LAGRANGIAN STATISTICS OF INERTIAL PARTICLES IN NEAR-WALL TURBULENCE

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ABSTRACT

We investigate Lagrangian nature of inertial particles by ensemble-averaging over a number of individual trajectories released at specific wall-normal locations in turbulent channel flow for a wide range of Stokes number. We performed direct numerical simulation of turbulent channel flow at a shear Reynolds number of 180, coupled with Lagrangian particle tracking. The particle dispersion, Lagrangian autocorrelation and time evolution of probability density function of particle wall-normal locations are analyzed. Furthermore, the effect of turbulence modification due to the presence of particles at sufficient mass loadings is investigated.

INTRODUCTION

Understanding the behavior of small, inertial particles in turbulent flows is of importance in many engineering applications, such as material transfer and pollution control. Despite numerous experimental and numerical studies regarding this kind of particle-laden turbulence, little is known about Lagrangian nature of inertial particles in near-wall turbulence.

Several investigators have reported Lagrangian statistics of fluid or inertial particles by releasing and tracking them in numerically simulated turbulent flows using interpolation schemes. Wang *et al.* (1995) performed large eddy simulation (LES) to obtain single-particle Lagrangian velocity autocorrelations and mean-square dispersion of fluid particles in turbulent channel flow. Choi *et al.* (2004) carried out direct numerical simulation (DNS) of turbulent channel flow and tracked fluid particles to show Lagrangian velocity and acceleration autocorrelations, velocity structure function and fluid particle dispersion.

Uijttewaal & Oliemans (1996) used LES of turbulent pipe flow to show Lagrangian velocity autocorrelations and squared particle displacement of inertial particles. Zhang & Ahmadi (2000) using DNS reported particle trajectory statistics for inertial particles in vertical and horizontal turbulent channel flows. Iliopoulos *et al.* (2003) calculated results for inertial particles using DNS and compared them with a stochastic model employing a modified Langevin equation to obtain the fluid turbulence at the position of inertial particles in horizontal channel flow.

In this study, we investigate Lagrangian properties of inertial particles by ensemble-averaging over a number of individual trajectories released at specific wall-normal locations in turbulent channel flow for a wide range of Stokes number. Furthermore, we present the effect of turbulence modification by particles on their Lagrangian statistics. A sufficient mass loading of particles in turbulence leads to significant turbulence modification (Balachandar & Eaton, 2010).

TURBULENT CHANNEL FLOW SIMULATION

The governing equations for incompressible flow are given by

$$\frac{\partial u_i}{\partial x_i} = 0, \tag{1}$$

$$\frac{Du_i}{Dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x_j} + \nu \frac{\partial^2 u_i}{\partial x_j \partial x_j} + \frac{1}{\rho} f_i, \tag{2}$$

where x_1, x_2 and x_3 indicate, respectively, the streamwise (x), wall-normal (y) and spanwise (z) directions and u_i is the fluid velocity in the x_i direction. p is the fluid pressure, ρ is the fluid density and ν is the kinematic viscosity. f_i represents particle reaction to the fluid in the x_i direction as

$$f_i(\vec{x}) = -\sum_k \frac{w(\vec{x} - (\vec{q})_k)}{\Delta V_k} (D_i)_k, \tag{3}$$

Table 1. Particle parameters for one-way coupling

St^+	d_p^+	ρ_p/ρ	$N_p{}^a$
0.0^b	0.0	0.0	1577848
0.2	0.02216	7333	1577848
1	0.04954	7333	1577848
5	0.11078	7333	1577848
25	0.24772	7333	1577848
125	0.55392	7333	1577848
625	1.23861	7333	1577848

[a] N_p is the number of particles simulated in each case.
[b] St=0.0 means fluid particles.

in which ΔV_k is the volume of the computational cell including the k-th particle and $(D_i)_k$ is the drag force acting on the particle in the x_i direction. According to the action-reaction principle, this force is exerted back on the fluid at the nearby eight grid points via the weight function w based on linear interpolation from the particle position $(\vec{q})_k$. This approach has been employed by others when particle size is smaller than the Kolmogorov length scale of the flow. At first, we assume that $f_i = 0$ in order to focus on the pure effect of turbulence on Lagrangian statistics of inertial particles. Next we consider the effect of turbulence modification due to the particle reaction force.

We perform direct numerical simulation of turbulent channel flow at a shear Reynolds number of $Re_\tau = 180$ by using a pseudo-spectral method to solve the above equations. Time advancing was performed using the third-order Runge-Kutta and Crank-Nicolson schemes for the nonlinear and viscous terms, respectively. For the homogeneous directions, periodic conditions were employed. At the walls, no-slip and impermeability conditions were used. The domain size is $4\pi\delta \times 2\delta \times 2\pi\delta$ in the streamwise, wall-normal and spanwise directions, where δ is the channel half width, and, correspondingly, $128 \times 129 \times 128$ grid points are used to resolve the flow domain.

LAGRANGIAN PARTICLE TRACKING

The particles simulated are solid spheres, smaller than the Kolmogorov length scale of the flow (i.e. $\rho_p/\rho \gg 1$ and $d_p < \eta$, ρ_p, d_p and η are the particle density, particle diameter and the Kolmogorov length scale of the flow, respectively). Thus, the Stokes drag force is most significant compared to other forces such as the pressure gradient, added mass and Basset forces (Armenio & Fiorotto, 2001). Furthermore, in the absence of gravity, the wall-normal lift force can be neglected (Arcen *et al.*, 2006). Therefore, the equation of particle motion can be

$$\frac{dq_i}{dt} = v_i, \tag{4}$$

$$D_i = m_p \frac{dv_i}{dt} = m_p \frac{1 + 0.15 Re_p^{0.687}}{\tau_p} \left(u_i(\vec{q},t) - v_i\right), \tag{5}$$

in which q_i and v_i are, respectively, the particle position and particle velocity in the x_i direction, m_p is the particle mass, $Re_p = |\vec{u} - \vec{v}| d_p/\nu$ is the particle Reynolds number and $\tau_p = d_p^2 \rho_p/(18\rho\nu)$ is the particle response time scale. In order to obtain the fluid velocity at particle position $\vec{u}(\vec{q},t)$, the four-point Hermite interpolation scheme in the streamwise and spanwise directions and the fifth-order Lagrange polynomial interpolation in the wall-normal direction were used. Time advancement for Lagrangian particle tracking was performed concurrently with that for the Navier-Stokes equation, using the third-order Runge-Kutta scheme.

Initially (i.e. at $t = 0$), particles are randomly scattered over the flow domain of fully developed channel turbulence, with the initial velocity identical to the interpolated fluid velocity at their position. For particles moving outside the computational domain, periodic boundary conditions are applied in the streamwise and spanwise directions. It is assumed that particles hit the wall without any energy loss and particle-particle collisions are negligible.

RESULTS AND DISCUSSION

In this study, we consider a wide range of Stokes numbers of $St^+ = 0.2, 1, 5, 25, 125$ and 625, where St^+ is the Stokes number based on wall units. Hereafter, the superscript $+$ indicates the quantities normalized by wall units of the flow. Detailed parameters are summarized in Tables 1 and 2.

Since the particle initial velocity is given artificially, it necessarily takes time for particles to reach their natural velocity. In order to obtain Lagrangian statistics of particles without this numerical artifact, we discard results up to 1250 wall time units. Note that 1250 wall time units are larger than the particle response time scales considered (i.e. at least two times larger than the largest particle time

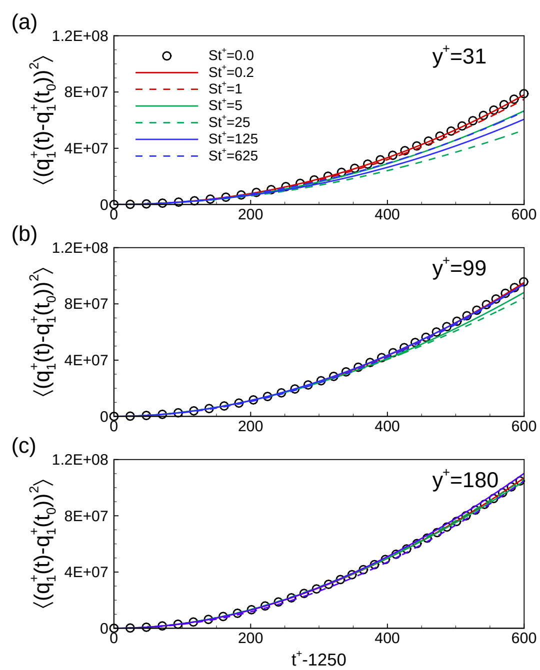

Figure 1. Particle dispersion in the streamwise direction for particles starting from the (x, z) plane at (a) $y^+ = 31$, (b) $y^+ = 99$ and (c) $y^+ = 180$ when $t^+ = 1250$.

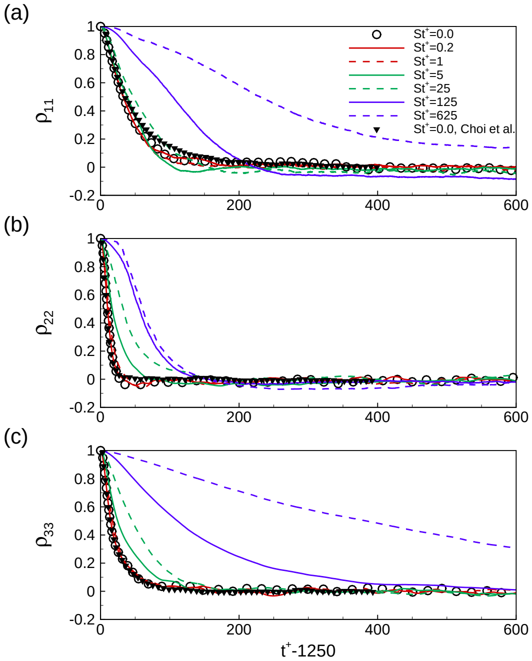

Figure 2. Lagrangian velocity autocorrelations of particles starting from the (x,z) plane at $y^+ = 31$. (a) Streamwise, (b) wall-normal and (c) spanwise component.

scale), and thus, the individual particle can adjust locally to the surrounding fluid before $t^+ = 1250$. When $t^+ = 1250$, we mark particles that lie within thin (x,z) slabs at each of three different locations of $y^+ \approx 5, 31, 99$ and 180 (i.e. the channel centerline) and then ensemble-average individual trajectories of the marked particles during a period of 600 wall time units.

Particle Lagrangian statistics In this section, we consider the case of $f_i = 0$ (i.e. one-way coupling) to investigate the pure effect of turbulence on particle Lagrangian statistics. Table 1 shows the particle parameters simulated in this section. Here, we consider copper particles suspended in air turbulence (i.e. $\rho_p/\rho = 7333$).

Figure 1 shows the streamwise dispersion of the particles starting at $y^+ = 31, 99$ and 180 (the channel centerline). The effect of Stokes number on the particle dispersion becomes pronounced as the wall is approached due to the presence of near-wall streamwise vortices which are responsible for turbophoretic drift of particles towards the wall (Marchioli & Soldati, 2002). Therefore, the dispersion for the particles starting at $y^+ = 31$ decreases compared to the dispersion of Lagrangian fluid particles (i.e. the case of $St^+ = 0.0$). In the Stokes number range considered in this study, particles with $St^+ = 25$ maximally accumulate towards the wall, as will be shown later, consistent with the previous observations by Marchioli *et al.* (2007) (e.g. they observed near-wall particle accumulation for a similar Stokes number range in zero-gravity and vertical turbulent channel flows). Therefore, the dispersion suppression is most significant for $St^+ = 25$. It should be noted that although the results from Fig. 1 are useful, they are limited only up to 600 wall time units. This time is too short for the distribution of the particles starting at a certain wall-normal location to be fully developed. It may be informative to investigate late-time behavior of the particle dispersion.

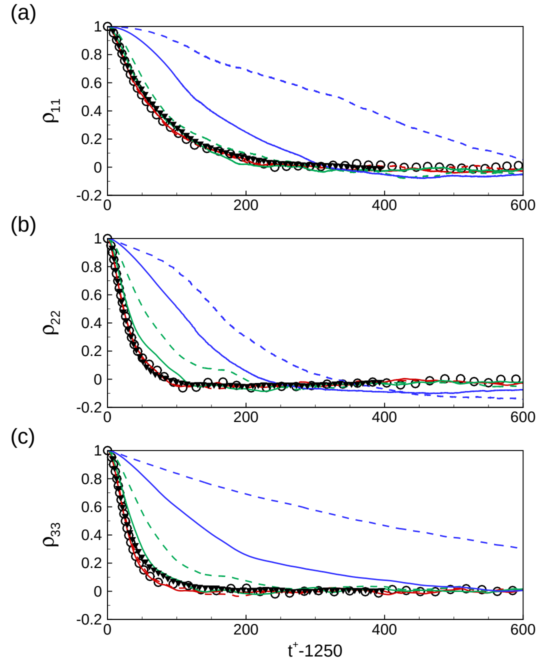

Figure 3. Lagrangian velocity autocorrelations of particles starting from the (x,z) plane at $y^+ = 99$. (a) Streamwise, (b) wall-normal and (c) spanwise component. Lines and symbols are the same as in Fig. 2.

Figures 2 and 3 show Lagrangian velocity autocorrelations of the particles starting at $y^+ = 31$ and 99, respectively. For comparison purpose, the previous results at $Re_\tau = 400$ by Choi *et al.* (2004) are also displayed in Figs. 2 and 3. Although our Reynold number is much smaller (i.e. $Re_\tau = 180$), the current results are in agreement with those by Choi *et al.* The Lagrangian particle velocity autocorrelations decay before $t^+ = 600$, except for the largest Stokes number case. Particles with $St^+ = 0.2$ and 1 almost follow the fluid velocity autocorrelation functions. As the Stokes number increases further, the Lagrangian particle velocity autocorrelations slowly decay compared to the fluid, since particles with larger Stokes numbers tend to maintain their velocity for a longer time. However, the decaying rate of the wall-normal velocity fluctuation for particles with $St^+ = 625$ does not decrease significantly compared to $St^+ = 125$ when they start at $y^+ = 31$, but ρ_{22} for $St^+ = 625$ rather decays faster than $St^+ = 25$ and 125 when particles

Table 2. Particle parameters for two-way coupling

St^+	d_p^+	ρ_p/ρ	mass loading	N_p
25	0.24772	7333	0%[a]	1577848
25	0.24772	7333	10%	1577848
25	0.49544	1833	20%	1577848
25	0.99088	458	40%	1577848

[a] 0% mass loading indicates one-way coupling simulation.

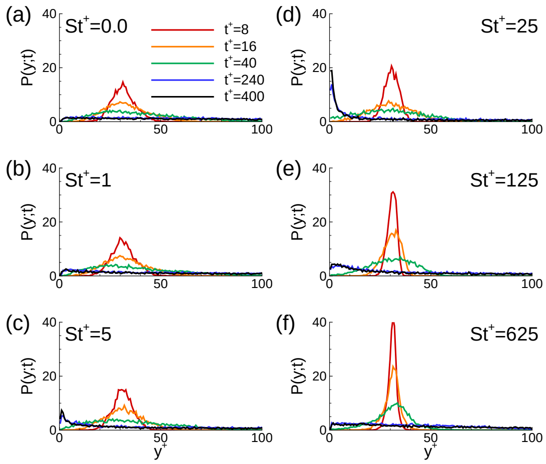

Figure 4. Time evolution of probability density functions of particles starting from the (x,z) plane at $y^+ = 31$.

start at $y^+ = 5$ (although not shown here). This may be due to particle-wall interaction. On the other hand, far from the wall, the increasing Stokes number appears to result in isotropy between three components of the autocorrelation since particles with larger Stokes numbers filter out the turbulent fluctuations (Fig. 3).

Figure 4 shows the time evolution of probability density functions of the wall-normal locations of the particles starting at $y^+ = 31$. Near-wall accumulation of particles due to turbophoresis is most pronounced for $St^+ = 25$, consistent with Fig. 1. It appears that as the Stokes number increases, particles slowly disperse from their starting position.

Effects of turbulence modification due to particles

In this section, we investigate the effect of turbulence modification by particles (i.e. two-way coupling) on particle Lagrangian statistics. We consider three different mass loadings of 0.1, 0.2 and 0.4. In all cases simulated, the number of particles is fixed at $N_p = 1577848$. Therefore, d_p increases and ρ_p/ρ decreases with increasing mass loading. However, this does not violate the assumption that $\rho_p/\rho \gg 1$ and $d_p < \eta$ within the mass loading range considered in this section. Detailed parameters for two-way coupling are summarized in Table 2.

It has been known that particles with an intermediate Stokes number ($St^+ = 25$) efficiently modify the surrounding turbulence (Zhao *et al.*, 2010; Lee & Lee, 2015). For example, they attenuate streamwise vortices and small scales related with high- and low-speed streaks near the wall. This effect is expected to affect Lagrangian statistics of inertial particles.

In Fig. 5, the effect of turbulence modification by particles is to reduce the decaying rate of the Lagrangian particle velocity autocorrelations. This becomes significant with increasing mass loading and most pronounced for the streamwise component.

In Fig. 6, the turbulence modification reduces turbophoresis, consistent with the previous observation (Nasr *et al.*, 2009). Furthermore, the particles starting at $y^+ = 31$ slowly disperse as the mass loading increases although the Stokes number based on the Kolmogorov time scale of the flow is slightly reduced due to the turbulence modification.

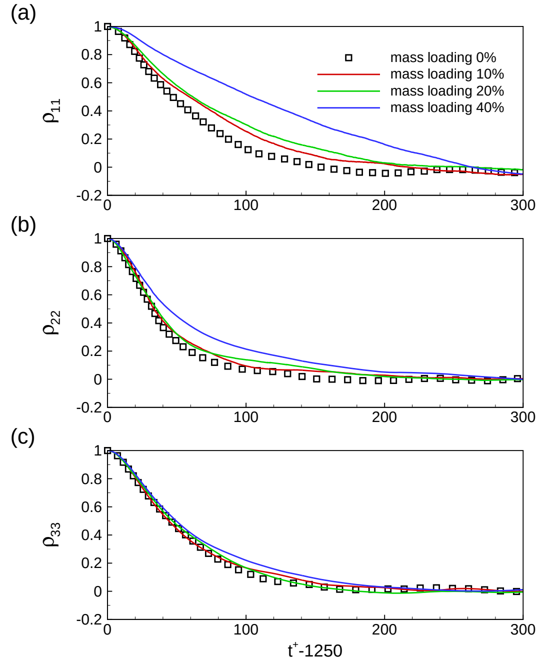

Figure 5. Lagrangian velocity autocorrelations of particles with $St^+ = 25$ starting from the (x,z) plane at $y^+ = 31$. (a) Streamwise, (b) wall-normal and (c) spanwise component.

CONCLUSIONS

In this study, we investigated Lagrangian statistics of inertial particles starting from different wall-normal locations for a wide range of Stokes number, using direct numerical simulation of turbulent channel flow with Lagrangian particle tracking. It is shown that the streamwise dispersion of particles decreases due to turbophoresis. As the Stokes number increases, the Lagrangian particle velocity autocorrelation increases, except for the wall-normal component near the wall due to particle-wall interaction. The time evolution of probability density function of particle locations shows that particles with larger Stokes numbers slowly disperse in the wall-normal direction.

We also investigated the effect of turbulence modifica-

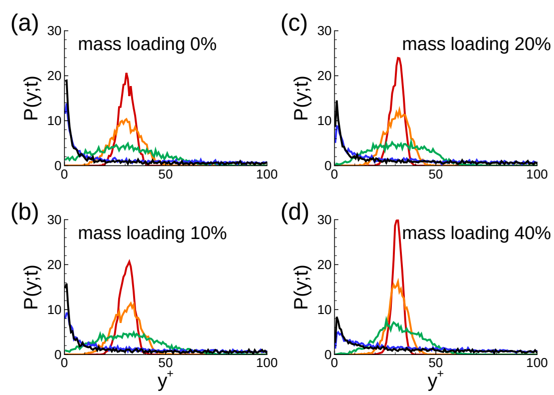

Figure 6. Time evolution of probability density functions of particles starting with $St^+ = 25$ from the (x,z) plane at $y^+ = 31$. (a) mass loading 0% (one-way coupling), (b) 10%, (c) 20% and (d) 40%. Lines are the same as in Fig. 4.

tion due to particle reaction force. The Lagrangian particle velocity autocorrelations slowly decay due to the suppression of turbulence. This is most pronounced for the streamwise component. Turbophoresis is reduced and particles slowly disperse in the wall-normal direction due to the turbulence modification.

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