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OSBORNE REYNOLDS' PIPE FLOW: DIRECT COMPUTATION FROM LAMINAR THROUGH TRANSITION TO FULLY-DEVELOPED TURBULENCE

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ABSTRACT

The most fundamental internal flow has been computed accurately from first-principle in laboratory framework. It exhibits a turbulence onset scenario that bears certain similarities to, and differences from, the bypass transition in the narrow sense found in the most basic external flow under free-stream turbulence, which has also been computed concurrently. In both flows, finite, weak, and well-controlled turbulent perturbations introduced at the inlet far away from the wall excite large semi-regular Lambda structures upstream of breakdown. Breakdown is directly caused by the formation of hairpin packets in the near-wall region. One major difference is that the transitional pipe flow exhibits a distinct overshoot in skin-friction over the corresponding turbulent flow value, whilst the transitional boundary layer does not. It is found that the energy norm associated with weak, localized, finite-amplitude perturbations in the fully-developed laminar pipe flow are capable of growing exponentially, despite the fact that infinitesimally small disturbances will not grow exponentially in this flow. This is the first time in fluid mechanics research that the Osborne Reynolds pipe flow has been accurately simulated starting from fully-developed laminar state, through the whole process of transition, then early turbulent region, and eventually arriving at the fully developed turbulent pipe flow state.

INTRODUCTION

Physics in the Osborne Reynolds pipe flow can be decomposed into a laminar to turbulent transition problem and a fully-developed turbulence problem. Here we primarily focus on the former, albeit not exclusively. The latter subject was extensively reviewed by Marusic et al. (2010), and Smits et al. (2011). Very recent papers on fully-developed turbulent pipe flow since these two reviews include the work of Hultmark et al. (2012) on advanced statistical measurement techniques, and Wu et al. (2012), Baltzer et al. (2013) on very-large-scale motions (VLSM).

Significant efforts were invested in the past decade by the physics community on the Osborne Reynolds pipe transition, albeit mostly concentrating on the relarminarization aspect of the problem.

Faisst and Eckhardt (2003) solved the incompressible Navier-Stokes equation and continuity equation in a pipe flow with Newton-Raphson marching in time, periodic Fourier modes in the azimuthal and axial directions, and Legenedre modes in the radial direction. A prescribed body force was first added to the momentum equations at low Reynolds number to produce the desired streamwise vortices. The body force was later gradually removed with a corresponding increase in Reynolds number to maintain the initially imposed streamwise vortices in the final solution. The pairs of streamwise vortices and streaks thus obtained in the solutions with zero body force, albeit unstable with respect to infinitesimal disturbances, are termed as travelling wave (TW) solutions for the pipe flow. The lowest Reynolds number at which they found TW is 1250.

Peixinho and Mullin (2006) noted, as in many previous experimental studies, that it was very difficult to uncover the detailed flow physics in pipe transition because the process is sudden and ``catastrophic" in its overall appearance. As such, instead of studying directly the laminar-to-turbulent transition in the Osborne Reynolds pipe flow, they, and many other authors, focused on the reverse problem, i.e., relaminarization of (artificially generated) turbulent puffs. Similar to Hof et al., turbulent puff was first generated through injection of perturbation jets through small holes at the wall. After the puff was 100D away from the injection holes, Re was reduced by decelerating a piston mounted to the exit of the pipe. They found that $\mathrm{Re}_c = 1750$ is a critical Reynolds number above which the probability of observing debris of the turbulent puff at a downstream location is always 1. The mean life of a turbulent puff was found to be $T_L \propto 1/(\mathrm{Re}_c - \mathrm{Re}),$ implying divergence at Re_c . Hof et al. (2006), on the other hand, reported the absence of such critical Reynolds number dependence in their puff relaminarization experiments in a pipe with 4mm diameter and 30m length. Instead, their experimental data indicated that the meanlife of pipe flow turbulence is of the form $T_L \propto \exp(b\,\mathrm{Re}),$ where b>0. Their deduction from this is that turbulence is merely transient. Given enough time all turbulence will relaminarize. Simulations by Willis and Kerswell (2007) on the reverse transition problem (relarminarization of turbulent puff) using a streamwisely periodic pipe of 100R-long confirmed the critical Reynolds number behavior of turbulent puff life-time, namely, $T_L \propto 1/(\mathrm{Re}_c - \mathrm{Re})$.

Moxey and Barkley (2010) simulated the pipe relaminarization problem using the streamwisely periodic boundary condition with a pipe length of 250R. Their results indicate that below $\mathrm{Re}_1 = 2300$, turbulent puffs are highly
localized: they only exist in a laminar environment without any mutual interaction. Above $\mathrm{Re}_1 = 2600$, fully continuous turbulence exists without indications of either puff or slugs. For intermediate Reynolds numbers between 2300 and 2600, turbulent puffs change with time, interact with each other, and
contaminate their environment.

These and other recent progresses on the Osborne Reynolds pipe transition problem have been surveyed in two Annual Review of Fluid Mechanics articles: Eckhardt et al. (2007) and Mullin (2011), see also the review by Willis et al. (2008). A special theme issue on the problem was compiled by the Philosophical Transactions of the Royal Society (Eckhardt and others 2009) to celebrate the 125th anniversary of Reynolds original paper published on the same venue. From these reviews, the lowest Reynolds number for TW-type of coherent structures to exist is currently believed to be 773. The lowest Reynolds number for a turbulent puff to be produced stands now at 1650. The lowest Reynolds number for an existing turbulent puff to persist is 1750. It is evident that remarkable progresses were made on the relamarization of turbulent puff
(lifetime, critical Reynolds number), the travelling-wave solution in otherwisely laminar flow, and the magnitude of side-wall injections required to induce turbulent puff. Yet, transition in the pipe flow remains ``abrupt and mysterious" (Mullin 2011).

Probably one of the reasons for the reverse problem (turbulent puff relaminarization) to have received so much attention in recent studies is that the forward problem (from weak inlet disturbance transitioning to continuous turbulence) is more difficult due to the related temporal and spatial complexities. Inspired by our spatially-developing DNS work of the flat-plate boundary layer starting from laminar through bypass transition to fully-turbulent (Wu and Moin 2009, 2010), we propose to tackle directly the forward Osborne Reynolds pipe transition problem using an analogous spatially-developing DNS approach. A finite amplitude yet weak and localized disturbance introduced at the laminar pipe inlet is anticipated to induce transition downstream of the pipe, eventually leading to a state of fully-developed turbulence. Confidence on this approach can be established by evaluating the acquired DNS statistics against analytical solutions in the early laminar region, and against experimental data in the fully-developed turbulent region. The thus validated DNS data fields can subsequently provide detailed flow physics in the transitional region that is bracked by these two verified ends, which may include the growth of the inlet disturbance, the breakdown of the perturbed laminar flow and the onset of turbulence. A key feature of the proposal is that the DNS will be done in the laboratory reference framework without invoking the traditional streamwisely periodic boundary condition. As such, connection between the present DNS to experiments will be un-ambiguous. It is hopeful this approach would open the door for the turbulence and transition simulation community to initiate more direct and systematic attacks on the Osborne Reynolds pipe flow problem.

The computer program is from the coaxial combustor code of Pierce and Moin (2004). It solves the governing equations for instantaneous velocity components and pressure in a cylindrical coordinate system with the fractional step method of Kim & Moin (1985). Spatial discretization is conservative, staggered central differencing. Conservation of kinetic energy in the inviscid limit is facilitated by the use of the staggering.
Time advancement is semi-implicit with nested inner iterations. Convection and diffusion terms that involve derivatives in the radial direction or azimuthal direction are treated implicitly. A third-order Runge-Kutta scheme is used for terms treated explicitly and a second-order Crank-Nicolson scheme is used for terms treated implicitly. Poisson equation was solved using a combined Fast Fourier Transform and Successive over Relaxation.

The wall boundary conditions used in the present study are Neumann condition for pressure, and no-slip Dirichlet condition for velocity. Outflow boundary is treated with standard convective condition. In the immediate vicinity of pipe axis all quantities except for the radial velocity component u_r are staggered in the r direction with respect to the centerline. As such, special centerline boundary treatment is required only for u_r. This is accomplished by averaging corresponding values across the centerline. For further details, see Pierce and Moin (2001).

Inflow condition deserves special attention because it is related to the specific cases being simulated, whereas all the other boundary conditions are case-independent.

A series of numerical tests were first carried out through a systematic variation of the inlet condition. The starting point was plug-flow inlet condition without any perturbation. The downstream velocity distribution develops into the expected parabolic profile of full-developed laminar pipe. Replacing the plug-inflow with an exact parabolic inflow simply maintained the downstream flow field as that of the inlet, also as expected. After these two purely laminar tests, perturbations were introduced at the inlet under the guiding principle that the disturbance magnitude should be finite yet well-controlled. Finite amplitude is needed because it is well-known that pipe flow is linearly stable with respect to infinitesimal disturbances. Yet only weak and localized inlet disturbance should be used in the DNS so as not to destroy the overall characteristics of the base (fully-developed laminar pipe) flow. For instance, the size of the contaminated area by the perturbation over the inlet cross-section should be limited. There should exist an extended streamwise region prior to breakdown over which the slightly perturbed flow agrees essentially with the fully-developed laminar solution.

Another guiding principle in these tests is that a fully-developed turbulent pipe flow state should be achieved in the planned DNS following the completion of transition. This will allow us to establish accuracy of the simulation by comparing statistics with analytical solution in the laminar end, and with accepted experimental data in the fully-developed turbulent end. For the present pipe transition tests, the inlet base flow was prescribed to be the fully-developed laminar profile. Reynolds number was initially chosen to be 5300 because this value was used in several previous streamwisely periodic turbulent pipe flow simulation, Eggels et al (1994), Wu and Moin (2008). Initially, the contaminated area by the perturbations at the inlet plane was confined within $0< r/R< 0.02$. Namely, a tiny circle at the centerline over which the exact laminar parabolic profile was to be replaced by the perturbation.This choice was inspired by the figure 16 of Osborne Reynolds (1883). Regarding the nature of the prescribed inlet perturbation, the first tested type was isotropic grid turbulence. This was motivated by our previous work on boundary layer simulations (Wu and Moin 2009, 2010). It turned out that this type of inlet perturbation disappeared quickly within a very short distance from the inlet, and did not excite any noticeable turbulence further downstream at the given Reynolds number. For the current pipe test, the radial dimension of the contaminated area is much smaller compared to the axial length scale of the pipe. For isotropic turbulence, all the three length scales are nearly the same. Probably the cause for the failure to excite turbulence at the given Reynolds number is due to this disparity between the pipe geometrical scales and the isotropic turbulence length scale. Regarding the nature of the prescribed inlet perturbation, the first tested type
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We subsequently replaced the grid turbulence imposed over the area $0< r< 0.02$ with instantaneous velocity fields extracted from a separate, streamwisely periodic simulation on the fully-developed turbulent pipe flow at Re=5300. This new inlet disturbance excited a turbulent puff in the mid-section of the pipe. The puff grew in size with time, however, with the progression of the iteration, it was flushed out of the exit of the computational domain, and the flow field returned to laminar. It was felt that the contaminated area by the perturbation at the inlet might be too small. However, additional tests by increasing the contaminated area over the inlet cross-sectional plane from $0< r< 0.02$ to $0< r< 0.05$ and further to $0< r/R< 0.10$ also failed to produce a full transition and continuous turbulence. Increasing the Reynolds number from 5300 to 8000 did not produce a full transition, either. Of course, needless to say, if one further increases the Reynolds number, continuous transition will eventually be obtained under this disturbance. The constraints due to resolution and time required by DNS prevent us from raising the Reynolds number further. Our finite yet localized weak disturbance principle also prevents us from enlarging the contaminated area further.

Inspired by the absolute instability of tangential discontinuity in general shear flows (Landau and Lifshitz 1959), it was decided to make the perturbation area over the inlet plane into a narrow ring instead of a small full circle. Within this ring the exact parabolic velocity profile was replaced by the instantaneous fully-developed turbulent field at Re_D=5300. Under this new inflow condition, it was found that at Re_D=5300 the flow still returned to laminar.

Successful transition is obtained at Re_D=8000 for perturbations imposed at the inlet plane over the area $0.4< r/R< 0.42$. The contaminated area is well localized and accounts for merely 1.6% of the cross-sectional area. The perturbations are again instantaneous velocity fields obtained from a separate, streamwisely periodic simulation on fully-developed turbulent pipe flow at Re=5300. Under this inflow boundary condition, the (base) fully-developed laminar pipe flow gradually breaks down, and eventually develops into a fully-developed turbulent state downstream.

Most of the pipe flow results in the present paper are from this successful case. After the successful run at Re_D=8000, it was felt that effort should be made to find the lowest Reynolds number for transition under this particular imposed disturbance. A full simulation at Re_D=6000 was carried out under the identical inflow conditions. It was found that at Re_D=6000 full-developed turbulent field could still be obtained. Therefore, the lowest Reynolds number corresponding to this particular inlet perturbation is within the narrow window between 5300 and 6000.

In the present spatially-developing pipe simulation, unlike the approach used in conventional streamwisely periodic pipe simulations, there is no imposed streamwise pressure gradient. The inlet mass flux varies very slightly due to the temporal nature of the imposed turbulent perturbations over the ring. Global mass conservation is satisfied if the total outflow mass flux balances the total inflow mass flux. This is enforced at each time step by adding a very small correction to the outflow velocity (Pierce and Moin 2001).

The pipe length is 250R, and the computational mesh size is 8192 $\times$ 200 $\times$ 256 in the axial, radial and azimuthal directions, respectively. At Re_D = 8000, $R^+ = 258.5,\ u_\tau = 0.06462 U_b$.

To increase confidence on the accuracy of the spatially-developing pipe simulation results, auxiliary simulation on the fully-developed turbulent pipe flow at Re_D=8000 and Re_D=6000 were also performed using the conventional streamwise periodic boundary condition over a domain whose length is 30R.

In summary, selected results from four pipe flow DNS cases will be presented: case 8KS (Re_D=8000, spatial simulation); case 8KP (Re_D=8000, periodic simulation); case 6KS (Re_D=6000, spatial simulation); case 6KP (Re_D=6000, periodic simulation).

An overview of the four pipe flow cases is given in figures 1 and 2 using two-dimensional contours of the streamwise velocity. They demonstrate qualitatively that laminar state is maintained for considerable distance from the inlet prior to breakdown, and downstream of transition the flow develops towards a fully-developed turbulent state.

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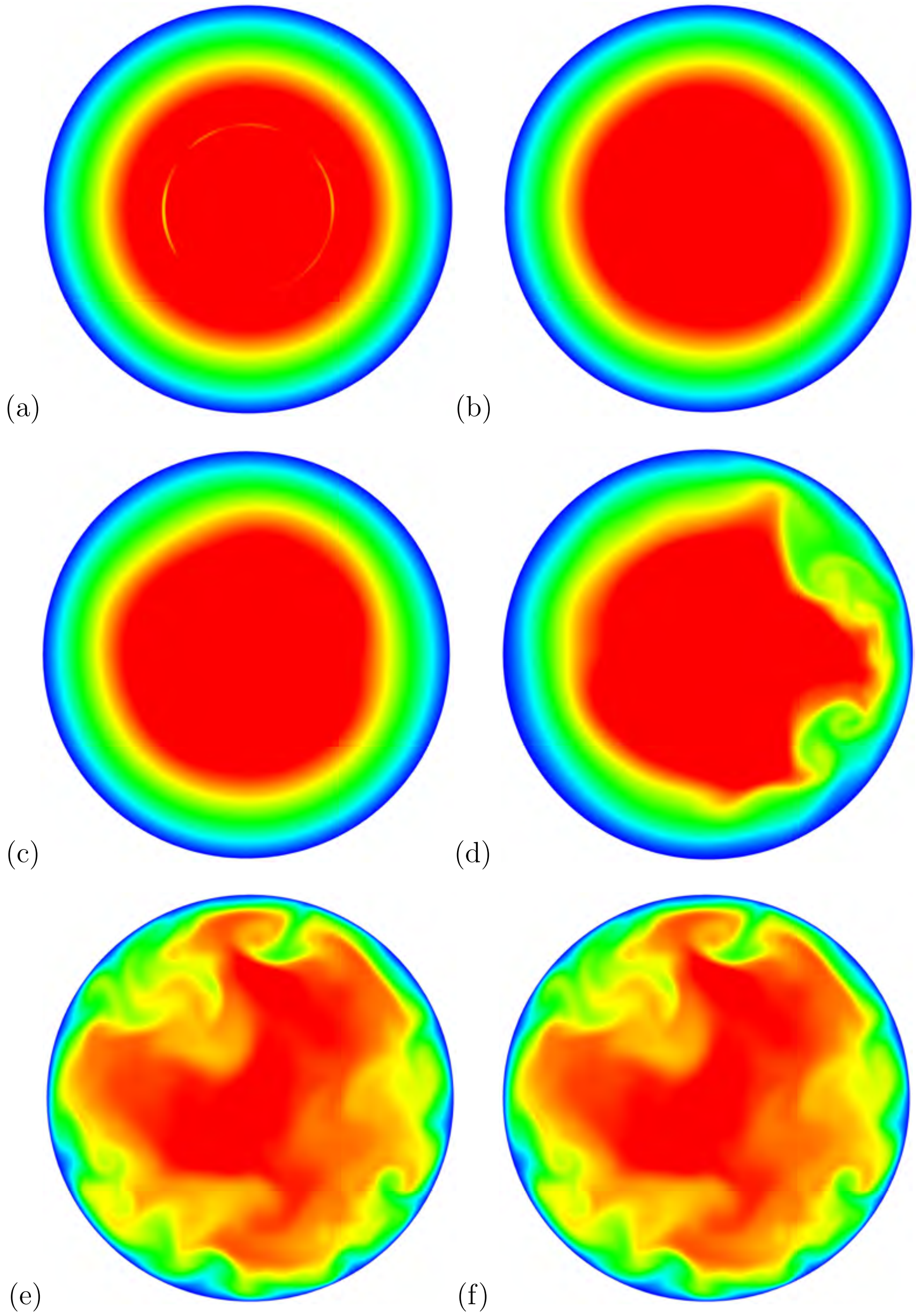

Figure 1: Overview of case 6KS using u at a random instant over selected axial planes. (a) $z = 0$; (b) $z = 20R$; (c) $z = 30R$; (d) $z = 65R$; (e) $z = 220R$; (f) additional pipe flow DNS with streamwise periodicity, $z = 15R$ (case 6KP). Color varies continuously from blue ($u = 0$) to red ($u \geq 1.3U_{\mathrm{bulk}}$).

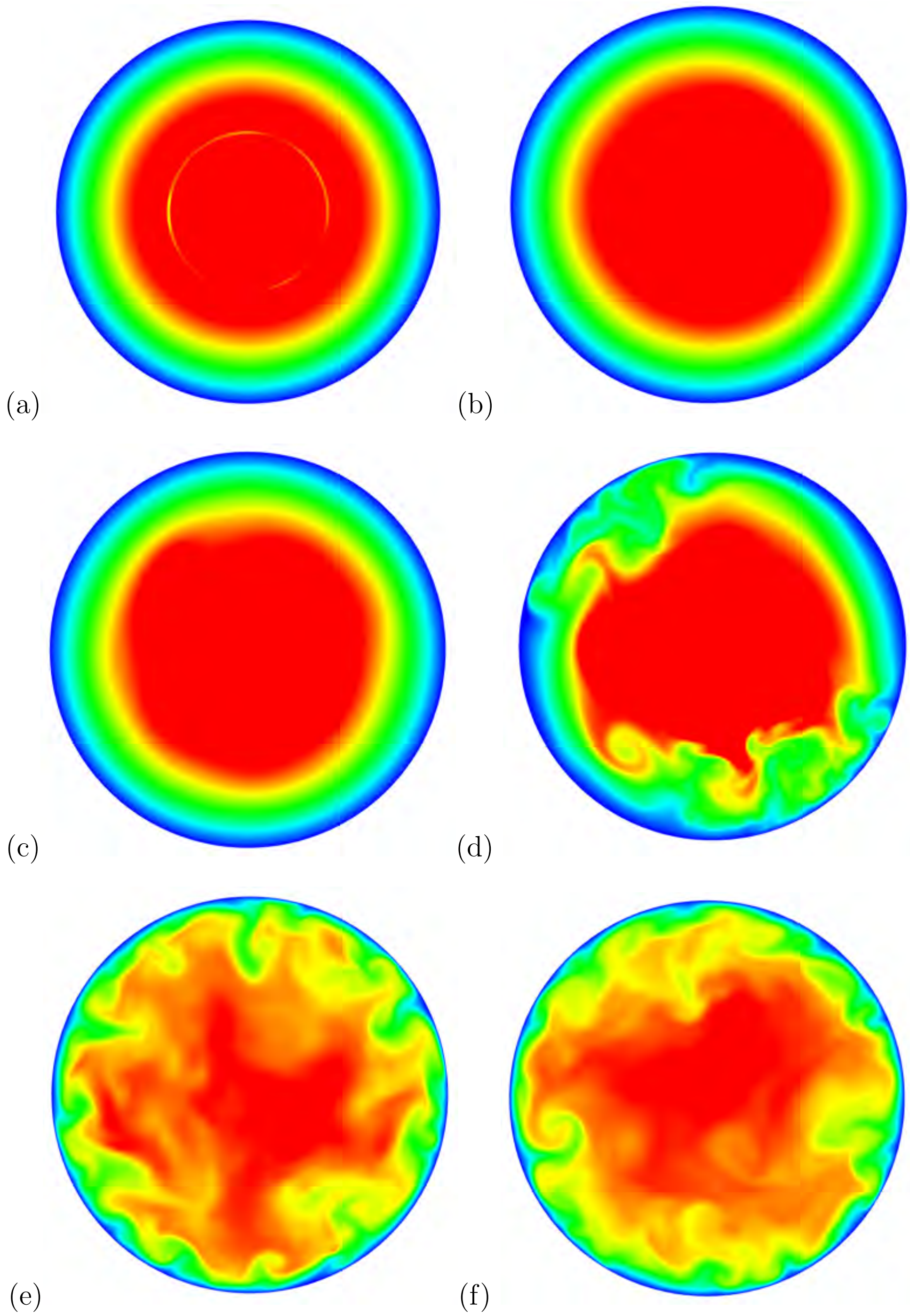

Figure 2: Overview of case 8KS using u at a random instant over selected axial planes. (a) $z = 0$; (b) $z = 20R$; (c) $z = 30R$; (d) $z = 40R$; (e) $z = 220R$; (f) additional pipe flow DNS with streamwise periodicity, $z = 15R$ (case 8KP). Color varies continuously from blue ($u = 0$) to red ($u \geq 1.3U_{\text{bulk}}$).