FINITE REYNOLDS NUMBER EFFECTS ON THE PRESSURE SPECTRUM IN ISOTROPIC TURBULENCE FREE DECAY

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ABSTRACT

The time evolution of the pressure spectrum E_p in freely decaying homogeneous isotropic turbulence (HIT) is investigated via Eddy-Damped Quasi-Normal Markovian (EDQNM) computations. It is well known that physical quantities associated to the energy spectrum evolve through power laws in HIT decay. For both low and high values of Re_λ, the associated power law exponents of these laws are known to depend on the initial conditions, such as the slope of the energy spectrum at the large scales σ, with $E(k \to 0, 0) \propto k^\sigma$. Batchelor (1951) and Lesieur *et al.* (1999) proposed theoretical frameworks to evaluate the power law exponents associated to the pressure decay statistics. These formulae, which relate pressure and energy decay, have been recovered by the use of several underlying hypothesis, such as $Re_\lambda \to +\infty$ and the Joint Gaussian Assumption (JGA). Such hypothesis are not completely satisfied in experiments and numerical simulations, the departure from the theoretical background being caused by a number of effects such as saturation, intermittency and finite Reynolds numbers (FRN). As a consequence, theoretical predictions are rarely completely matched by experimental/numerical results.

In the present work, FRN effects over the prediction of the pressure spectrum are quantified in order to recover information about pressure fluctuations in HIT decay. The first issue investigated is the presence of a plateau in the Kolmogorov (1941) compensated pressure spectrum. This plateau, which has been observed at very high Re_λ ($Re_\lambda = O(10^4)$ in practise), disappears approaching moderate Re_λ. More specifically, the appearance of a $-5/3$ region instead of the classical $-7/3$ Kolmogorov scaling in the pressure spectrum at very small scales is observed. This result justifies the lack of agreement of the Kolmogorov $-7/3$ scaling with several DNS reported in literature, which were performed at moderate Re_λ.

The ratio between the pressure and velocity Taylor microscales λ_p/λ is also analysed, the results being in very good agreement with the predictions by Batchelor (1951) and with the experiments available in open literature. Both large and small Re_λ behaviours are analysed, and the relevance of the FRN effects is quantified.

INTRODUCTION

One of the most classical test cases investigated in the field of Fluid Mechanics is the free decay of Homogeneous Isotropic Turbulence (HIT), because of its relevance in understanding the physical behaviour of turbulent flows and its implications in turbulence modelling. In HIT, the physical quantities associated to the energy spectrum, such as the turbulent kinetic energy $\overline{u^2}$, the integral length scale l and the energy dissipation rate ε, evolve in time by power laws. After the seminal works by Taylor (1935), several comprehensive reviews have been published (e.g. Batchelor (1953); Hinze (1975); Davidson (2004); Sagaut & Cambon (2008)). Nevertheless, some basic aspects, such as the quantification of the power law exponent related to the decay of $\overline{u^2}$, have not been fully understood at the present time.

In HIT, the initial conditions play an important role in the emergence and evolution of different stable decay regimes. The two classical cases investigated in the literature are referred to as Saffman Turbulence ($E(k \to 0, 0) \propto k^2$) and Batchelor turbulence ($E(k \to 0, 0) \propto k^4$). Comte-Bellot & Corrsin (1966) (CBC) proposed analytical formulae to recover the power law exponents as a function σ, which is the slope of the energy spectrum at the large scales. These formulae, which are reported in Table 1, are in very good agreement with numerical simulation results, when corrected by a coefficient α taking into account the breakdown of Permanence of Large Eddies (i.e. $E(k,t) = E(k,t_0), k \ll k_l$) due to non local energy transfers (Eyink & Thomson (2000); Lesieur (2008)). For integer values of σ, $\alpha = 0$ for $\sigma = 1, 2, 3$ and $\alpha \approx 0.52$ for $\sigma = 4$. These formulae have been extended to non-integer values of σ by Meldi & Sagaut (2012): $\alpha = \max[0, 0.65(\sigma - 3.2)]$. They are in excellent agreement with numerical results in open literature (Lesieur (2008); Meldi *et al.* (2011)). Conversely, the comparison of numerical results and theoretical predictions with experiments is a difficult task. In fact, it is not possible to impose a chosen shape of the energy spectrum at very large scales in grid experiments. Moreover, the large-scale spectrum shape is not directly measured, but it is recovered through the application of theoretical formulae to the sampled data, which are usually taken in the inertial/dissipative region.

The attention of the scientific community on HIT decay has been mostly focused on the energy transfer and on the decay law of the velocity-based statistical quantities. Conversely, a limited number of papers in open literature are devoted to the analysis of the pressure spectrum and the related statistics. In some early works, Heisenberg (1948) investigated the pressure gradient variance $\overline{(\nabla p)^2}$ and Batchelor (1951) proposed a seminal work based on the Joint Gaussian Assumption (JCA) of the velocity field. In his work, Batchelor highlighted the correlation between the pressure and velocity field. Casting aside the JGA hypothesis, Hill & Wilczak (1995) derived a theory relating the pressure structure function to fourth-order velocity structure functions. The statistics related to the energy spectrum being extremely sensitive to its shape at large energetic scales, i.e. near the spectrum peak, we can assume that the pressure statistics should exhibit a significant sensitivity to the shape of the energy spectrum.

A number of papers studying pressure statistics through experiments (Uberoi & Corrsin (1953); Pearson & Antonia (2001); Tsuji & Ishihara (2003)) and direct numerical simulation (Schumann & Patterson (1978); Pumir (1994); Gotoh & Fukayama (2001); Yeung *et al.* (2012); Donzis *et al.* (2012)) has been published and general consensus about the nature of the correlation between pressure and velocity fields has been reached. Nevertheless, several open issues are still debated at the present time, one of them being the scaling of the pressure spectrum in the inertial region. Kolmogorov (1941) derived through dimensional analysis the theoretical values of the slope of the energy and pressure spectra in the inertial range for $Re_\lambda \to +\infty$, which are $-5/3$ and $-7/3$, respectively. While there is a strong experimental and numerical evidence of the accuracy of Kolmogorov scaling for the energy spectrum, the results observed in several DNS (see Gotoh & Rogallo (1999); Cao *et al.* (1999); Vedula & Yeung (1999)) cast doubts about the Kolmogorov scaling for the pressure spectrum. Indeed, the numerical results show that the slope of the pressure spectrum in the inertial region is included in the range $[-7/3, -5/3]$.

The deviation of the experimental/numerical results from theory are usually attributed to the fact that a realistic physical configuration is investigated. Effects such as intermittency and Finite Reynolds Number (FRN), which are not accounted for in the traditional theoretical approaches, are assumed to be responsible for the discrepancies observed. While these effects are usually advocated to justify the validity of the recovered results, a few analysis reported in the open literature are devoted to the their assessment and quantification.

Pressure fluctuations have a strong impact on several physical phenomena such as turbulent sound generation, particle dispersion and droplet growth. Several domains of research can gain advantage of a better understanding of the statistics related to the pressure spectrum. The results reported in literature at the present time appear fragmentary, the experiments being affected by not reducible epistemic uncertainties and the DNS results being limited to moderate Re_λ and unsatisfactory resolution at very large scales.

In the present paper, the FRN effects over pressure fluctuations are investigated by the use of an Eddy-Damped Quasi-Normal Markovian (EDQNM) model. This model, which does not account for intermittency effects, allows for a clear analysis of the FRN effects on the pressure spectrum and its statistical quantities.

The paper is divided as follows. In Section II, details about the numerical implementation of the EDQNM model and the set-up of the test case are given. In Section III, the characteristics of the pressure spectrum E_p are investigated. In particular, FRN effects over the slope of E_p in the inertial range are quantified, the results being compared with experimental and numerical results in the literature. In Section IV, the well known relation between the Taylor microscale λ and its counterpart for the pressure spectrum λ_p is analysed. Two asymptotic limits, in agreement with the seminal work by Batchelor (1951), are observed. Finally, in Section V the conclusions are drawn.

	$n_{\overline{u^2}}$	n_ε	$n_{\overline{(\nabla p)^2}}$
High Re_λ	$-2\frac{\sigma-p+1}{\sigma-p+3}$	$-\frac{3(\sigma-p)+5}{\sigma-p+3}$	$-\frac{9(\sigma-p)+15}{2(\sigma-p+3)}$
Low Re_λ	$-\frac{\sigma+1}{2}$	$-\frac{\sigma+3}{2}$	$-\frac{(\sigma+2)}{1}$

Table 1. Analytical formulae for the prediction of the power-law exponents of the decay of the main HIT statistical quantities. The high Re_λ formulae are proposed by Comte-Bellot & Corrsin (1966) and revisited by Meldi & Sagaut (2012), while the low Re_λ formulae are elaborated by Clark & Zemach (1998).

EDQNM MODEL & SET UP OF THE SIMULATIONS

The EDQNM model is a quasi-normal closure based on the discretisation of the Lin equation, which is the spectral counterpart of the Karman-Howarth equation. The model accurately describes the triadic energy transfer in wave number space and can be used to evaluate several statistics in turbulence, up to three-point fourth-order correlations. It has proved to be a reliable, robust and efficient method to investigate HIT free decay (e.g. Meldi *et al.* (2011); Tchoufag *et al.* (2012)). The reader is addressed to the works of Orszag (1970), Lesieur (2008) and Sagaut & Cambon (2008) for an exhaustive discussion.

A database of computations has been generated, imposing as initial energy spectrum a simplified version the energy spectrum formulated by Pope (2000):

$$E(k) = \begin{cases} A\,k^\sigma & kl \ll 1 \\ C_k\,\varepsilon^{2/3}k^{-5/3}f_l(kl) & kl \gg 1 \end{cases} \tag{1}$$

with

$$f_l(kl) = \left(\frac{kl}{[(kl)^{c_1}+c_2]^{1/c_1}}\right)^{5/3+\sigma} \tag{2}$$

The free parameter c_1 in Equation 2 is set to 1.5, while the parameter c_2 is computed in order to recover $l(0) = 1$. An initial condition $Re_\lambda = 10^6$ has as well been imposed. Defining $k_l(0)$ the initial position of the peak of the energy spectrum, the maximum resolution at the large scales is $k_0 = 10^{-10} k_l(0)$. The minimum resolution is instead set to $k_{\max} = 10/\eta(0)$, being $\eta(0)$ the initial value of the Kolmogorov scale. The database consists of four simulations, corresponding to integer values of the parameter $\sigma = 1, 2, 3$ and 4. Using these simulations, the HIT decay has been described in the range $10^{-4} \leq Re_\lambda \leq 10^5$ with an excellent resolution at the large scales. All the results are referred to the normalised time scale $\tau = t/t_0$, with $t_0 = \overline{u^2}(0)/\varepsilon(0)$ being the initial characteristic turnover time.

The EDQNM model has been used to compute the triadic energy transfer and, simultaneously, to compute the pressure spectrum E_p (Meldi & Sagaut, 2013). Starting from the relation between the pressure fluctuations and the fourth-order velocity correlations, the application of the JGA approximation allows to recover the following form of the pressure spectrum:

$$E_p(k) = \frac{k^2}{4\pi} \int_{\mathbf{r}+\mathbf{q}=\mathbf{k}} E(r) E(q) \frac{\sin^4 \beta}{r^4} \, d\mathbf{q} \tag{3}$$

where $[\mathbf{k}, \mathbf{r}, \mathbf{q}]$ is the vectorial base used to compute the energy triadic interactions in the spectral space, and β is the angle facing $\mathbf{r}$ in the triangle formed by the three vectors. The mean-square pressure fluctuation $\overline{p^2}$ and the pressure gradient $\overline{(\nabla p)^2}$ have been respectively recovered as:

$$\overline{p^2} = \int_0^\infty E_p(k) \, dk \tag{4}$$

$$\overline{(\nabla p)^2} = \int_0^\infty k^2 E_p(k) \, dk \tag{5}$$

These quantities have been as well sampled in the range $10^{-4} \leq Re_\lambda \leq 10^5$.

FRN EFFECTS ON THE PRESSURE SPECTRUM AND THE KOLMOGOROV SCALING LAW

The characteristics of the pressure spectrum $E_p(k,\tau)$, recovered by the EDQNM model, are now analysed. First, the interest will be restricted to high Reynolds HIT ($Re_\lambda \geq 10^3$) and to Saffman and Batchelor turbulence.

The time evolution of the pressure spectrum is shown in Figure 1 (a) and (b) for the two considered cases. The presence of an extended range for which $E_p(k,\tau) = A_p(\tau) k^2$ is observed in the large scales region for both cases. This range has been predicted theoretically and observed numerically by Lesieur *et al.* (1999) and it exhibits a constant slope coefficient 2. The slope of this range is independent of the parameter σ. Conversely, the time evolution of the coefficient $A_p(\tau)$ is driven by the initially imposed energy spectrum. Dimensional analysis allows to recover the following expression:

$$A_p(\tau) \simeq \frac{8}{15} \int_0^{+\infty} \frac{E^2(k,\tau)}{k^2} dk \tag{6}$$

Equation 6 is in very good agreement with the numerical results by Lesieur *et al.* (1999) and Meldi & Sagaut (2013).

The compensated pressure spectrum $E_p^s = E_p/(\varepsilon^{4/3} k^{-7/3})$ is now investigated, in order to derive clear information about the slope of the pressure spectrum in the inertial range. E_p^s is displayed in Figure 2 (a) for Saffman turbulence. The results of Batchelor turbulence simulations are here omitted, as the information deducible is the same observed for Saffman turbulence. In this case, we consider pressure spectra in the range $100 \leq Re_\lambda \leq 10^5$. At very high Re_λ, it is possible to observe a fully developed plateau for more than three decades. A similar plateau, but much less developed, has been observed in the DNS results by Pumir (1994). Moreover, a bump in the compensated pressure spectrum is observed approaching the dissipation region. This phenomena, which is classically referred to as *bottleneck effect* when dealing with the energy spectrum, has been as well observed in the pressure spectrum by Gotoh & Fukayama (2001). The present results indicate that the slope of this bump is about $1/4$ at very high Reynolds numbers. Decreasing the Reynolds number investigated, the slope of the bump progressively increases up to $3/10$ for $Re_\lambda \approx 600$. This behaviour can be observed in Figure 2 (b), were the local slope of the pressure spectra is reported. If even lower Re_λ are considered, the plateau of the compensated spectra progressively disappears and the bump region degenerates into a small secondary plateau. Moreover, the slope becomes progressively steeper at lower Re_λ numbers, with an asymptotic value of $-5/3$.

A global picture can be drawn by the analysis of the EDQNM results. At very large Re_λ, the pressure spectrum complies with the $-7/3$ Kolmogorov scaling. Moreover, a bottleneck near the Kolmogorov scale can be observed. At very small Re_λ, the Kolmogorov inertial range is no longer present, but the bottleneck near the Kolmogorov scale evolves into a small (less than one-decade long) $-5/3$ range. This behaviour, which can be clearly appreciated in Figure 2 (b) for moderate Reynolds numbers, is independent of the shape of the energy spectrum E. The present results support the discussion by Tsuji & Ishihara (2003), who argued that the lack of agreement between most of the DNS results reported in literature and the Kolmogorov scaling law is due to the FRN effects. More specifically, Re_λ presently investigated in DNS are not sufficient for a fully developed plateau of the compensated spectrum to be observed. While Tsuji & Ishihara (2003) indicate a minimum limit of $Re_\lambda = 600$ to neglect FRN effects, EDQNM results suggests that $Re_\lambda \geq O(10^4)$ is the minimum threshold limit to observe the Kolmogorov scaling. Indeed, the EDQNM results proposed are not reachable by present DNS, due to the prohibitive amount of computational resources needed to simulate a flow at $Re_\lambda = 10^6$.

SENSITIVITY OF THE RELATION BETWEEN THE ENERGY-BASED AND PRESSURE-BASED TAYLOR MICROSCALES TO FRN EFFECTS

During the past decades the relation between the decay of the Taylor microscale $\lambda^2 = 10\nu\overline{u^2}/\varepsilon$ and the equivalent scale for the pressure spectrum $\lambda_p^2 = \rho^2 \left(\overline{u^2}\right)^2 / \overline{(\nabla p)^2}$ has been extensively studied (e.g. Batchelor (1951); Vedula & Yeung (1999); Pearson & Antonia (2001)). Relying on the JGA hypothesis, Batchelor (1951) derived the following re-

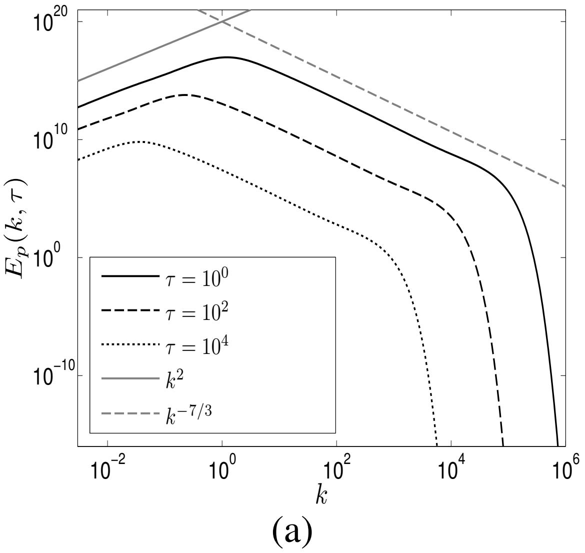

Figure 1. Evolution of the pressure spectrum $E_p(k,\tau)$ in time, in the case of high Reynolds number ($Re_\lambda > 10^3$). The cases investigated are (a) Saffman turbulence and (b) Batchelor turbulence, respectively. Two ranges at $E_p \propto k^2$ and $E_p \propto k^{-7/3}$ are observed at large scales and in the inertial range.

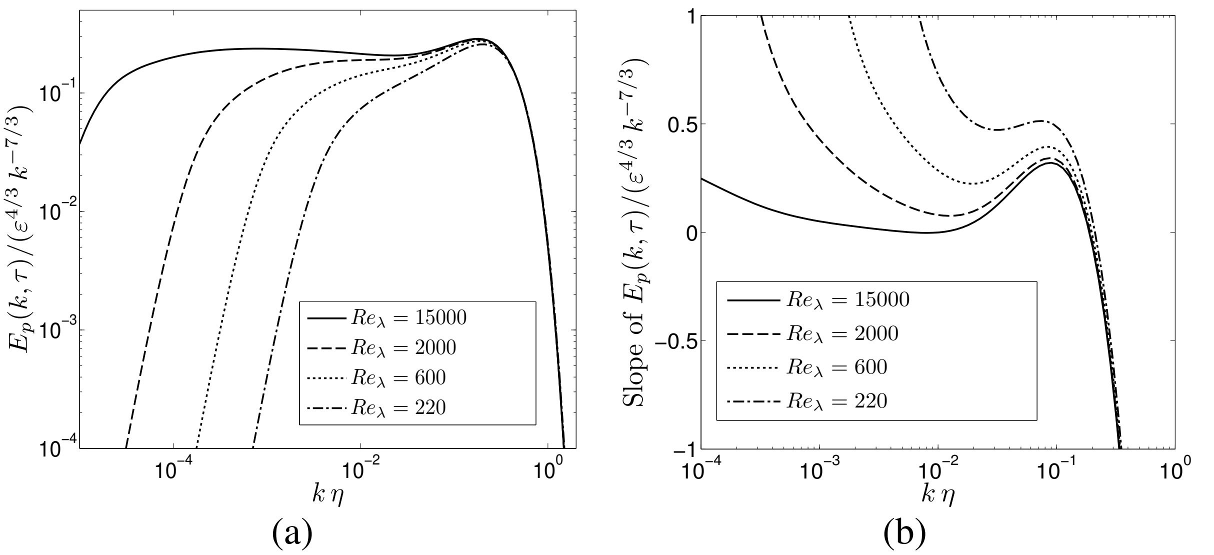

Figure 2. (a) Compensated pressure spectrum $E_p^s = E_p/(\varepsilon^{4/3}\,k^{-7/3})$ and (b) local slope of the compensated pressure spectrum in the case of Saffman turbulence.

lations in the case of high and low Re_λ:

$$\begin{cases} \lambda_p/\lambda &= 0.11 Re_\lambda^{1/2}, \quad Re_\lambda \to \infty \\ \lambda_p/\lambda &= 0.81, \qquad\quad Re_\lambda \to 0 \end{cases} \tag{7}$$

These laws can be deduced as well by CBC dimensional analysis, for both high and low Re_λ. If λ, λ_p and Re_λ are described by classical power laws, such as the ones proposed by Comte-Bellot & Corrsin (1966), and recalling that $Re_\lambda = \sqrt{\frac{20}{3}}\frac{\overline{u^2}}{\sqrt{\nu\varepsilon}}$, Equation 7 can be rewritten as:

$$\tau^{0.5(2n_{\overline{u^2}} - n_{\overline{(\nabla p)^2}})}\,\tau^{-0.5} \propto \tau^{0.5(n_{\overline{u^2}} - 0.5(n_\varepsilon))} \tag{8}$$

for high Re_λ and

$$\tau^{0.5(2n_{\overline{u^2}} - n_{\overline{(\nabla p)^2}})}\,\tau^{-0.5} \propto \tau^0 \tag{9}$$

for low Re_λ. Substituting the values of $n_{\overline{u^2}}$, n_ε and $n_{\overline{(\nabla p)^2}}$ with the corresponding CBC formulae values reported in Table 1, it is possible to observe that the left and right term of Equation 8 and 9 decay with the same power law exponent, for every value of $\sigma \in [1,4]$. Moreover, if $\sigma = 1$, the power law exponent is 0 and the ratio between λ_p and λ is constant in time both at high and low Re_λ.

The coefficient $C_p = (\lambda_p/\lambda)/Re_\lambda^{1/2}$ characterising the decay at high Re_λ is now investigated. If the CBC decay power laws are exactly recovered in EDQNM simulations, the coefficient C_p will be an invariant in HIT decay and its value will be determined by the parameter σ. The results are plotted against Re_λ in Figure 3 (a) for the classical decay regimes of Saffman and Batchelor turbulence. The analysis of the results shows that at very high Re_λ the value of C_p is almost constant, in agreement with Batchelor (1951). Nevertheless, the value recovered by the EDQNM simulations is smaller. In the range $10^3 \leq$

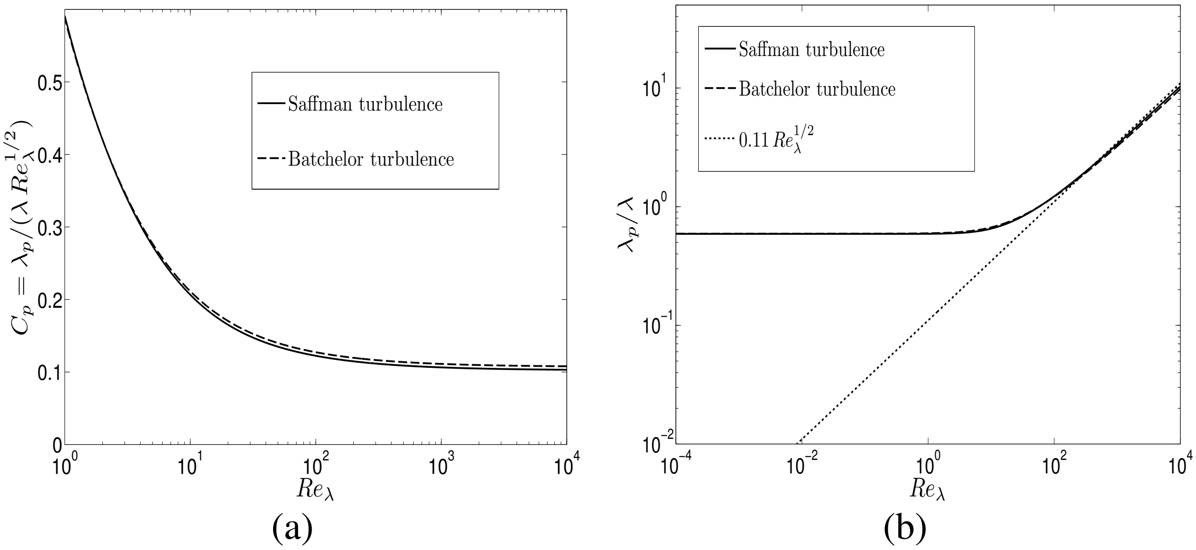

Figure 3. (a) Coefficient $(\lambda_p/\lambda)/Re_\lambda^{1/2}$ in the high Re_λ formula by Batchelor (1951) and (b) ratio of the Taylor microscales λ_p/λ for $10^{-4} \le Re_\lambda \le 10^5$.

$Re_\lambda \le 10^5$, $C_p \in [0.103, 0.106]$ for Saffman turbulence and $C_p \in [0.107, 0.111]$ for Batchelor turbulence. C_p value appears to increase progressively faster due to the FRN effects at moderate Re_λ, leading the ratio λ_p/λ to diverge from the high Reynolds behaviour $\approx 0.11 Re_\lambda^{1/2}$ and to reach the low Reynolds asymptotic limit. Moreover, the curves λ_p/λ for Saffman and Batchelor turbulence converge toward an asymptotic value of $\lambda_p/\lambda = 0.6$ for low Re_λ, as it can be clearly appreciated in Figure 3 (b). This result is in qualitative agreement with Batchelor (1951) and in very good agreement with the experimental results Pearson & Antonia (2001); Uberoi & Corrsin (1953).

CONCLUSIONS

The effects of finite Reynolds number over the time evolution of the pressure spectrum and the related statistics have been investigated by numerical EDQNM simulations in the range $10^{-4} \le Re_\lambda \le 10^5$. In order to quantify the sensitivity of E_p to the slope of the energy spectrum E at large scales, the initial conditions have been chosen such that $\sigma = 1,2,3,4$.

The analysis of the sampled pressure spectra confirms the presence of two ranges. The first range, $E_p(k,\tau) = A_p(\tau)k^2$, is observed in the large scales region. The power law exponent driving the time evolution of the coefficient $A_p(\tau)$ shows a sensitivity to the parameter σ. The second range, which represents the inertial region of the pressure spectrum, can be approximated as $E_p(k) \propto \varepsilon^{4/3} k^{-7/3}$. This last range is progressively less clear to observe for decreasing Re_λ, as it merges with the pseudo-bottleneck region close to the Kolmogorov scale. The presence of a short range exhibiting a $-5/3$ scaling near the Kolmogorov scale, which has been reported in several DNS, originates in the pseudo-bottleneck and is thus due to the FRN effects.

As a last point, the ratio between the Taylor microscales λ_p/λ has been investigated. The asymptotic behaviours derived by Batchelor (1951) for high and low Re_λ are recovered by the present EDQNM results. For the high Re_λ case, the coefficient $C_p = \lambda_p/(\lambda\, Re_\lambda^{1/2}) \in [0.105, 0.11]$ and exhibits a mild sensitivity to the parameter σ. Moreover, the comparison with DNS data confirms that finite Reynolds number effects are significant in the determination of C_p for $Re_\lambda \le 10^4$, and that they lead to deviation of the results from the theoretical JGA behaviour. The ratio λ_p/λ converges to a universal value of ≈ 0.6 at very low Re_λ.

An important conclusion that can be drawn by the present study is that, for $Re_\lambda \le 10^4$, intermittency is not the sole physical mechanisms yielding the occurrence of anomalous exponents, e.g. for λ_p/λ. Thus, DNS of HIT decay for $Re_\lambda O(10^5)$ should be performed in order to recover clear information about pure intermittency effects.

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