency of $C_S$ obtained exactly from the DNS data. Here, the coefficient $C_s$ is obtained at different $\Delta/\eta$ so that the GS-SGS energy transfer predicted by the Reynolds term with Smagorinsky model agrees with the energy transfer evaluated from the DNS data. $C_S$ increases with $\Delta/\eta$ and the asymptotic value ($C_\infty$) is about 0.15 which is smaller than the general value of 0.2 in HIT since Smagorinsky model is evaluated as the model only for Reynolds term in the present formulation. The profile of $C_S$ does not depend on Reynolds number in high Reynolds number turbulence. Therefore, the constants, $\alpha$ and $\beta$, in Eq. (2) are determined by least square method.

A new method to predict $\Delta/\eta$ by using resolved scale variables is proposed. From the assumption of local equilibrium and the fact that $C_S$ is the function of $\Delta/\eta$, it is found that $\Delta/\eta$ can be predicted by an indicator which consists of only GS variables, as shown in Eq. (3). The constants, $a$ and $b$ are also determined by least square method from DNS database of HIT in the range of $Re_\lambda$ from 256.1 to 344.1 and the maximum error is less than about 10%. Equation (3) is effective to predict $\Delta/\eta$ accurately. To verify the scale self-recognition mixed SGS model, the correlation coefficient between the total GS-SGS energy transfer obtained from filtered DNS data and predicted by the proposed model is investigated. In any size of $\Delta$, the correlation coefficient is higher than 0.8. The scale self-recognition mixed SGS model based on the universal representation of Kolmogorov length by GS variables can predict spatial distribution of GS-SGS energy transfer with higher accuracy.

To demonstrate superiority of the scale self-recognition mixed SGS model, LES of decaying HIT at initial $Re_\lambda = 175.4$ is dynamically conducted. The results of Smagorinsky model with $C_S = 0.2$, Bardina model with $C_B = 1.0$ and the present model are compared. The initial $\Delta/\eta_{init}$ is 11.4 or 22.7 which is 4 and 8 times as large as the DNS gird size, respectively. Temporal history of turbulent kinetic energy is shown with the filtered DNS for comparison in Fig. 5(a). Without dependency on $\Delta/\eta$, Smagorinsky model overpredicts decay of turbulence at an early stage and the decay rate becomes slower with time. It is noted that dynamic Smagorinsky model with average process of $C_S$ in homogenous directions shows the similar tendency. Bardina model predicts decay of turbulent kinetic energy well at $\Delta/\eta_{init} = 11.4$, but the temporal development at larger $\Delta/\eta_{init}$ is very different with the filtered DNS results. The prediction accuracy by Bardina model strongly depends on LES grid size. On the other hand, the prediction by the scale self-recognition mixed SGS model agrees with the DNS results better than the other models. To investigate spectral characteristics of turbulence in detail, Fig. 5(b) shows the energy spectra predicted by LES and obtained from DNS results, which are normalized by energy dissipation, $\varepsilon$, and kinetic viscosity, $\nu$, at $t = 3.0$. Smagorinsky model underpredicts the energy in higher wavenumber range for both LES grid sizes. Bardina model underpredicts the kinetic energy in lower wavenumber and overpredicts in higher wavenumber especially in the case of $\Delta/\eta_{init} = 22.7$. On the other hand, the proposed model agrees with the filtered DNS much better than the other models. The present model can represent the GS-SGS energy transfer more accurately.

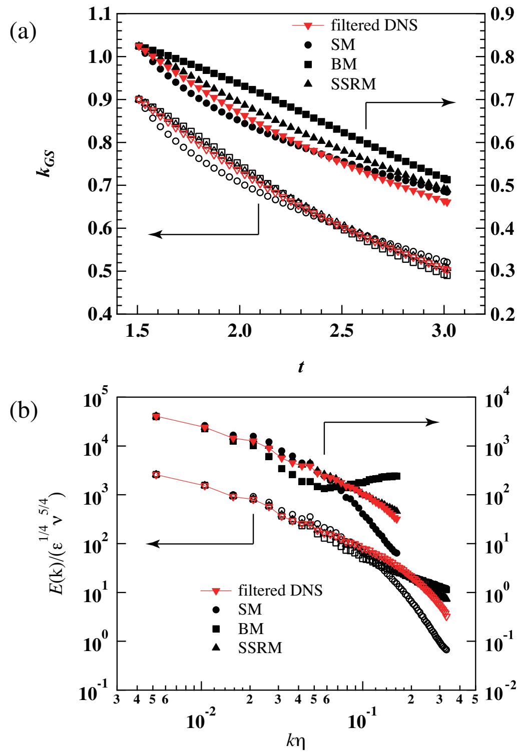


Figure 5. Temporal history of turbulent kinetic energy (a) and Energy spectra of turbulent kinetic energy (b) at $t = 3.0$ and $Re_\lambda = 175.4$ with $\Delta/\eta_{init} = 11.4$ (open) and 22.7 (solid) from filtered DNS and SGS models.

To compare local prediction accuracy among the models, instantaneous 2D distributions of one in-plane component of velocity are shown in Fig. 6. Smagorinsky model shows smoother distribution than the DNS results since kinetic energy in higher wavenumber decays too much. The oscillations in higher wavenumber are observed for Bardina model due to the model properties as mentioned above. The distribution predicted by the scale self-recognition mixed SGS model is very close to the filtered DNS results. Moreover, in Fig. 7, instantaneous distributions of vortical structures of GS component, positive GS-SGS energy transfer (forward scatter) and negative GS-SGS energy transfer (backward scatter) are visualized from filtered DNS and LES results. As mentioned in Fig. 1, regions of strong forward and backward scatter are distributed around vortical structures of GS component from filtered DNS database in Fig. 7(a). Since Smagorinsky model inherently predicts only forward scatter, regions of strong forward scatter are found around vortical structures of GS component in Fig.

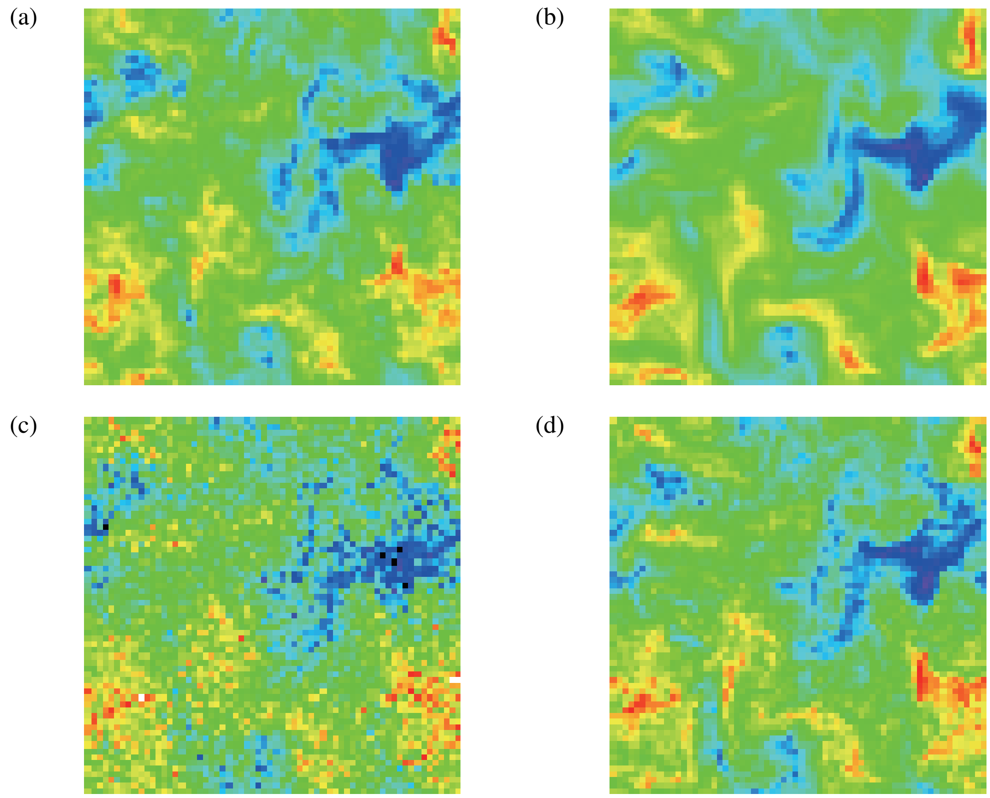


Figure 6. Instantaneous 2D distributions of one in-plane component of velocity at $t = 3.0$ and $Re_\lambda = 175.4$ with $\Delta/\eta_{init} = 22.7$: (a) filtered DNS, (b) Smagorinsky, (c) Bardina and (d) scale self-recognition mixed models.

7(b). In Bardina model, small-scale distributions of vortical structures of GS component, forward and backward scatter are observed due to the characteristics of underprediction in lower wavenumber and overprediction in higher wavenumber in Fig. 7(c). The scale self-recognition mixed SGS model predicts both forward and backward scatter well and the distributions of strong forward and backward scatter indicate a similar tendency to the filtered DNS results in Fig. 7(d).

Finally, the applicability of the present scale self-recognition mixed SGS model is demonstrated in a turbulent channel flow. Filtered DNS results and dynamic predictions by SGS models are compared. The details of DNS condition and method can be referred to Tanahashi *et al.* (2004). Figure 8 shows mean velocity and RMS of streamwise turbulent velocity at $Re_\tau = 800$ obtained from filtered DNS and predicted by SGS models with $\Delta = 2\Delta_{\mathrm{DNS}}$. Here, SMVD denotes Smagorinsky model with van Driest damping function. The scale self-recognition mixed SGS model predicts both of the profiles accurately without the damping function, while Smagorinsky model can predict turbulent statistics accurately in wall turbulence only when the damping function is used.

From the results, it is evident that the present model gives the best prediction not only of turbulent statistics but also of instantaneous local turbulent properties among the models.

## CONCLUSIONS

A new SGS model, a scale self-recognition mixed SGS model, is proposed. The model formulation is similar to the conventional mixed model, but the Smagorinsky coefficient, $C_S$, is not a constant but the function of $\Delta/\eta$. This is based on the fact that Smagorinsky model can predict energy transfer by Reynolds term well for large $\Delta/\eta$ where Reynolds term is dominant and Bardina model has a potential to predict cross term well especially for small $\Delta/\eta$ where cross term is dominant by analyzing the DNS database of homogenous isotropic turbulence at relatively high Reynolds numbers. Moreover, a new method to predict a ratio of filter width to Kolmogorov length by using only resolved scale is developed based on the assumption of local equilibrium and the fact that the Smagorinsky coefficient is the function of $\Delta/\eta$. Finally, the superiority of the scale self-recognition mixed SGS model is demonstrated through static and dynamic tests in homogenous isotropic turbulence (HIT) and a turbulent channel flow. In HIT, the correlation coefficient between the total GS-SGS energy transfer obtained from filtered DNS data and statically predicted by the proposed model is very high with any size of $\Delta$. The present model dynamically gives the best prediction not only of turbulent statistics but also of instantaneous local turbulent properties among the models. In the application to a turbulent channel flow, it is demonstrated that the present model can predict turbulent statistics accurately without damping function in wall turbulence.

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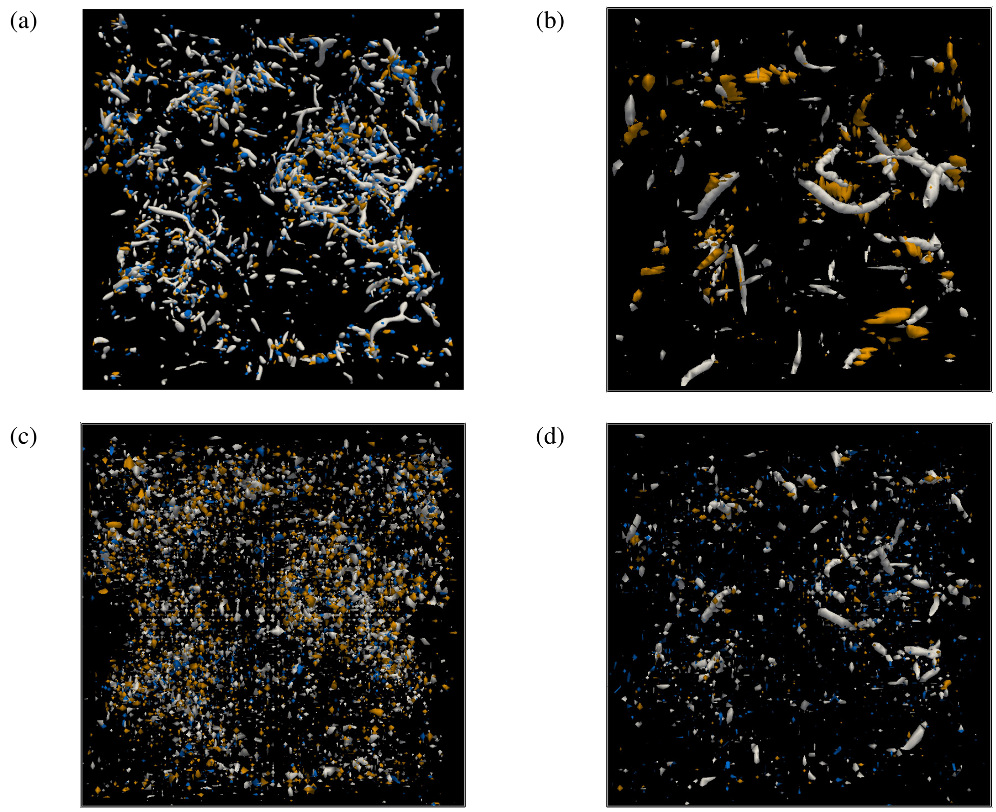


Figure 7. Instantaneous distributions of vortical structures of GS component (white), positive GS-SGS energy transfer (yellow) and negative GS-SGS energy transfer (blue) at $t = 3.0$ and $Re_\lambda = 175.4$ with $\Delta/\eta_{init} = 22.7$: (a) filtered DNS, (b) Smagorinsky, (c) Bardina and (d) scale self-recognition mixed models.

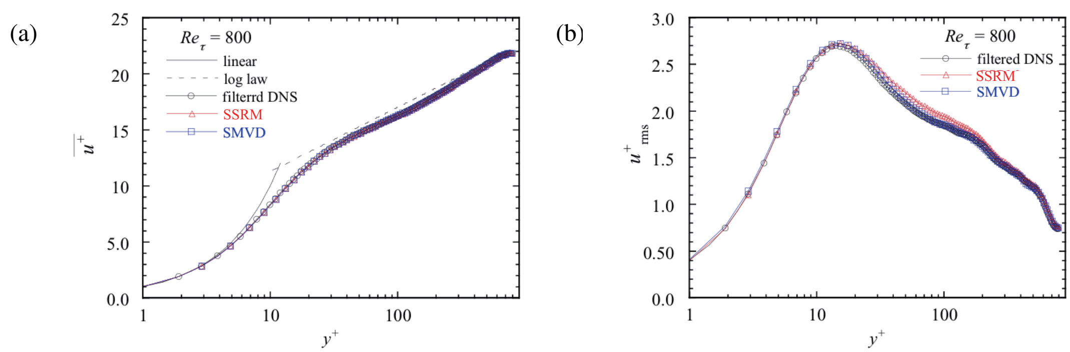


Figure 8. Mean velocity (a) and RMS of streamwise turbulent velocity (b) at $Re_\tau = 800$ in turbulent channel flow from filtered DNS and SGS models with $\Delta = 2\Delta_{\mathrm{DNS}}$.


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