MULTISCALE ANALYSIS OF TURBULENT VELOCITY FIELD BY USING SCALE-DECOMPOSITION METHOD BASED ON A SECOND GENERATION WAVELETS

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ABSTRACT

To understand the multiscale coherent structures of the turbulent velocity fields, a scale-decomposition method based on a second generation wavelets using lifting scheme is promoted. It is used to analyse the experimental time-series velocity of a homogeneous turbulence at $Re_\lambda = 720$, velocity of a zero-pressure-gradient boundary layer at $Re_\theta = 7705$ and velocity signals from direct numerical simulation (DNS) of a turbulent channel flow at $Re_\tau = 206$. Each scale velocity component contributes to the total spectra at its corresponding frequency range.It is found that at the largest scale, the scale velocities at different wall distance have very large correlation coefficients at different time delays. Under the similarity assumption, a reconstruction model is promoted to rebuild fine scale velocity for homogeneous turbulence by using its neighbour larger scale velocities. The reconstructed scale velocity component has correct energy spectra compared with that of experimental one.

INTRODUCTION

Turbulent flow consists of self-similar structures with a wide range of length scales. Multi-scale decomposition has gained increasing interest in the turbulence research, and has been proven to be useful for understanding the evolution of eddies and the interaction between turbulent flow structures at different scales.

The classical Fourier spectrum gives the energy distribution of a signal in a frequency domain and is evaluated over the entire time interval, which losses the localization of transient features and spatial information (Domaradzki & Liu (1993)) and is limited to periodic boundary problem. HuangY.X. & F.G. (2007) proposed an Empirical Mode Decomposition (EMD) method, and introduced a concept of Hilbert spectrum. However, EMD cannot correctly represent the instantaneous frequency of an intrinsic mode function. And the Hilbert spectrum has no obvious physical meaning. Meneveau (1991) used continuous wavelets transformation to analyse turbulent flow. In recent years, wavelets have found increasing use for scale decomposition investigation. Classical construction of orthogonal or biorthogonal wavelets on the infinite real line is based on the Fourier transform (FT) and is carried out in the frequency domain. This introduces considerable constraints on the implementation of the wavelets for practical flow analysis.

The second generation wavelets constructed by using lifting scheme (Sweldens (1996)) does not resort to the FT, and hence, the derived basis functions are not necessarily translations and dilations of a mother function, which makes it suitable for problems defined in bounded domains, analysis of data on curves or surfaces, weighted approximations, and irregular grids. Additional important benefits are the fast implementation, which is fully in-place calculation, and perfect reconstruction. Therefore, in this investigation, based on the second generation wavelets, a scale decomposition method achieve a hierarchical decomposition of the turbulent velocity field in the scale space. Under the scale similarity assumption, a fine scale reconstruction model is proposed.

SCALE DECOMPOSITION METHOD AND RECONSTRUCTION MODEL

Scale Decomposition Method

The lifting scheme is independent of FT and has three stages: split, predict, and update. An original signal $\lambda_{0,k}$ can be splitted into two subsets:

$$(\lambda_{0,2k}, \lambda_{0,2k+1}) = split(\lambda_{0,k}) \quad (1)$$

and let $\lambda_{-1,k} = \lambda_{0,2k}$. The wavelet coefficients $\gamma_{-1,k}$ are obtained in the prediction step as:

$$\gamma_{-1,k} = \lambda_{0,2k+1} - P(\lambda_{0,2k}). \quad (2)$$

where P is a prediction operator. Finally, scaling coefficients $\lambda_{-1,k}$ are obtained in the update step as:

$$\lambda_{-1,k} = \lambda_{-1,k} + U(\gamma_{-1,k}). \quad (3)$$

where U is an update operator. Then repeat this process to get wavelet and scaling coefficients at large spatial scales. If n levels of scale are decomposed and denote the forward second generation wavelets transformation as 'SWT', then the n times forward transformation can be represented as:

$$(\lambda_{-n,k}, \gamma_{-j,k}) = SWT^n(\lambda_{0,k}), (j = 1 \cdots n, k \in Z). \quad (4)$$

The scale decomposition is realized by a layer inverse transformation. The low frequency residual s_{-n} having the same signal length in physical space is calculated from inverse transformation of scaling coefficients $\lambda_{-n,k}$:

$$s_{-n} = SWT^{-n}(\lambda_{-n,k}). \quad (5)$$

And a high frequency detail d_{-j} at level j is obtained by inverse transformation of corresponding wavelets coefficients $\gamma_{-j,k}$:

$$d_{-j} = SWT^{-n}(\gamma_{-j,k}), j = 1 \cdots n. \quad (6)$$

Finally the original signal can be perfectly reconstructed as:

$$\lambda_{0,2k} = s_{-n} + \sum_{j=1}^{n} d_{-j} = SWT^{-n}(\lambda_{-n,k}, \gamma_{-j,k}), j = 1 \cdots n. \quad (7)$$

Reconstruction Model

Based on the view of energy cascade and scale similarity hypothesis for homogeneous turbulence, the finer wavelet coefficients $\lambda_{-j,k}$ at level j can be expected as a function of all the coefficients levels as:

$$\gamma_{-j,k} = f(\lambda_{-n,k}, \gamma_{-j-1,k} \cdots \gamma_{-n,k}), \quad (8)$$

where f is an unknown function. In the consideration of localization of fine scales, it is supposed that the predicted wavelet coefficients $\gamma^p_{-j,k}$ by a model are just linear function of its neighbour coarse level coefficients $\gamma_{-j-1,k}$ and $\gamma_{-j-2,k}$. Therefore, a wavelet coefficient at time t_l can be predicted by using its two neighbours γ_{-j-1,t_m} and γ_{-j-2,t_n} in a time range feom t_m to t_n as:

$$\gamma^p_{-j,t_l} = \frac{t_n - t_l}{t_n - t_m} * \gamma_{-j-1,t_m} + \frac{t_l - t_m}{t_n - t_m} * \gamma_{-j-2,t_n}. \quad (9)$$

where suppose $t_m < t_l < t_n$. Then the predicted high frequency detail d^p_{-j} can be obtained by inverse transformation of $\gamma^p_{-j,k}$ as:

$$d^p_{-j} = SWT^{-n}(\gamma^p_{-j,k}), j = 1 \cdots n. \quad (10)$$

Using this model, a higher frequency detail $\gamma^p_{0,2k}$ can be predicted and reconstructed from two highest frequency details $\gamma^p_{-1,k}$ and $\gamma^p_{-2,k}$. Therefore, a new turbulent velocity signal $\lambda^p_{1,2k}$ using $\gamma^p_{0,k}$ and $\lambda_{0,k}$ can be built . As $\lambda^p_{1,2k}$ is two times of length of $\lambda_{0,k}$, hence, the length of $\lambda_{0,k}$, $\gamma_{-1,k}$ and $\gamma^p_{-2,k}$ should be doubled. Then the finest detail $\gamma^p_{0,2k}$ is predicted as:

$$\gamma^p_{0,2k} = f(\gamma_{-1,k}, \gamma_{-2,k}). \quad (11)$$

Finally, the new fine turbulent velocity signal can be obtained as:

$$\lambda^p_{1,2k} = SWT^{-1}(\gamma^p_{0,2k}, \lambda^p_{0,2k}). \quad (12)$$

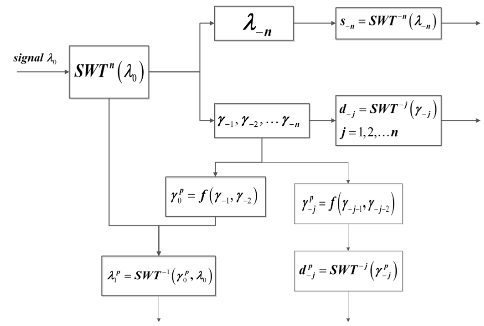

Figure 1. Flow chart of scale decomposition method and reconstruction.

This procedure is outlined in Figure 1, which can be easily extended to two dimension. The only required input is an original signal. All the high frequency detail d_{-j}, low frequency residual s_{-n}, predicted high frequency detail d_{-j} and new fine turbulent velocity signal can be obtained at a time.

RESULTS AND DISCUSSIONS

Homogeneous turbulence

An original data is obtained from the measurements of isentropic grid turbulence at Taylor Reynolds number $Re_\lambda = 720$(Kang *et al.* (2003)).The velocity signal is decomposed into n=9 levels, that is, a series of high frequency details $d_{-j}(j = 1 \cdots 9)$ and a low frequency residual s_{-9}, as shown in Figure 2. Each high frequency detail represents a series of velocity at certain scale, which is named scale velocity. The low frequency residual represents the largest scale velocity, which represents the trend of the original signal as shown in Figure 3. The correlation coefficients between original velocity signals and scale velocity is shown in Table 1. It is obvious that the correlation coefficients of larger scale velocity are much larger than those of smaller scale velocity. To know the kinetic energy characteristics, FT is done for each scale velocity and original velocity, as shown in Figure 4. It can be found that each scale velocity component contributes to the total spectra at its corresponding frequency range. As the scale becomes smaller, the corresponding frequencies become larger and the energy spectra become smaller.To approach the $-5/3$ Kolmogorov spectrum, one by one Fourier spectrum of $d_{-j}(j = 1 \cdots 9)$ has been adding together to reach low frequency in the inertial range as shown in Figure 5. It can be seen that the velocity signal is decomposed into three terms: the small scales corresponding to a dissipation range, the large scales corresponding to the energy carrying structures and the moderate scales corresponding to a inertial subrange.

Zero-pressure-gradient Boundary Layer

An original data of a zero-pressure-gradient boundary layer is obtained from the instantaneous velocity fields in a streamwise-wall-normal plane at Reynolds number $Re_\theta = 7705$ (Adrian & Hart (1998)). It contains the streamwise velocity component (u) and the normal turbulent velocity component (v). The fluctuation velocity signals u and v (mean-velocity subtracted) are decomposed into 5 levels. The vorticity can be obtained from the scale velocity field which is the combination of the scale velocity compo-

Table 1. Correlation coefficients between original velocity signals and scale-velocity.

d_{-1}	d_{-2}	d_{-3}	d_{-4}	d_{-5}
0.0314	0.0365	0.0717	0.1132	0.1390
d_{-6}	d_{-7}	d_{-8}	d_{-9}	s_{-9}
0.1896	0.2113	0.2322	0.3363	0.8075

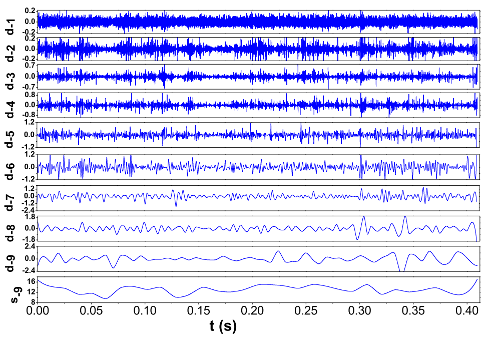

Figure 2. Scale velocities obtainded at n=9 for homogeneous turbulent velocity at $Re_\lambda = 720$.

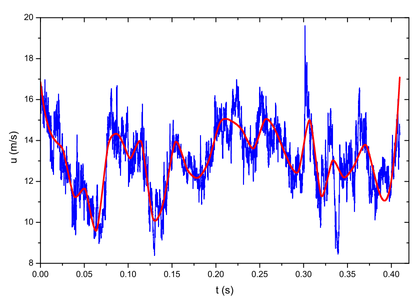

Figure 3. Comparison of original velocity and the largest scale velocity component s_{-9}.

nents and their coordinates. The vorticity contour of scale-velocity is shown in Figure 6. Clearly, both the vorticity of scale-velocity and the scale of vortex tend to increase from scale velocities d_{-1} to d_{-5}. s_{-5} is the largest scale-velocity containing the largest scale eddy.

The vorticities of scale velocity have been added together and is compared with that of the original data as shown in Figure 7. The difference between them is also shown in 8. It can be seen that the vorticities of original velocity are same as that of the scale velocity and the differences can be ignored due to their little magnitudes. It shows that the vorticity also can be reconstituted. The fluctuation velocity signals are decomposed into several scale velocities, and the vortex has been separated into series of smaller scale vortex and a bigger scale vortex. Moreover, the corresponding vorticity can be decomposed.

Turbulent Channel Flow

Nine series of the velocity are obtained at nine different wall distances in a DNS of a turbulent channel flow

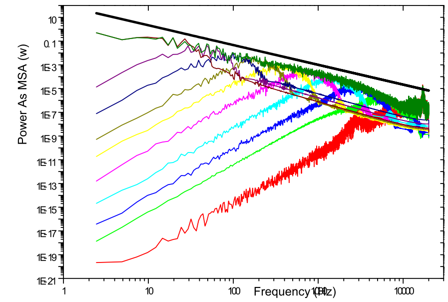

Figure 4. Fourier spectrum of d_{-j} $(j = 1 \cdots 9)$. The reference line has slop $-5/3$.

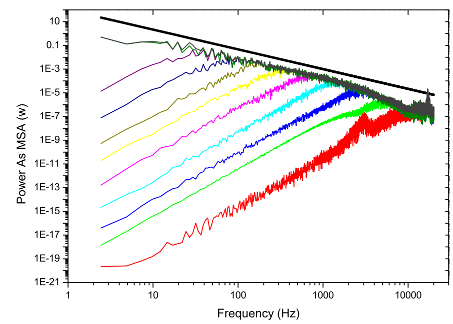

Figure 5. Fourier spectrum of the sum of d_{-j} from d_{-1} to d_{-9}, s_{-9}. It shows a clear asymptotic behavior.

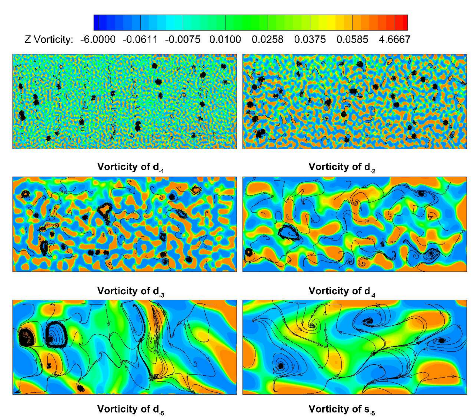

Figure 6. Vorticity contour of scale-velocity component.

at $Re_\tau = 206$. The fluctuation velocity signals (mean velocity subtracted) are decomposed into 11 levels. To know the relationship between the largest scale velocities s_{-11} at different wall distances, they are compared in Figure 9. It can be clearly seen that they have similarity in the buffer and logarithmic regions and that there are time delays from bottom to channel centre. The correlation coefficients of s_{-11} and original velocity signals at $y^+ = 97.96$ to those at other wall distances are shown in Figure 10. It is obvious that the correlation coefficients of s_{-11} are much larger than those of original velocity signals. In the consideration of all these large scale velocities are related to a large coherent structure across the half channel, the max correlation coefficients are also computed and presented. All the cor-

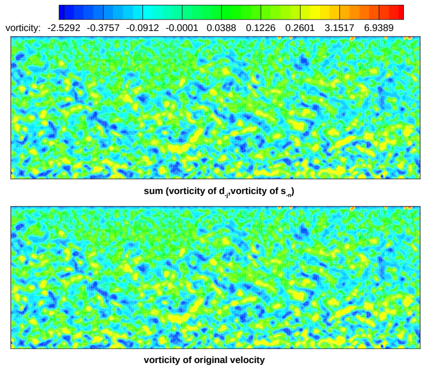

Figure 7. Sum of vorticity of scale velocity and difference from original velocity .

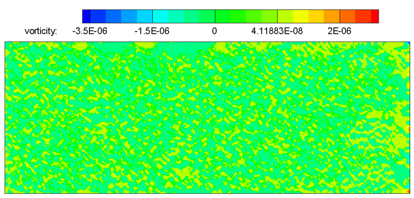

Figure 8. Difference between voticity of original velocity and sum of vorticity of scale velocity.

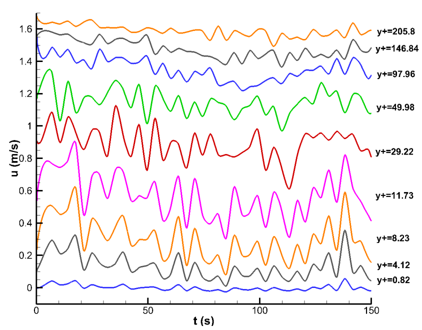

Figure 9. Comparison of s_{-11} at different wall distances, each one is shifted 0.2 upwards from $y^+ = 4.12$ for clarity.

relation coefficients increase, except for that at $y^+ = 97.96$ as shown in Figure 10. The analysis above is consistent with the experimental results of Marusic (MATHIS *et al.* (2009)) at high Reynolds number that the large scales in the logarithmic region have modulation effect on the near wall motion.

Four series of instantaneous streamwise velocities in x-z plane are obtained at four different wall distances. The velocity signals are decomposed into 5 levels. From the contour of scale-velocity as shown in Figure 11, Figure 12 and Figure 13, the streak of high and low velocities can be clearly seen. In buffer layer, the streak of high and low velocity appears in smaller scale velocity like d_{-3}, but the velocities in logarithmic region do not have clear steak. In logarithmic region, the scale-velocity d_{-4} tend to be streak, yet unclearly, and the scale-velocity s_{-5} shows the clear streak.It can be clearly seen that the contour of smaller scale velocity like d_{-3} is similar in the same region while less similar in the different layer. However, the contour of larger scale-velocity like s_{-5} is similar not only in same region but also in different layers.

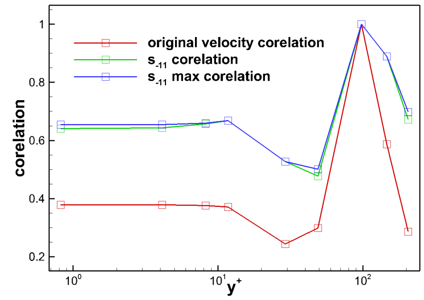

Figure 10. Correlation coefficients of s_{-11} and original velocity signals between at $y^+ = 97.96$ and at other y^+.

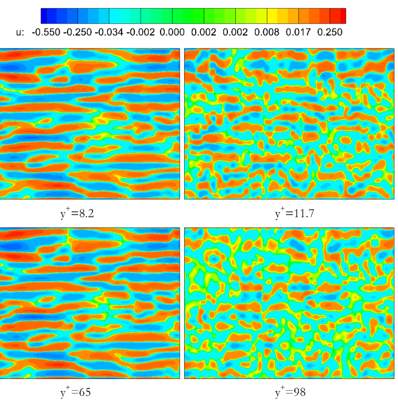

Figure 11. Velocity contour of d_{-3} at different wall distances.

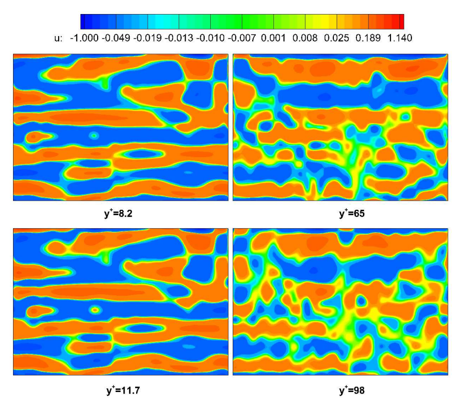

Figure 12. Velocity contour of d_{-4} at different wall distances.

High Frequency Detail Prediction

For homogeneous turbulence the finer details can be predicted from its neighbour coarse levels. The details at levels -1, -4 and -6 are reconstructed and their power spectra are compared with the original decomposed scale velocities, as shown in Figure 14. There are small discrepancies between the predicted and original ones, especially at energy containing frequency.

The original signal can also be extended to finer one

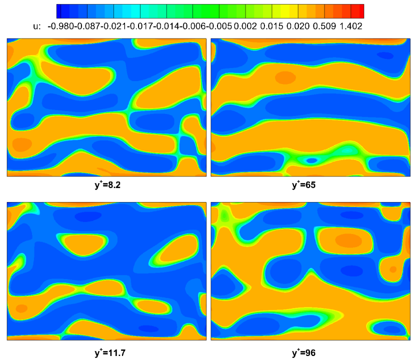

Figure 13. Velocity contour of s_{-5} at different wall distances.

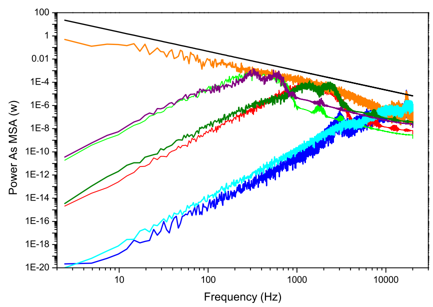

Figure 14. Comparisions of Fourier spectrum of reconstructed and original scale velocity. From bottom to top, d^p_{-1}, d^p_{-4}, d^p_{-6}.

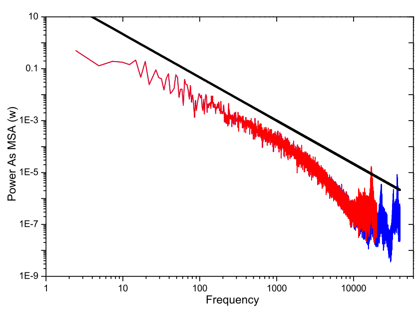

Figure 15. Comparison of Fourier spectrum for λ_1^p(blue) and λ_0(red).

using this model without data contamination as shown in Figure 15 and Figure 16.

CONCLUSION

A scale decomposition method based on the second generation wavelets method using lifting scheme is constructed for a homogeneous and wall bounded turbulent velocity signals. The turbulent velocity signals are decomposed into series of scale-related velocities which have narrow bandwidth. From the power spectra analysis of the scale velocity, it is found that each scale velocity component contributes to the total spectra at its corresponding frequency range. From the vorticity analysis of the scale velocity, it is found that the velocity signals can be decomposed into

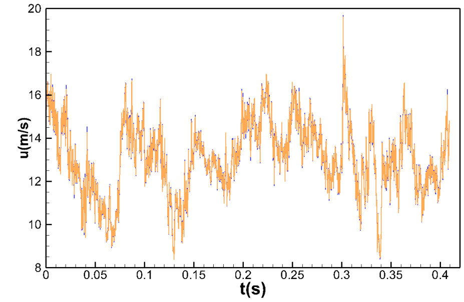

Figure 16. Comparison of λ_1^p(blue) and λ_0 (yellow).

several scale velocities, and the vortex has been separated into a series of smaller scale vortex and a bigger scale vortex. From the correlations of velocities with the same scale level at different wall distances, it is found that the scale velocities have large correlation coefficients at the largest scale.

A reconstruction model promoted under the similarity assumption can be used to rebuild fine scale-velocity for homogeneous turbulence by using its neighbour larger scale velocities. The reconstructed scale velocities have correct energy spectra compared with that of experimental ones and have no data contamination. This reconstruction model has the potential to be applied to build subgrid scale model in the framework of Large-Eddy Simulation (LES) method by using resolved velocity filed.

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