A NEW INTERMITTENCY TRANSPORT EQUATION FOR BYPASS TRANSITION

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ABSTRACT
The new transport equation for intermittency is developed and proposed in this research work, based on the definition of intermittency and the existing transport equations of laminar and turbulent kinetic energy. Its performance is compared with the two existing transition models used in commercial CFD software: k_L model and $\gamma - \mathrm{Re}_\theta$ model, in case of bypass transition. It is found that the proposed model can accurately predict the mean streamwise velocity in the transition zone. For C_f, k and $-\overline{u'v'}$, the proposed model has the same performance as the $\gamma - \mathrm{Re}_\theta$ model.

INTRODUCTION
During the last decade, there have been two RANS-based transition models used in commercial CFD software: k_L model (Walters and Cokljat, 2008) and $\gamma - \mathrm{Re}_\theta$ model (Langtry and Menter, 2009). The $\gamma - \mathrm{Re}_\theta$ model was constructed based on correlations obtained from experimental data so that it is reliable only within a range of flow conditions that the experiment is set up to obtain such correlations. The k_L model was developed based on basic physical mechanisms and their interaction to capture the flow transition, e.g. redistribution term (process) to model energy transfer from laminar to turbulent stages so that it is more attractive in such a way that it can be applied to a wider range of flow conditions. However, γ, k and k_L are strongly related to each other by the definition of γ. Therefore, their transport equations should be developed in an interconnected manner. This research work is aimed to identify the incomplete modeling scheme of the k_L model which requires one more transport equation for γ to complete the relationship among γ, k and k_L, according to the definition of γ. Finally, the new transport equation for γ will be developed and proposed here.

DERIVATION OF A NEW INTERMITTENCY TRANSPORT EQUATION
To begin with, γ or the intermittency of laminar-to-turbulent flow transition is defined as *the fraction of time in which the flow is turbulent at a fixed point* (Schneider, 1995). According to its definition, γ can be formulated as follows:

$$\gamma = \frac{t}{t_{TOTAL}} \tag{1}$$

where

$$t_{TOTAL} = t + t_L \tag{2}$$

with t being the fraction of time in which the flow is turbulent at a fixed point and t_L being the fraction of time in which the flow is laminar at a fixed point. In laminar flow regime, there exists only k_L, or the laminar kinetic energy, in which case ε is the dissipation rate of k_L only and hence

$$t_L = \frac{k_L}{\varepsilon} \tag{3}$$

In fully turbulent flow regime, there exists only k, or the turbulent kinetic energy, in which case ε is the dissipation rate of k only and hence

$$t = \frac{k}{\varepsilon} \tag{4}$$

After substituting Eqs.(2)-(4) into Eq.(1), the definition of γ becomes

$$\gamma = \frac{k}{k + k_L} \tag{5}$$

This equivalent definition for γ was proposed by Lardeau et al. (2009). Obviously, Eq.(5) represents an algebraic relationship among γ, k and k_L. Therefore, one of them can be calculated from the others using Eq.(5). The value of k can be obtained from the transport equation of k in any k-based two-equation turbulence model in which case the SST k - ω model is chosen here. The value of k_L can be obtained from the transport equation of k_L proposed by Walters and Cokljat (2008). In principle, γ can be calculated directly from Eq.(5). The value of γ can then be used to control the production and destruction terms of the k-equation in order to account for the transitional effect on the mean flow as proposed by Langtry and Menter (2009).

However, the adopted k - and k_L-equations are only semi-empirical mathematical models so that the values of k and k_L obtained are not exact and hence the value of γ calculated from Eq.(5) is not truly its value. The only way to obtain the physically realistic value for γ is to create a new transport equation for γ by using its definition in Eq.(5), the k-equation and the k_L-equation. The question may arise why the new transport equation for γ is needed when the γ-equations proposed by Durbin (2012) and Langtry and Menter (2009) are already existing as choices. The answer is that those three transport equations for k, k_L and γ were created independently by different research groups at different times for different purposes so that they obviously cannot fit together within the definition of γ in Eq.(5). Moreover, the adopted transport equations of k and k_L are physics-based while the existing transport equation of γ in commercial CFD software is correlation-based which requires another transport equation for Re_θ to close the $\gamma - \mathrm{Re}_\theta$ transition model (Langtry and Menter, 2009). Therefore, the new transport equation for γ is essentially needed and its derivation is demonstrated below.

First of all, Eq.(5) must be re-arranged as follows:

$$k = \left(\frac{\gamma}{1-\gamma}\right) k_L \tag{6}$$

For simplicity to show how to derive the transport equation for γ, the flow problem considered at this stage is the boundary layer on a flat plate with zero pressure gradient. Based on RANS computation and boundary-layer approximation, the common standard form of the k transport equation can be written as

$$\bar{U}\frac{\partial k}{\partial x} + \bar{V}\frac{\partial k}{\partial y} = \frac{\partial}{\partial y}\left[\left(\nu + \frac{\nu_T}{\sigma_k}\right)\frac{\partial k}{\partial y}\right] + \nu_T\left(\frac{\partial \bar{U}}{\partial y}\right)^2 - \beta^* k\omega \tag{7}$$

After substituting Eq.(6) into Eq.(7), the resulting equation is

$$\frac{k_L}{(1-\gamma)^2}\left[\bar{U}\frac{\partial \gamma}{\partial x} + \bar{V}\frac{\partial \gamma}{\partial y}\right] + \frac{\gamma}{1-\gamma}\left[\bar{U}\frac{\partial k_L}{\partial x} + \bar{V}\frac{\partial k_L}{\partial y}\right] =$$
$$\frac{k_L}{(1-\gamma)^2}\frac{\partial}{\partial y}\left[\left(\nu + \frac{\nu_T}{\sigma_k}\right)\frac{\partial \gamma}{\partial y}\right] + \left(\nu + \frac{\nu_T}{\sigma_k}\right)\frac{\partial}{\partial y}\left[\frac{k_L}{(1-\gamma)^2}\frac{\partial \gamma}{\partial y}\right]$$
$$+ \frac{\gamma}{1-\gamma}\frac{\partial}{\partial y}\left[\left(\nu + \frac{\nu_T}{\sigma_k}\right)\frac{\partial k_L}{\partial y}\right] + \left(\nu + \frac{\nu_T}{\sigma_k}\right)\frac{\partial}{\partial y}\left[\frac{\gamma}{1-\gamma}\frac{\partial k_L}{\partial y}\right]$$
$$+ \nu_T\left(\frac{\partial \bar{U}}{\partial y}\right)^2 - \frac{\gamma}{1-\gamma}\beta^* k_L\omega \tag{8}$$

For bypass transition, the transport equation of k_L proposed by Walters and Cokljat (2008) is given as follows:

$$\bar{U}\frac{\partial k_L}{\partial x} + \bar{V}\frac{\partial k_L}{\partial y} = \frac{\partial}{\partial y}\left[\nu\frac{\partial k_L}{\partial y}\right] + \nu_{T,\ell}\left(\frac{\partial \bar{U}}{\partial y}\right)^2$$
$$-\nu\left(\frac{\partial \sqrt{k_L}}{\partial y}\right)^2 - R_{BP} \tag{9}$$

Multiplying Eq.(9) by $\frac{\gamma}{1-\gamma}$ and then subtracting the resulting equation from Eq.(8) give

$$\frac{k_L}{(1-\gamma)^2}\left[\bar{U}\frac{\partial \gamma}{\partial x} + \bar{V}\frac{\partial \gamma}{\partial y}\right] = \frac{k_L}{(1-\gamma)^2}\frac{\partial}{\partial y}\left[\left(\nu + \frac{\nu_T}{\sigma_k}\right)\frac{\partial \gamma}{\partial y}\right]$$
$$+ \left(\nu + \frac{\nu_T}{\sigma_k}\right)\frac{\partial}{\partial y}\left[\frac{k_L}{(1-\gamma)^2}\frac{\partial \gamma}{\partial y}\right] + \frac{\gamma}{1-\gamma}\frac{\partial}{\partial y}\left[\frac{\nu_T}{\sigma_k}\frac{\partial k_L}{\partial y}\right]$$
$$+ \left(\nu + \frac{\nu_T}{\sigma_k}\right)\frac{\partial}{\partial y}\left[\frac{\gamma}{1-\gamma}\frac{\partial k_L}{\partial y}\right] + \left[\nu_T - \frac{\gamma}{1-\gamma}\nu_{T,\ell}\right]\left(\frac{\partial \bar{U}}{\partial y}\right)^2$$
$$- \frac{\gamma}{1-\gamma}\left[\beta^* k_L\omega - \nu\left(\frac{\partial \sqrt{k_L}}{\partial y}\right)^2\right] + \frac{\gamma}{1-\gamma}R_{BP} \tag{10}$$

After multiplying Eq.(10) by $\frac{(1-\gamma)^2}{k_L}$, the new transport equation for γ can be obtained as follows:

$$\bar{U}\frac{\partial \gamma}{\partial x} + \bar{V}\frac{\partial \gamma}{\partial y} = \frac{\partial}{\partial y}\left[\left(\nu + \frac{\nu_T}{\sigma_k}\right)\frac{\partial \gamma}{\partial y}\right]$$

$$+\frac{(1-\gamma)^2}{k_L}\left(\nu+\frac{\nu_T}{\sigma_k}\right)\frac{\partial}{\partial y}\left[\frac{k_L}{(1-\gamma)^2}\frac{\partial\gamma}{\partial y}\right]$$

$$+\frac{\gamma(1-\gamma)}{k_L}\frac{\partial}{\partial y}\left[\frac{\nu_T}{\sigma_k}\frac{\partial k_L}{\partial y}\right]$$

$$+\frac{(1-\gamma)^2}{k_L}\left(\nu+\frac{\nu_T}{\sigma_k}\right)\frac{\partial}{\partial y}\left[\frac{\gamma}{1-\gamma}\frac{\partial k_L}{\partial y}\right]$$

$$+\frac{(1-\gamma)^2}{k_L}\left[\nu_T-\frac{\gamma}{1-\gamma}\nu_{T,\ell}\right]\left(\frac{\partial \bar{U}}{\partial y}\right)^2$$

$$-\frac{\gamma(1-\gamma)}{k_L}\left[\beta^* k_L\omega-\nu\left(\frac{\partial\sqrt{k_L}}{\partial y}\right)^2\right]+\frac{\gamma(1-\gamma)}{k_L}R_{BP} \qquad (11)$$

Re-arranging Eq.(11) yields

$$\bar{U}\frac{\partial\gamma}{\partial x}+\bar{V}\frac{\partial\gamma}{\partial y}=\frac{\partial}{\partial y}\left[\left(\nu+\frac{\nu_T}{\sigma_k}\right)\frac{\partial\gamma}{\partial y}\right]$$

$$+\frac{(1-\gamma)^2}{k_L}\left\{\left(\nu+\frac{\nu_T}{\sigma_k}\right)\frac{\partial}{\partial y}\left[\frac{k_L}{(1-\gamma)^2}\frac{\partial\gamma}{\partial y}\right]\right.$$

$$+\left(\nu+\frac{\nu_T}{\sigma_k}\right)\frac{\partial}{\partial y}\left[\frac{\gamma}{1-\gamma}\frac{\partial k_L}{\partial y}\right]$$

$$\left.+\left[\nu_T-\frac{\gamma}{1-\gamma}\nu_{T,\ell}\right]\left(\frac{\partial \bar{U}}{\partial y}\right)^2\right\}$$

$$-\frac{\gamma(1-\gamma)}{k_L}\left\{\beta^* k_L\omega-\nu\left(\frac{\partial\sqrt{k_L}}{\partial y}\right)^2\right.$$

$$\left.-\frac{\partial}{\partial y}\left[\frac{\nu_T}{\sigma_k}\frac{\partial k_L}{\partial y}\right]-R_{BP}\right\} \qquad (12)$$

Since $\frac{(1-\gamma)^2}{k_L}=\frac{\gamma(1-\gamma)}{k}$ with the aid of Eq.(6), Eq.(12) can be written as follows:

$$\bar{U}\frac{\partial\gamma}{\partial x}+\bar{V}\frac{\partial\gamma}{\partial y}=\frac{\partial}{\partial y}\left[\left(\nu+\frac{\nu_T}{\sigma_k}\right)\frac{\partial\gamma}{\partial y}\right]$$

$$+\frac{\gamma(1-\gamma)}{k}\left\{\left(\nu+\frac{\nu_T}{\sigma_k}\right)\frac{\partial}{\partial y}\left[\frac{k_L}{(1-\gamma)^2}\frac{\partial\gamma}{\partial y}\right]\right.$$

$$\left.+\left(\nu+\frac{\nu_T}{\sigma_k}\right)\frac{\partial}{\partial y}\left[\frac{\gamma}{1-\gamma}\frac{\partial k_L}{\partial y}\right]+\left[\nu_T-\frac{\gamma}{1-\gamma}\nu_{T,\ell}\right]\left(\frac{\partial \bar{U}}{\partial y}\right)^2\right\}$$

$$-\frac{\gamma(1-\gamma)}{k_L}\left\{\beta^* k_L\omega-\nu\left(\frac{\partial\sqrt{k_L}}{\partial y}\right)^2\right.$$

$$\left.-\frac{\partial}{\partial y}\left[\frac{\nu_T}{\sigma_k}\frac{\partial k_L}{\partial y}\right]-R_{BP}\right\} \qquad (13)$$

However, it is found that the second and third terms on the right-hand side of Eq.(13) can be combined into one term as follows:

$$\left(\nu+\frac{\nu_T}{\sigma_k}\right)\frac{\partial}{\partial y}\left[\frac{k_L}{(1-\gamma)^2}\frac{\partial\gamma}{\partial y}\right]+\left(\nu+\frac{\nu_T}{\sigma_k}\right)\frac{\partial}{\partial y}\left[\frac{\gamma}{1-\gamma}\frac{\partial k_L}{\partial y}\right]$$

$$=\left(\nu+\frac{\nu_T}{\sigma_k}\right)\frac{\partial^2 k}{\partial y^2} \qquad (14)$$

After substituting Eq.(14) into Eq.(13), Eq.(13) becomes

$$\bar{U}\frac{\partial\gamma}{\partial x}+\bar{V}\frac{\partial\gamma}{\partial y}=\frac{\partial}{\partial y}\left[\left(\nu+\frac{\nu_T}{\sigma_k}\right)\frac{\partial\gamma}{\partial y}\right]$$

$$+\frac{\gamma(1-\gamma)}{k}\left\{\left(\nu+\frac{\nu_T}{\sigma_k}\right)\frac{\partial^2 k}{\partial y^2}+\left[\nu_T-\frac{\gamma}{1-\gamma}\nu_{T,\ell}\right]\left(\frac{\partial \bar{U}}{\partial y}\right)^2\right\}$$

$$-\frac{\gamma(1-\gamma)}{k_L}\left\{\beta^* k_L\omega-\nu\left(\frac{\partial\sqrt{k_L}}{\partial y}\right)^2\right.$$

$$\left.-\frac{\partial}{\partial y}\left[\frac{\nu_T}{\sigma_k}\frac{\partial k_L}{\partial y}\right]-R_{BP}\right\} \qquad (15)$$

Eq.(15) can be re-arranged to separate sink terms from source terms as follows:

$$\bar{U}\frac{\partial\gamma}{\partial x}+\bar{V}\frac{\partial\gamma}{\partial y}=\frac{\partial}{\partial y}\left[\left(\nu+\frac{\nu_T}{\sigma_k}\right)\frac{\partial\gamma}{\partial y}\right]$$

$$+\frac{\gamma(1-\gamma)}{k}\cdot\left(\nu+\frac{\nu_T}{\sigma_k}\right)\frac{\partial^2 k}{\partial y^2}+\frac{\gamma(1-\gamma)}{k}\cdot\nu_T\left(\frac{\partial \bar{U}}{\partial y}\right)^2$$

$$+\frac{\gamma(1-\gamma)}{k_L}\cdot\nu\left(\frac{\partial\sqrt{k_L}}{\partial y}\right)^2+\frac{\gamma(1-\gamma)}{k_L}\cdot\frac{\partial}{\partial y}\left[\frac{\nu_T}{\sigma_k}\frac{\partial k_L}{\partial y}\right]$$

$$+\frac{\gamma(1-\gamma)}{k_L}\cdot R_{BP}$$

$$-\frac{\gamma^2}{k}\cdot\nu_{T,\ell}\left(\frac{\partial \bar{U}}{\partial y}\right)^2-\frac{\gamma(1-\gamma)}{k_L}\cdot\beta^* k_L\omega \qquad (16)$$

However, it is found that $\frac{\gamma^2}{k}=\frac{\gamma(1-\gamma)}{k_L}$ with the aid of Eq.(5) and $1-\gamma=\frac{k_L}{k+k_L}$. Therefore, all the source and sink terms in Eq.(16) can be grouped into the physical mechanisms involved with k and k_L separately as follows:

$$\bar{U}\frac{\partial\gamma}{\partial x}+\bar{V}\frac{\partial\gamma}{\partial y}=\frac{\partial}{\partial y}\left[\left(\nu+\frac{\nu_T}{\sigma_k}\right)\frac{\partial\gamma}{\partial y}\right]$$

$$+\frac{\gamma(1-\gamma)}{k}\left\{\left(\nu+\frac{\nu_T}{\sigma_k}\right)\frac{\partial^2 k}{\partial y^2}+\nu_T\left(\frac{\partial \bar{U}}{\partial y}\right)^2\right\}$$

$$+\frac{\gamma(1-\gamma)}{k_L}\left\{\nu\left(\frac{\partial\sqrt{k_L}}{\partial y}\right)^2+\frac{\partial}{\partial y}\left[\frac{\nu_T}{\sigma_k}\frac{\partial k_L}{\partial y}\right]+R_{BP}\right.$$

$$\left.-\nu_{T,\ell}\left(\frac{\partial \bar{U}}{\partial y}\right)^2-\beta^* k_L\omega\right\} \qquad (17)$$

Finally, Eq.(17) can be generalized for unsteady three-dimensional incompressible flow in terms of tensor notation as follows:

$$\frac{D\gamma}{Dt}=\frac{\partial}{\partial x_j}\left[\left(\nu+\frac{\nu_T}{\sigma_k}\right)\frac{\partial\gamma}{\partial x_j}\right]$$

$$+\frac{\gamma(1-\gamma)}{k}\left\{\left(\nu+\frac{\nu_T}{\sigma_k}\right)\frac{\partial^2 k}{\partial x_j^2}+\nu_T S^2\right\}$$

$$+\frac{\gamma(1-\gamma)}{k_L}\left\{\nu\left(\frac{\partial\sqrt{k_L}}{\partial x_j}\right)^2+\frac{\partial}{\partial x_j}\left[\frac{\nu_T}{\sigma_k}\frac{\partial k_L}{\partial x_j}\right]+R_{BP}\right.$$

$$\left.-\nu_{T,\ell}S^2-\beta^* k_L\omega\right\} \qquad (18)$$

The physical meaning of each term on the right-hand side of Eq.(18) is listed as follows:

- $\frac{\partial}{\partial x_j}\left[\left(\nu+\frac{\nu_T}{\sigma_k}\right)\frac{\partial\gamma}{\partial x_j}\right]$ is the diffusion term for γ.
- $\frac{\gamma(1-\gamma)}{k}\cdot\left(\nu+\frac{\nu_T}{\sigma_k}\right)\frac{\partial^2 k}{\partial x_j^2}$ and $\frac{\gamma(1-\gamma)}{k}\cdot\nu_T S^2$ are the source terms involving the physical mechanisms of laminar and turbulent diffusion, and production contributed from the turbulent kinetic energy during the transition process respectively.
- $\frac{\gamma(1-\gamma)}{k_L}\cdot\nu\left(\frac{\partial\sqrt{k_L}}{\partial x_j}\right)^2$, $\frac{\gamma(1-\gamma)}{k_L}\cdot\frac{\partial}{\partial x_j}\left[\frac{\nu_T}{\sigma_k}\frac{\partial k_L}{\partial x_j}\right]$ and $\frac{\gamma(1-\gamma)}{k_L}\cdot R_{BP}$ are the source terms involving the physical mechanisms of laminar diffusion, turbulent diffusion, and redistribution contributed from the laminar kinetic energy during the transition process respectively.
- $\frac{\gamma(1-\gamma)}{k_L}\cdot\nu_{T,\ell}S^2$ and $\frac{\gamma(1-\gamma)}{k_L}\cdot\beta^* k_L\omega$ are the sink terms involving the physical mechanisms of production and dissipation contributed from the laminar kinetic energy during the transition process respectively.
- $\gamma(1-\gamma)$ is the ON/OFF switch, which is ON only in the transition zone.

SHEAR-SHELTERING EFFECT

In order to account for the shear-sheltering effect, the shear-sheltering function f_{SS} is used to damp or promote the influence of bypass transition mechanism by controlling the production term contributed from the turbulent kinetic energy, which is one of the main energy sources to promote bypass transition mechanism, as follows:

$$\frac{\gamma(1-\gamma)}{k}\cdot f_{SS}\cdot\nu_T S^2 \qquad (19)$$

where the function f_{SS} was proposed by Walters and Cokljat (2008) in the following form:

$$f_{SS}=\exp\left[-\left(C_{SS}\frac{\nu\Omega}{k}\right)^2\right] \qquad (20)$$

which is adopted here without any modification in which case C_{SS} is the model constant and $\Omega=\sqrt{2\Omega_{ij}\Omega_{ij}}$ is the magnitude of mean rotation rate. Therefore, the final form of the new transport equation for γ with the shear-sheltering effect can be written as follows:

$$\frac{D\gamma}{Dt}=\frac{\partial}{\partial x_j}\left[\left(\nu+\frac{\nu_T}{\sigma_k}\right)\frac{\partial\gamma}{\partial x_j}\right]$$

$$+\frac{\gamma(1-\gamma)}{k}\left\{\left(\nu+\frac{\nu_T}{\sigma_k}\right)\frac{\partial^2 k}{\partial x_j^2}+f_{SS}\nu_T S^2\right\}$$

$$+\frac{\gamma(1-\gamma)}{k_L}\left\{\nu\left(\frac{\partial\sqrt{k_L}}{\partial x_j}\right)^2+\frac{\partial}{\partial x_j}\left[\frac{\nu_T}{\sigma_k}\frac{\partial k_L}{\partial x_j}\right]+R_{BP}\right.$$

$$\left.-\nu_{T,\ell}S^2-\beta^* k_L\omega\right\} \qquad (21)$$

The effect of the shear-sheltering function on the transition mechanism is demonstrated in Fig. 1 where, without the shear-sheltering effect, i.e. the function $f_{SS}=1$, the laminar flow will become turbulent rapidly because the free-stream turbulence is fully allowed to penetrate the boundary layer. Therefore, the presence of the shear-sheltering function f_{SS} in Eq.(21) is served to make the new γ -equation applicable to both natural transition and bypass transition (Juntasaro et al., 2013).

OPTIMUM VALUES FOR MODEL CONSTANTS

For the redistribution term R_{BP} from the laminar kinetic energy which is one of the main energy sources to promote the bypass transition proposed by Walters and Cokljat (2008), the model constants ($C_{BP,crit}$, C_{λ}) are optimized by tuning their values using the experimental data of T3A. The optimum value of $C_{BP,crit}$ is found to be 4.0 as shown in Fig. 1 whereas C_{λ} = 0.87 is chosen because its corresponding line $C_{\lambda}d$ properly intersects with the turbulent length scale curve λ_T at the edge of boundary layer where d is the normal distance to the nearest wall.

MODEL IMPLEMENTATION

The new γ transport equation in Eq.(21) is implemented into our in-house CFD code of elliptic type which is developed based on the RANS equations and finite volume method. The new γ transport equation is used in cooperation with the original k_L -equation proposed by Walters and Cokljat (2008) and the SST $k-\omega$ turbulence model of Menter (1994). To bring the transition mechanism into effect, the production and destruction terms in the $k-$ equation are controlled by weighting (multiplying) them with the intermittency factor γ following the concept of Langtry and Menter (2009) and Menter et al. (2005).

RESULTS

In cases of T3B and T3A, the predicted results using the proposed model are compared with those of the k_L model of Walters and Cokljat (2008) and those of the $\gamma - \mathrm{Re}_{\theta}$ model of Langtry and Menter (2009) for the skin friction coefficient C_f in Figs. 2-3, the mean streamwise velocity in wall units U^+ in Figs. 4-5, the turbulent kinetic energy normalized by free-stream velocity k/U_0^2 in Figs. 6-7, and the Reynolds shear stress normalized by free-stream velocity $-\overline{u'v'}/U_0^2$ in Figs. 8-9.

CONCLUSION

Based on basic physical mechanisms, the proposed γ -equation in cooperation with the original k_L -equation of Walters and Cokljat (2008) and the SST $k-\omega$ turbulence model of Menter (1994) can accurately predict the mean streamwise velocity in the transition zone. For the skin-friction coefficient, turbulent kinetic energy and Reynolds shear stress, the proposed model has the same performance as the $\gamma - \mathrm{Re}_{\theta}$ model of Langtry and Menter (2009) which is the correlation-based transition model.

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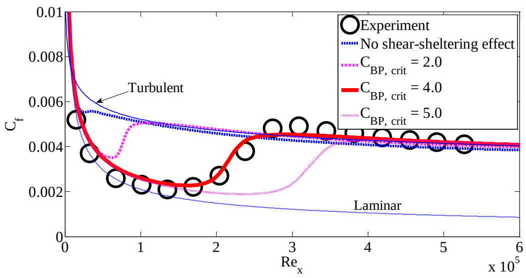

Figure 1. Shear-sheltering effect and $C_{BP,crit}$ optimization.

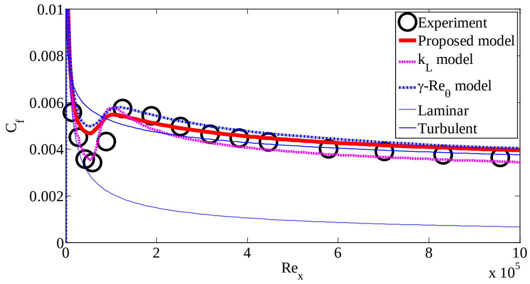

Figure 2. C_f for T3B.

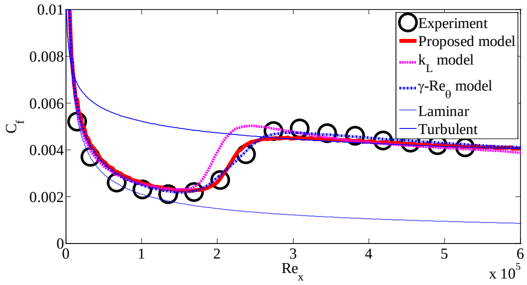

Figure 3. C_f for T3A.

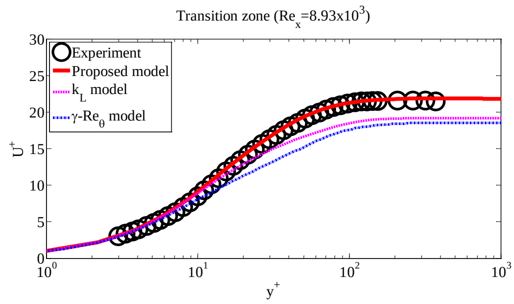

Figure 4. Mean streamwise velocity in wall units for T3B.

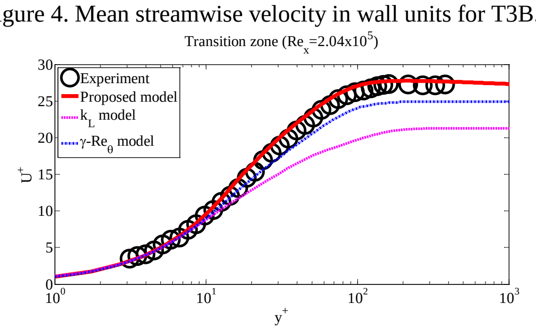

Figure 5. Mean streamwise velocity in wall units for T3A.

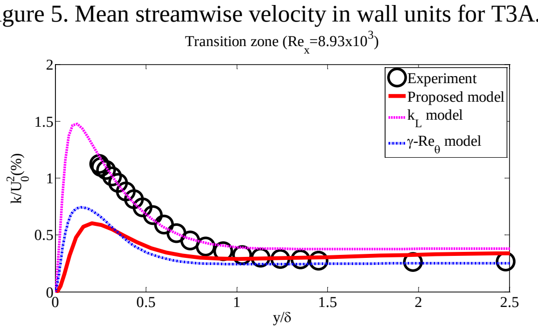

Figure 6. Turbulent kinetic energy for T3B.

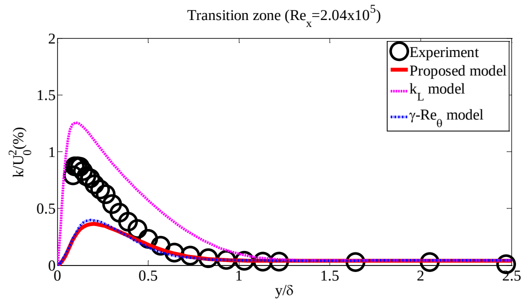

Figure 7. Turbulent kinetic energy for T3A.

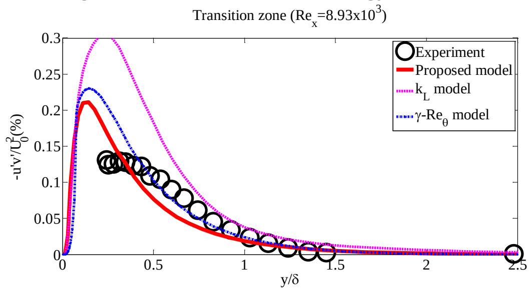

Figure 8. Reynolds shear stress for T3B.

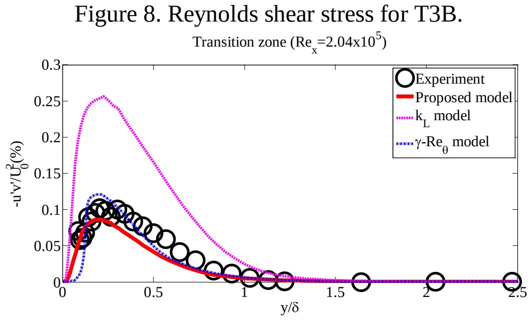

Figure 9. Reynolds shear stress for T3A.