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EXPLICIT REINITIALIZATION AND EXTENDING ALGORITHMS FOR LEVEL-SET BASED SHARP-INTERFACE METHOD

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ABSTRACT

We propose explicit reinitialization and extending algorithms, which are critical for the level-set based interface-tracking methodology. By introducing Wendland function, the level-set field is reinitialized with a one-step "forward tracing" algorithm. Furthermore, the fluid states are extrapolated or extended across the interface through a one-step "backward tracing" algorithm. Both algorithms, which can be regarded as variations of the ray-tracing technique, avoid multiple boundary information exchanges that are necessary for previous iterative reinitialization and extending approaches within multi-block framework. This property is crucial for efficient large-scale simulations on distributed-memory clusters. The new algorithms, in combining with the multi-resolution conservative sharp-interface method, have been validated by an extensive benchmark study. Demonstrated accuracy and robustness suggest that the proposed algorithms offer a highly efficient alternative to previous reinitialization and extending approaches in the level-set methodology.

Introduction

Reinitialization serves to recover the signed-distance property of a transported level-set function. Sussman *et al.* (1994) develops Eikonal equation for this purpose. The signed-distance property is approximately satisfied in terms of an iterative procedure. Generally the Reinitialization equation is solved on Eulerian grid leading to many boundary-information exchanges within a multi-block framework and therefore inevitably affects parallel efficiency. Sethian (1996) proposes the "Fast Marching Methods" for efficient solution of the static Hamilton-Jacobi equations. However, "Fast Marching Methods" implies data dependencies when the procedure loops from "close" to "far" cells. Such a sequential order is inefficient on distributed-memory architecture Wright *et al.* (1996). Adalsteinsson & Sethian (1995) present the so called "Narrow band" technique which greatly decreases the computational cost while maintaining accuracy. As all the mentioned methods involve the solution of a transport equation, for algorithms based on them, it is hard to obtain high efficiency on distributed-memory architecture. Fortmeier & Bcker (2011) recently develops a parallel reinitialization method on distributed unstructured tetrahedral grids. The technique still has global data dependency when searching for the minimum distance function. Cho *et al.* (2011) proposes a direct reinitialization approach for incompressible two-phase flow. The calculations involve the determination of all end points of interface segments and distances from the target cells to these end points. The direct computation of minimum distance function becomes extremely complex when this approach is extended to three dimensions.

When the computational domain is separated by the interface represented by zero level-set function, unknown ghost quantities on either side of the interface should be extrapolated from the real fluid properties. This extrapolation plays pivotal role for different ghost-fluid methods. Chang *et al.* (2013) intends to avoid the definition of such ghost cell values by introducing a reconstruction procedure near the interface. This method is prohibitively difficult to extend to three dimensions and may suffer from numerical instabilities. Fedkiw *et al.* (1999*a*) defines them from a constant extrapolation by solving a transport equation. Aslam (2003) further formulates this method to achieve higher order of polynomial extrapolation. The sharp interface model Hu *et al.* (2006) also exploits constant extrapolation. Ghost cell quantities provide the interpolation stencil for reconstruction near interface and allow for solving the interface condition. However, the transport equation must be solved

iteratively which implies similar drawbacks as reinitialization procedure with respect to parallel computation. According to our knowledge, the resolution of this problem is yet open.

In this paper, we propose an approach which belongs to the general type of semi-Lagrangian methods to complete the reinitialization and extending procedures in one step. The methods involve mainly simple geometrical algebraic calculations but recovers the solution accurately. Reinitialization is achieved by a "forward tracing" method. Extending of primitive variables to the other side of the interface is accomplished by a "backward tracing" method. These methods have three main advantages: (i) no iterations are needed; (ii) all the data operations are local on a computational-block packet including boundary ghost-cell information; (iii) extension to three-dimensions is straightforward and fairly simple. These properties eliminate the drawbacks of previous methods and enable large scale simulations on a distributed-memory architecture with high parallel efficiency.

Basic numerical methods

Provided that the fluid is inviscid and compressible neglecting the viscous, gravity and surface tension effects, the hyperbolic systems of conservation laws for single phase can be written as follows

$$\frac{\partial \mathbf{U}}{\partial t} + \nabla \cdot \mathbf{F} = 0, \tag{1}$$

where $\mathbf{U} = [\rho, \rho u, \rho v, E]^T$ represents the density of conservative variables of mass, momentum and total energy, $E = \rho e + \frac{1}{2}\rho(u^2 + v^2)$, $\mathbf{F}$ denotes the physical flux function of $\mathbf{U}$. This set of equations is closed by equations of state (EOS) appropriate to the particular type of fluid. Here, we consider the ideal-gas equation of state and the Tait's equation Fedkiw *et al.* (1999*a*) for gas and water phase respectively.

Considering two-phase fluid simulation, the computational domain Ω is separated by an interface Γ into Ω^1 and Ω^2. Following the sharp interface method proposed by Hu et al. Hu *et al.* (2006), each phase can be discretized individually. Discrete conservation is achieved by careful treatment of the cut cell.

In order to compute the volume fractions and cell-face apertures, we first define the interface. The level-set technique is adopted due to its sharp inherent representation and implicit handling of interface topology. A signed-distance function ϕ is established and the $\phi = 0$ contour represents the interface Γ

$$\Gamma = \{(x,y) | \phi(x,y,t) = 0\}. \tag{2}$$

The entire domain is divided into three parts: $\phi > 0$ represents the positive phase, $\phi < 0$ represents the negative phase and the interface Γ. Furthermore, interface characteristic parameters such as interface normal direction $\mathbf{N}$ and curvature κ can be calculated directly by finite-difference approximations.

The time evolution of ϕ corresponding to the adection of interface Γ is described by

$$\phi_t + \mathbf{v} \cdot \nabla \phi = 0. \tag{3}$$

This equation can be solved on the same Cartesian mesh as fluid evolution and by similar high order discretization such as fifth-order WENO scheme. The transport velocity $\mathbf{v}$ in Eq. (3) can be chosen in many different ways. As mentioned in Hu & Khoo (2004), it can be obtained from the interaction of real and ghost fluid through solving a Riemann problem. This method avoids spurious pressure oscillations and advects the level-set field smoothly and stably.

The level-set ϕ field evolution by Eq. (3) may lose the signed-distance function property. In order to recover this property, a reinitialization equation Sussman *et al.* (1994)

$$\phi_\tau + sgn(\phi)(|\nabla \phi| - 1) = 0, \tag{4}$$

where τ represents pseudo time and $sgn(\phi)$ represents sign function, is solved to steady state after certain physical time intervals. In current implementation, this procedure is performed at each time step right after completing all Runge-kutta substeps. The stable discretization of Eq. (4) is not straightforward.

In sharp-interface methods based on level-set technique Fedkiw *et al.* (1999*b*)Hu *et al.* (2006), the fluid state for each fluid phase is extrapolated to the other side of interface by solving the extending equation

$$q_\tau \pm \mathbf{N} \cdot \nabla q = 0 \tag{5}$$

within the narrow band. Here, $+\mathbf{N}$ is used to extend quantities from sub-domain of $\phi < 0$ to sub-domain of $\phi > 0$; while $-\mathbf{N}$ is utilized along the opposite direction.

Numerical results

Interface deformed by a single vortex

The two-dimensional version of this case is taken from Bell et al. Bell *et al.* (1989) and has ever been widely used to validate interface advection and reinitialization algorithms So *et al.* (2011).

The computational domain is [0, 1]×[0, 1]. A circular interface with radius 0.15 is initialized at (0.5, 0.75). The interface is stretched by the vortex velocity field and develops into long filaments around the vortex center. With application of the reversed velocity field since $t = 3$, it should recover to the original disk shape at $t = 6$. We run this case at two resolutions: 320×320 and 640×640. As shown in Fig.1, the present reinitialization method leads to slightly better resolution of the small structure and less mass loss than the standard iterative procedures used in Han *et al.* (2014).

In the three-dimensional version of the case LeVeque (1996)Hieber & Koumoutsakos (2005)Enright *et al.* (2005)Chenadec & Pitsch (2013), the computational domain is [0, 1]×[0, 1]×[0, 1]. A sphere with radius 0.15 is initially placed at (0.35, 0.35, 0.35). It is deformed by the velocity field above and develops into a thin film at $t = 1$. After that the reversed velocity field is imposed to check the recovery of the initial sphere at $t = 2$. A uniform mesh with resolution 320×320×320 is used. As shown in Fig. 2, both the present algorithm and the iterative method resolve the thin interface quite well. Again the present method reveals a slightly delayed rupture and slightly better resolution of small structures.

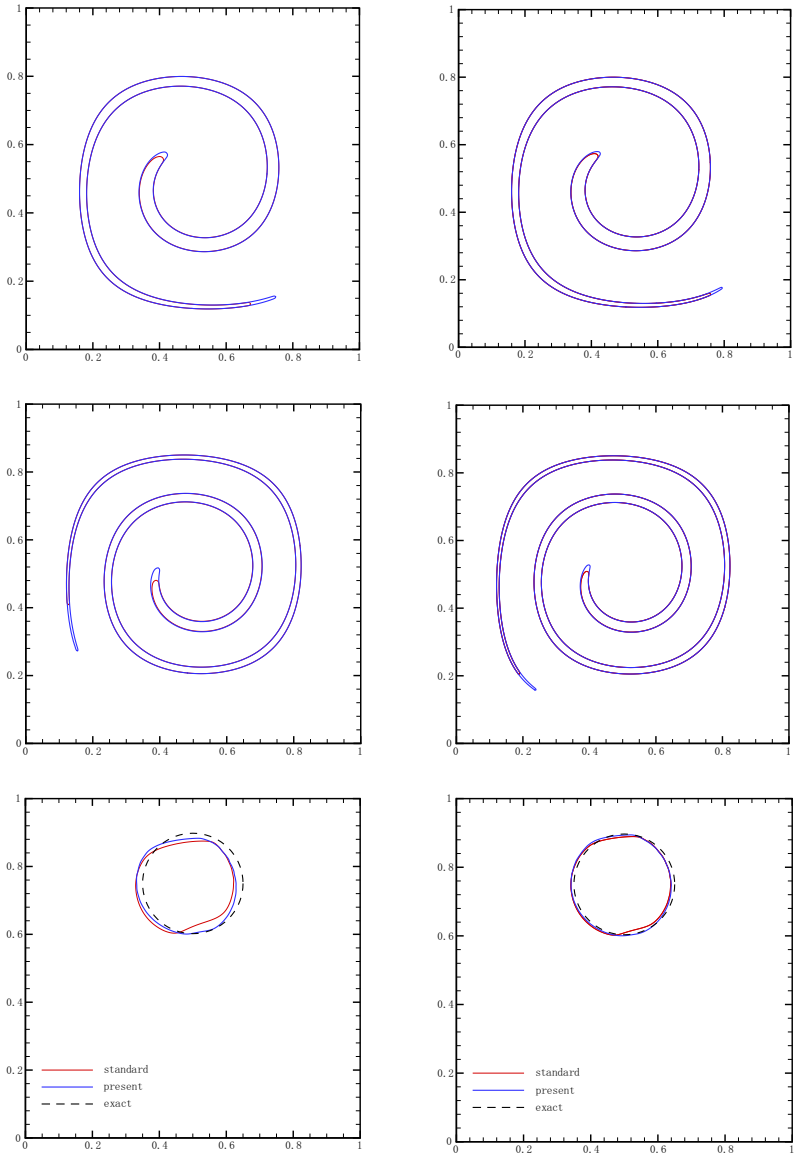


Figure 1. Interface shape: 320×320 (left) and 640×640 (right)

Underwater explosion

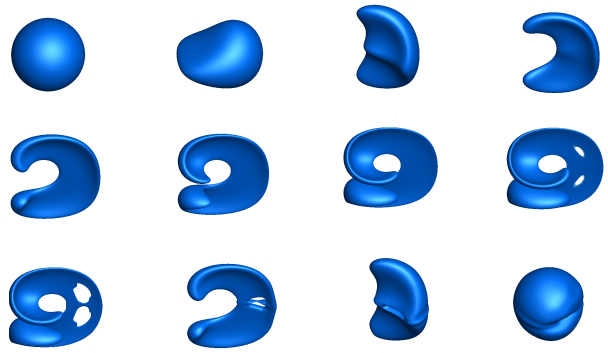
Here, we simulate a two-dimensional problem in which the interface remains smooth. This case has been investigated in Hu *et al.* (2009)Han *et al.* (2014). The simulation setup is the same as in Hu *et al.* (2009)

Reference density and pressure are $1000kg/m^3$ and $1atmosphere$ respectively. The computational domain is [0, 4]×[0, 4]. There are two gas water interfaces: air-water interface located at $y = 1.5$ and a circular gas bubble immersed in the water at $(2.0, 1.2)$ with the radius 0.12. We set maximum refinement level of 6 for the multi-resolution procedure corresponding to an effective resolution 1024×1024. Fig. 3 shows the Schlieren-type images of density gradient $|\nabla\rho|$ from 0.2ms to 2.0ms. It is observed that the result from present methods is almost the same as in Han *et al.* (2014) (their Fig. 24). As indicated in Fig. 4 which shows a contour line of the level-set field at t = 2.0ms, the present reinitialization method has reproduced very smooth level-set field.

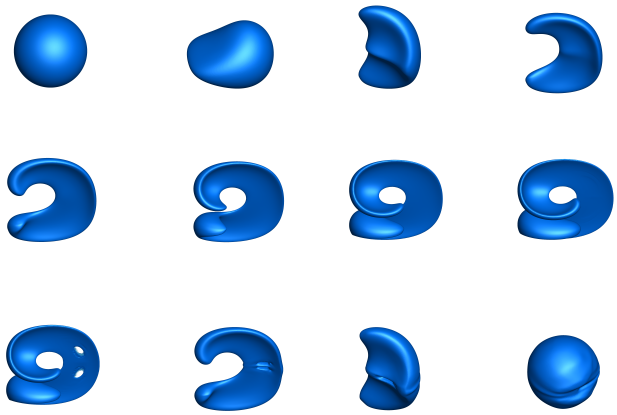
Shock and double water-columns interaction

In this case, a Mach 6 shock hits a double water-columns in gas. This case exhibits complex interface evolution and strong shock water-column interaction. It has been investigated in Hu *et al.* (2009) for method validation. Here, we adopt the exact setting as Han *et al.* (2014) for the convenience of comparisons.

Reference density and pressure are $1kg/m^3$ and $1atmosphere$ respectively. The computational domain is [0, 2.8]×[0, 2.8]. Two water-columns: one located at (1.5, 1.0) with radius 0.25 and the other located at (0.8, 1.4) with radius 0.32 are settled in pre-shocked air. The effective resolution is 2048×2048 at the finest level. Fig. 5 and Fig. 6 show the Schlieren-type images of density gradient $|\nabla\rho|$ and the interface shape evolution from simulation time 3 μs to 24 μs . It can be observed that the overall results agree very well with that of Han *et al.* (2014) (their Fig. 23). Specially, at the early time stage, the fluid details are identical to that in Han *et al.* (2014) as the interface is considerably smooth. On the other hand, at late time stage, while the s-



(a) by solving transport equation



(b) by the present method

Figure 2. Interface shape at t = 0.00, 0.20, 0.40, 0.53, 0.67, 0.80, 0.93, 1.00, 1.07, 1.33, 1.60, 2.00

mall interface structures obtained by the present method are different from those in Han *et al.* (2014) due to the inviscid model, the large interface structures are however in quite good agreement. Even the "Karman Vortex Street" at 24μ is resolved indicating our numerical results are physically reasonable. The density values of ghost cells for positive phase at 9 μs are illustrated in Fig. 7. Our new extending procedure results in smooth distributions of primitive variables and achieves the intended constant extrapolation.

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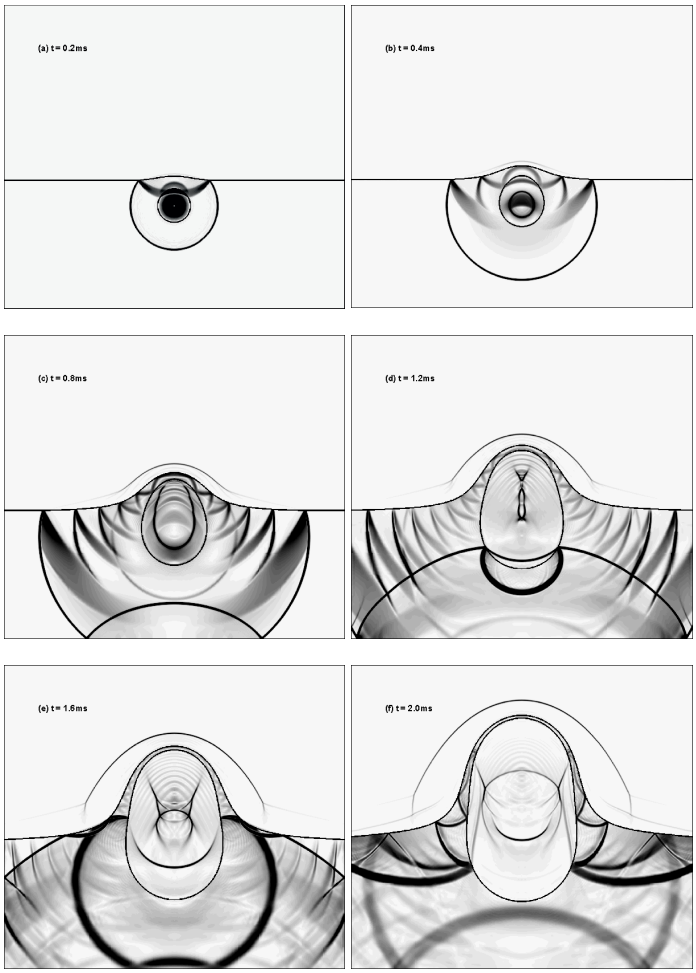

Figure 3. Underwater explosion: schlieren-type images of density gradient $|\nabla\rho|$.

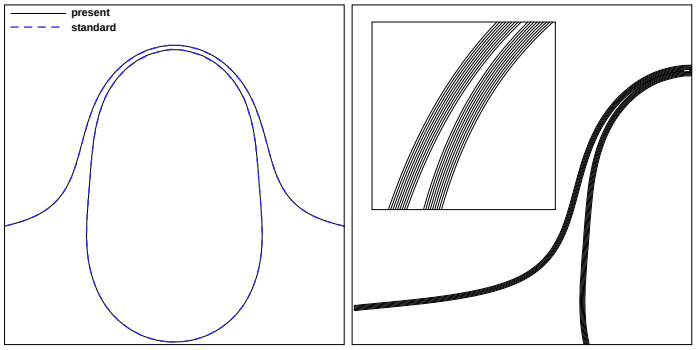

Figure 4. Details of level-set field at 2.0ms: interface position (left figure) and contour lines of narrow band near the interface (right figure).

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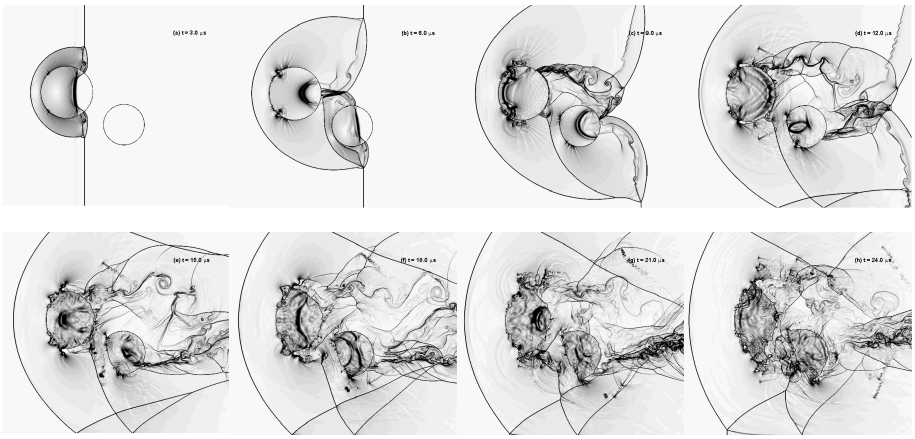

Figure 5. Shock and double water-columns interaction: schlieren-type images of density gradient $|\nabla\rho|$ from simulation time 3 μs to 24 μs .

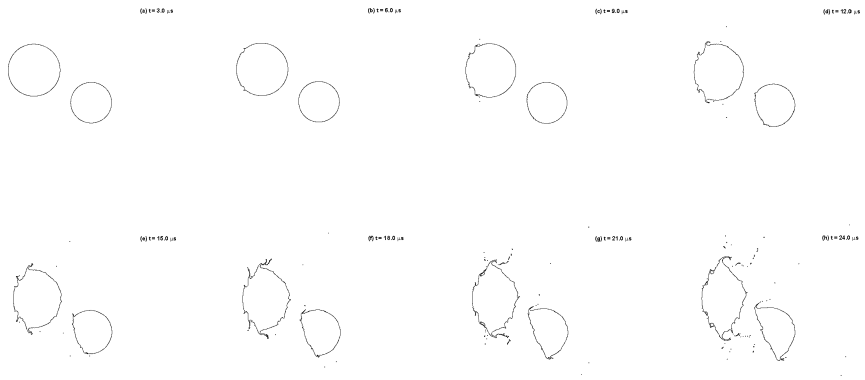

Figure 6. Shock and double water-columns interaction: interface shape evolution from simulation time 3 μs to 24 μs .

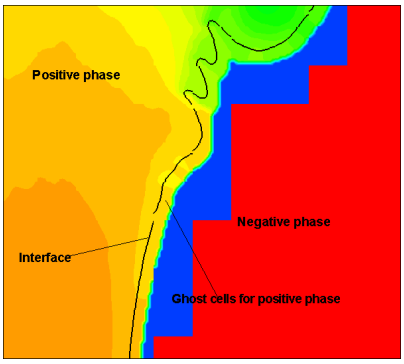

Figure 7. Density values of ghost cells for positive phase at $9\mu s$. The blue region is left to the initialized values as the cells are more than four-cell away from the interface, thus not critical for the positive phase. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

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