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A-PRIORI VALIDATION OF EDDY VISCOSITY SUBGRID-SCALE MODELS FOR WALL BOUNDED TURBULENCE WITH PRESSURE GRADIENT

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ABSTRACT

In the present work, the a priori analysis of three subgrid scale models is conducted using a database of Direct Numerical Simulation (DNS) of converging-diverging channel flow at Reynolds number $Re_\tau = 617$ based on friction velocity at inlet. In addition to the Gaussian filter, a Least Square Spline filter of fifth order (Lss-5^{th}) is used to investigate both the effects of filter scale and filter type on the energy transfer between sub-filter and resolved scales and the subgrid scales (SGS) energy dissipation. Then, a priori estimates of the coefficients of three subgrid scale models are computed on the full computational domain subjected to both favorable and adverse pressure gradients. The coefficients of the models are found to be sensitive to the sudden production of turbulent kinetic energy observed in near wall adverse pressure gradient (APG) region.

INTRODUCTION

The turbulent boundary layer flow subjected to an adverse pressure gradient induced by curvature is of crucial importance for many applications including aerodynamics of airfoils, ground vehicles or turbine blades. Significant progress is needed in understanding the near wall turbulence in order to improve numerical models. The available statistical models usually fail to predict flows at the onset of separation as they are based on scalings which are no more valid with pressure gradient. Large Eddy Simulations (LES) are expected to provide better results on such flows but at larger computational cost. Subgrid scale modeling of wall bounded turbulence is an active research subject. However, most detailed analysis of the energy transfer and models performances are done on flows without pressure gradient. The physics of turbulence is in fact significantly modified under adverse pressure gradient. Laval *et al.* [4] have evidenced in a DNS of converging diverging channel flow an intense generation of coherent structures corresponding to a localized peak of turbulent kinetic energy production which is due to a rise of low-speed streaks instabilities. From a modeling point of view, this phenomena could be associated with a strong backward energy transfer from the subgrid scales to the resolved scales (back scatter), which is not modeled by the standard turbulent viscosity SGS models [3].

Several other LES were performed recently on similar flows with various adverse pressure gradients leading to attached or detached flows [1] but only few were focused on energy transfer and the accuracy of subgrid scale models. Therefore a detailed a priori analysis on channel flow with pressure gradient, which is expected a promising research, is an opportunity to make progress in understanding the physics of such flows.

Kuban *et al.* [3] conducted research on the same flow and reported that the influence of the grid size related to the filter scale is more important for optimization of subgrid models. For this given geometrical configuration, the optimal model coefficient is not universal because none of the tested coefficients for the particular cases leads to good near-wall agreement throughout the simulation domain. This can be explained by the fact that the SGS energy transfer probably exhibits very different properties within the full domain, and especially in the region of minimum friction velocity. Kuban *et al.* [3] confirms that performing wall-resolved LES calculations without initial calibration of the SGS model coefficient is hazardous.

Therefore, the aim of the present work is to provide some useful information on scale interactions and energy transfer in near wall turbulence with pressure gradient. The

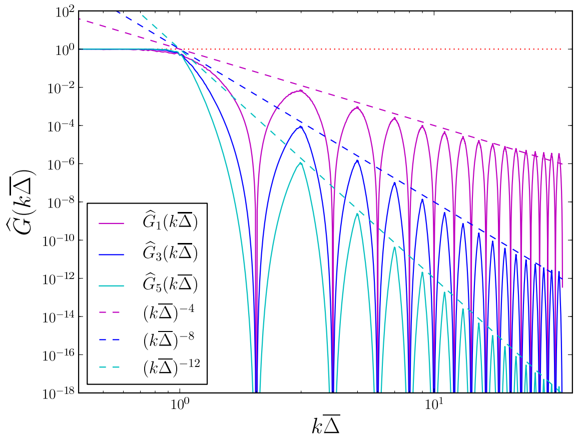

Figure 1. Transfer function $\widehat{G}_p$ of the Least Square Spline Filter (LSS) with 3 different spline orders p=1,3,5 and the corresponding slope of their asymptotic behavior

effect of filter scale and filter type on SGS energy transfer, and coefficients of subgrid models will be investigated using Gaussian filter with a smooth transfer function and Lss-5^{th}. The latter filter, which is novel for subgrid scale modeling, has a better flexibility to easily filter on both homogeneous and non-homogeneous grids. The derivation and a routine of Lss-5^{th} are provided by Dierckx [2]. The order of this filter is function of the order of the spline as can be seen from Fig. 1. The subgrid models investigated are the Smagorinsky model (SM), Wall-Adapting Local Eddy-viscosity model (WALE) and the σ model which has been tested on isotropic turbulence and flat channel flow by Nicoud *et al.* [9]. The a priori estimation of the SM model will be compared to the Dynamic Smagorinsky model (DSM).

DNS DATA BASE AND NUMERICAL METHOD

The analysis is conducted using the database of converging diverging channel flow documented in [4]. The simulation domain is $4\pi \times 2 \times \pi$ with spatial resolution of $2304 \times 385 \times 576$ in the streamwise, normal and spanwise direction respectively. The Reynolds number based on inlet velocity and half channel width is $Re_\tau = 617$. At this Reynolds the flow slightly separates at the lower curved wall and is at the onset of separation at the upper wall. Therefore, two different configurations of pressure gradient as well as the effect of wall curvature can be investigated and compared. The adverse pressure gradient region starts at $x = -0.2$ at the lower wall and is slightly shifted downstream ($x = +0.3$) at the upper wall, see Fig. 2.

As described in [4], a second peak of turbulent kinetic energy appears at the beginning of the APG region and moves away from the wall. The second peak, which corresponds to the generation of vortices much stronger than typical vortices in the buffer and log region of a boundary layer without pressure gradient, is present at the two walls but is more intense at the lower wall where it is generated within a short streamwise region near $x = 0.5$, slightly downstream of the bump summit. The starting location of these intense vortices coincide with the increase of the turbulent kinetic energy peak (see Fig. 2).

The analysis are performed using a specific numerical code parallelized with MPI in order to process the large database. The spatial derivatives are approximated by an eighth-order compact finite-difference scheme. The filtering operations are performed in streamwise-x and spanwise-z directions. No filtering is applied in vertical direction.

ENERGY TRANSFER MECHANISM

The main role of a subgrid-scale model is to reproduce the dissipative effect of the unresolved scales on the resolved ones. In order to better understand the interaction between resolved and unresolved scales, the transport equation of the resolved kinetic energy ($\overline{q}^2$) can be written as the following:

$$\underbrace{\frac{\partial \overline{q}^2}{\partial t} + \frac{\partial}{\partial x_j}(\overline{q}^2\overline{u}_j)}_{Advection\ of\ \overline{q}^2} = -\underbrace{\frac{\partial}{\partial x_j}(\overline{pu}_j)}_{Press.\ Diff.\ of\ \overline{q}^2} \tag{1}$$

$$+\underbrace{\frac{\partial}{\partial x_j}\left(\nu\frac{\partial \overline{q}^2}{\partial x_j}\right)}_{Visc.\ Diff.\ of\ \overline{q}^2} - \underbrace{\frac{\partial}{\partial x_j}(\tau_{ij}\overline{u}_i)}_{SGS\ Diff.} - \underbrace{\nu\frac{\partial \overline{u}_i}{\partial x_j}\frac{\partial \overline{u}_i}{\partial x_j}}_{Visc.\ Diss.\ of\ \overline{q}^2} + \underbrace{\tau_{ij}\overline{S}_{ij}}_{SGS\ Diss.}$$

where $\overline{u}_i$ is the filtered velocity field on specific filtering scale $\overline{\Delta_x^+}$, $\overline{\Delta_z^+}$ in streamwise and spanwise direction respectively and $|\overline{S}| = \sqrt{2\overline{S}_{ij}\overline{S}_{ij}}$ is the modulus of the deformation tensor. In this equation, $\tau_{ij} = \overline{u_i u}_j - \overline{u}_i\overline{u}_j$ is the subgrid tensor that needs to be modeled. Equation (1) shows that the energy exchange between resolved and unresolved scales is driven by two mechanisms: the SGS dissipation (ε_{sgs}) and the SGS energy transfer T_{sgs}.

$$\varepsilon_{sgs} = \tau_{ij}\overline{S}_{ij}, \quad T_{sgs} = \frac{\partial(\overline{u}_i\tau_{ij})}{\partial x_j} - \varepsilon_{sgs} \tag{2}$$

Both contributions can be positive or negative. The positive (resp. negative) value of ε_{sgs} (resp. T_{sgs}) can be interpreted as a 'back scatter' of energy from SGS to resolved scales.

With different filtering properties of the Gaussian filter and Lss-5^{th}, the sensibilities of the SGS energy transfer and SGS energy dissipation with respect to the filter type and filter width could be investigated. Up to 10 combinations of streamwise and spanwise filter scales (in the range $50 < \overline{\Delta_x^+} < 200$ and $20 < \overline{\Delta_z^+} < 80$) were tested for each filter type. However, in the present paper, only three configurations are presented.

In Fig. 3, very small SGS energy dissipation appear in the favorable pressure gradient region for the two filter types and the two filter scales. A region with slightly negative values of $-\langle\varepsilon_{sgs}\rangle$ is even perceptible with the Gaussian filter and the larger filter scale. Regions with intense positive values of $-\langle\varepsilon_{sgs}\rangle$ are observed in APG regions at both walls. They are more intense in near wall regions of lower wall where the instability of streaks and the generation of intense vortices have been demonstrated to be the most active (see Fig. 2). The SGS energy transfer T_{sgs} exhibits the same trend except that a clear region of backscatter (positive T_{sgs}) is only present on the summit of the bump with the widest Gaussian filter (The white spots occurring with the LSS filter are meanly due to a lack of convergence). As compared to the region of intense negative ε_{sgs}, the regions of maximum forward scatter of T_{sgs} are thinner and more detached from the two walls in the downstream part of the trails. In order to complement the energy transfer analysis, the fraction of points that experience positive ε_{sgs} and negative T_{sgs} were also computed (see Fig. 4). The probability of energy backscatter ($P(T_{sgs} < 0) > 0.5$ is restricted to near wall region at all streamwise positions except in the

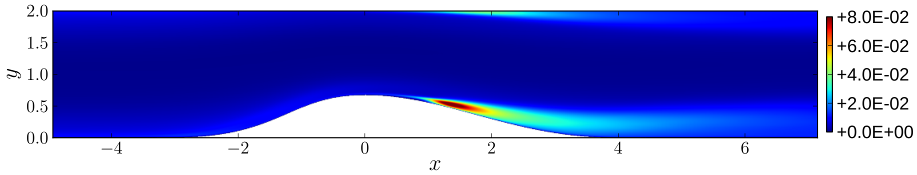

Figure 2. Turbulent kinetic energy $k = \frac{1}{2}\langle u_i' u_i' \rangle$

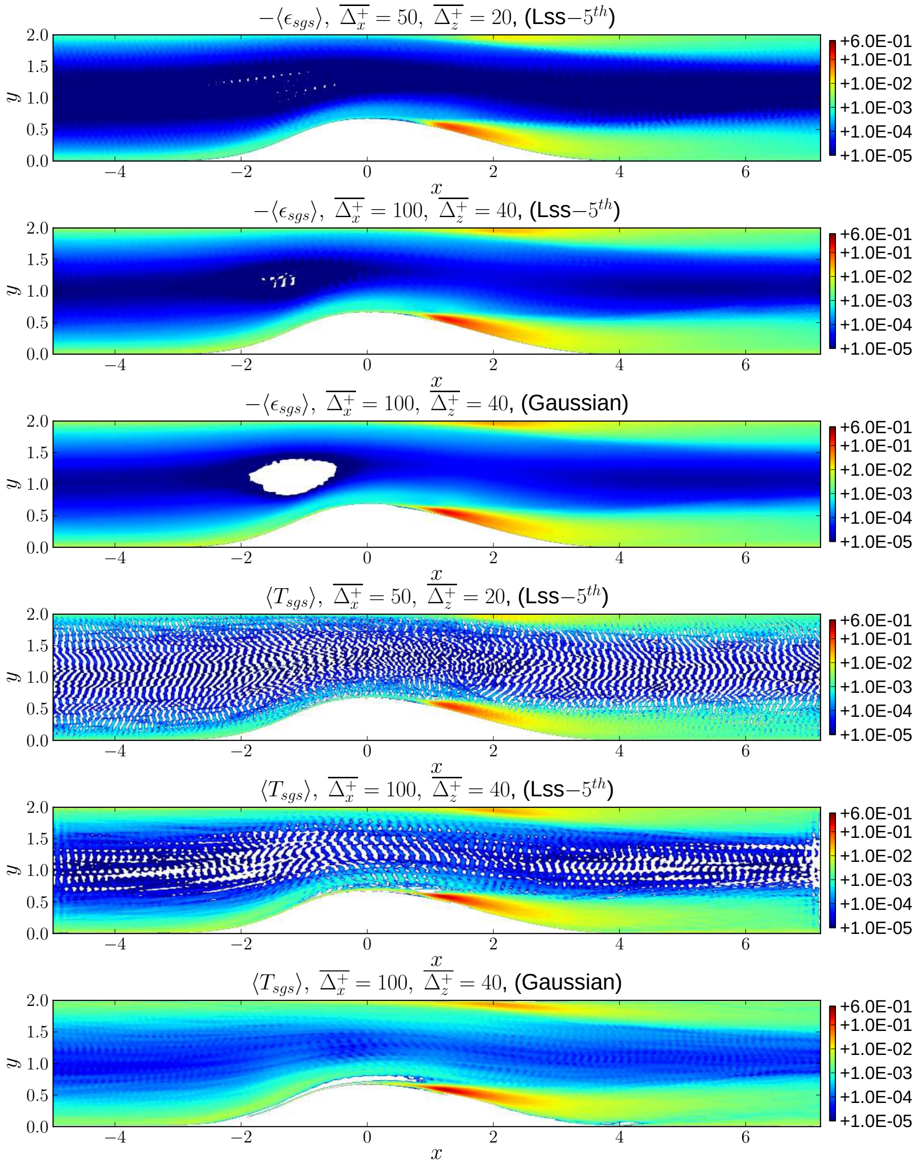

Figure 3. SGS dissipation $-\langle \varepsilon_{sgs} \rangle$ (top 3 plots) and SGS energy transfer T_{sgs} on different filter scale with Lss-5^{th} and Gaussian filters

region of strong adverse pressure gradient (essentially at the lower wall). Due to the different "smoothness" properties of the two investigated filters the probability statistics with the Lss-5^{th} filter are closer to the equal probability distribution than for the Gaussian filter at equivalent filter width. However the distribution exhibits the same feature. The situation is different for the probability of subgrid scale dissipation as high probability of $P(\varepsilon_{sgs} > 0)$ is only present on the summit of the bump and in the converging central part of the channel for the largest Gaussian filter. The observations in Fig. 4 are in good agreement with the average of ε_{sgs} and T_{sgs} meaning that the mean values are not affected by intermittent strong events of energy transfer that could affect our analysis of the main feature of the flow in the region of intense values of energy transfer.

A PRIORI EVALUATION OF MODEL COEFFICIENTS

In order to evaluate the performance of each SGS model, we consider a necessary conditions to produce accurate results that is the ability of the model to predict the SGS transfer rate of resolved kinetic energy ([7], [6], [8]):

$$\left\langle \tau_{ij}^{mod} \overline{S}_{ij} \right\rangle = \langle \tau_{ij} \overline{S}_{ij} \rangle \quad (3)$$

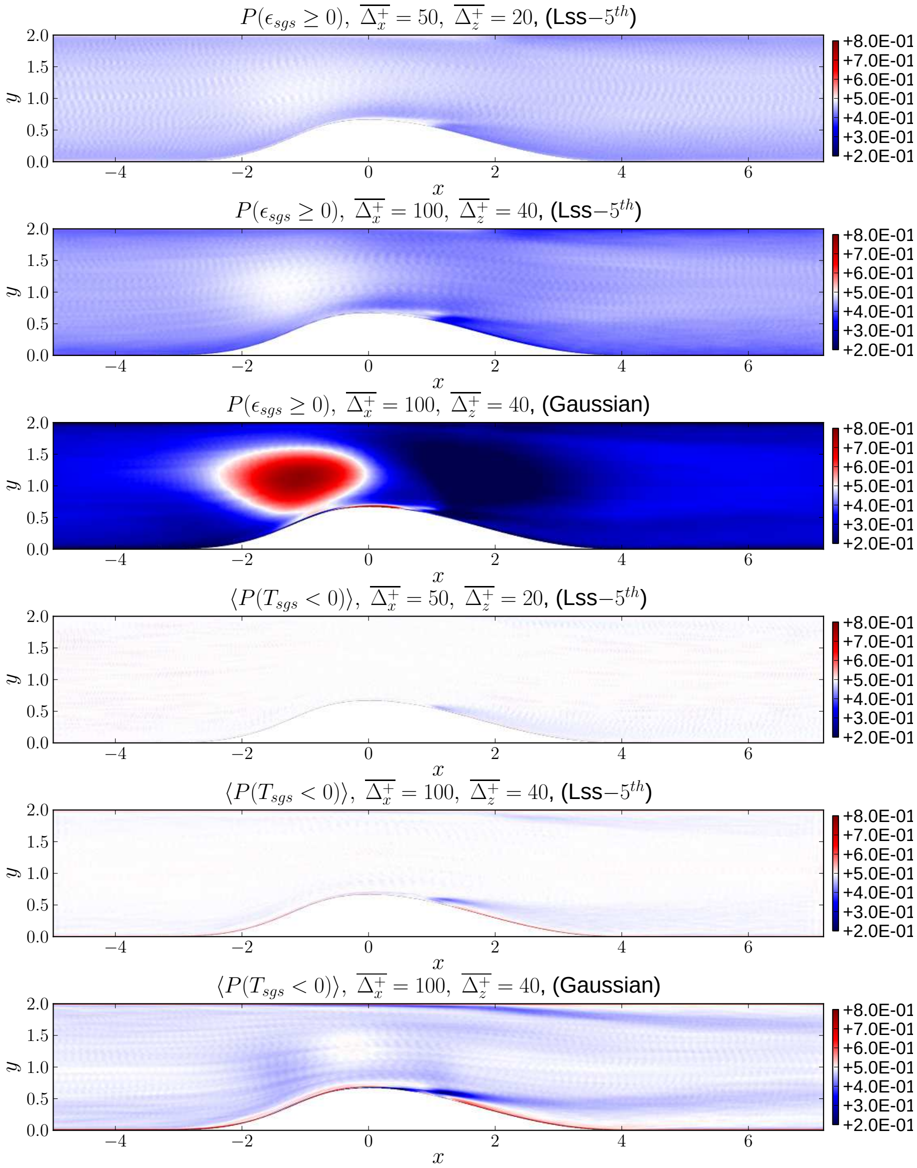

Figure 4. Percentage of points experiencing positive ε_{sgs} (top 3 plots) and energy backscatter (negative T_{sgs}) on different filter scale with Lss-5^{th} and Gaussian filters

where τ_{ij}^{mod} is the modeled SGS tensor. Therefore, a priori estimates of the model coefficients C_s of SM, C_w of WALE, C_σ of σ model, are respectively evaluated by matching the measured and modeled SGS dissipation as:

$$C_s^2 = \frac{\langle \tau_{ij}\overline{S}_{ij}\rangle}{-2(\overline{\Delta})^2\langle|\overline{S}|\overline{S}_{ij}\overline{S}_{ij}\rangle} \quad (4)$$

$$C_w^2 = \frac{\langle \tau_{ij}\overline{S}_{ij}\rangle}{-2(\overline{\Delta})^2\left\langle \frac{(S_{ij}^d S_{ij}^d)^{3/2}\overline{S}_{ij}\overline{S}_{ij}}{(\overline{S}_{ij}\overline{S}_{ij})^{5/2}+(S_{ij}^d S_{ij}^d)^{5/4}}\right\rangle} \quad (5)$$

$$C_\sigma^2 = \frac{\langle \tau_{ij}\overline{S}_{ij}\rangle}{-2(\overline{\Delta})^2\left\langle \frac{\sigma_3(\sigma_1-\sigma_2)(\sigma_2-\sigma_3)}{\sigma_1^2}\right\rangle} \quad (6)$$

The grid size $\overline{\Delta}$ is evaluated by $\overline{\Delta} = \sqrt{\overline{\Delta}_x^2 + \overline{\Delta}_x^2 + \Delta_{gridy}^2}$ taking into account the implicit filtering in y direction with the grid size in wall-normal direction Δ_{gridy}. One important results is the disparity of the model coefficients in space and the influence of the filter scales and the filter type.

The coefficients of Smagorinsky model distribute differently in the whole region as can be seen from Fig.5. The negative values (the blue region) or very low positive values which correspond to a source of kinetic energy or region of almost no SGS dissipation appear mainly in the center of convergent region and, at much lower level, in the vicinity of the two walls experiencing adverse pressure gradient. A region of high value of C_s is located in the central part of the adverse pressure gradient region outside from the region of strong turbulent kinetic energy. For the statistics with the LSS filter, C_s only slightly increases with the filter scale. Even if the main feature are comparable, the values of C_s with the Gaussian filter is slightly larger in absolute value. The a-priori estimation of C_s can be compared with the Lilly dynamic estimation C_d of the Smagorinsky coefficient [5] using only resolved scales and a test filter with a scale twice as large as the explicit filter in each filtered directions. The main features are comparable except the presence of small negative regions of C_d observed with the LSS filter at the exact location of intense turbulent kinetic energy production at the lower wall and larger values in magnitude in the center of the channel.

The coefficient of the WALE model C_w computed with

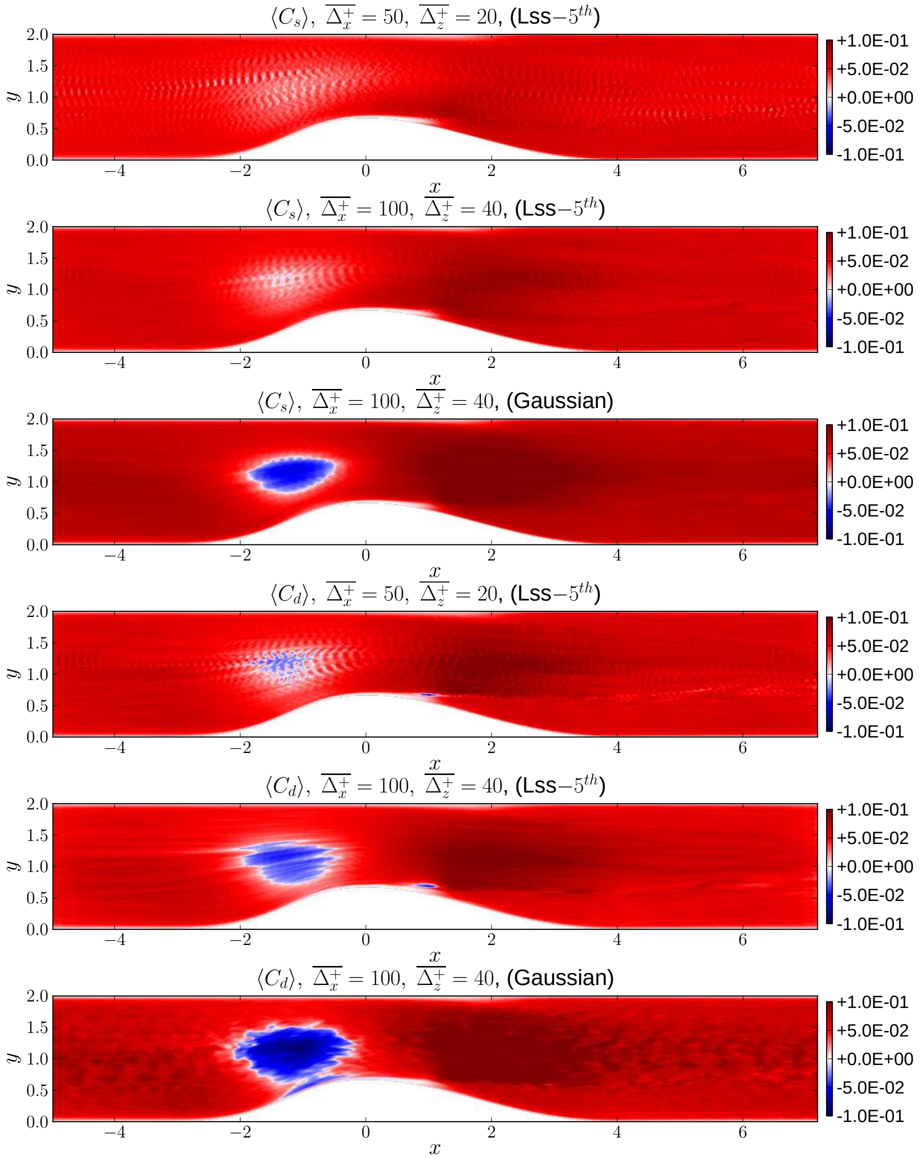

Figure 5. Coefficients of Smagorinsky model C_s (top 3 plots) and Dynamic Smagorinsky model C_d computed with a Gaussian and a LSS-5th filter and two different filter scales

smallest LSS filter is almost constant within the full computational domain down to the wall with an averaged value closed to 0.2 except in the converging part where it drops to almost zero (see Fig. 6). The same global behavior is observed with the largest LSS filter but a short layer of higher coefficient appears in a region starting at the bump summit. Using the Gaussian filter, the same region of higher values is present but a thin layer of negative coefficient appears on the bump summit which extends for higher filter width (not shown). The spatial distribution of the coefficient for the SIGMA model is similar to that of the WALE model but the averaged value is more sensitive to the filter width.

CONCLUSIONS

The effects of filter scale and filter type on SGS energy dissipation and energy transfer between resolved scales and unresolved scales are investigated on turbulence with pressure gradient using least square spline filter and Gaussian filter. The results demonstrate how forward scatters increases when increasing the filter scale. Similar behavior are observed with the Gaussian and LSS filters even if the intensity of the energy transfer may differ significantly in the near wall region. An important forward energy transfer in the adverse pressure gradient region is shown to be restricted to a thin layer moving away from the wall when a backscatter regenerates in the near wall region further downstream. The filter-scale and filter-type dependence of the coefficients of four subgrid models (SM, DSM, WALE, σ model) are also investigated. These coefficients, which distribute differently in the whole region, have significantly larger values with Gaussian filter. It is demonstrated that the a-priori estimation of C_s agree reasonably well with its dynamic computation. The a-priori estimation with the WALE and SIGMA model seems less dependent on the filter within the region of complex physics and intense turbulent kinetic energy transfer on the downstream part of the bump. These first results on a-priori estimation of the model coefficients in this flow with strong pressure gradient is able to explain why LES results with constant values of the Smagorinsky and WALE model coefficients are not able to accurately predict the near wall statistics for grid spacing equivalent to the filter scales of the present a-priori analysis. [3].

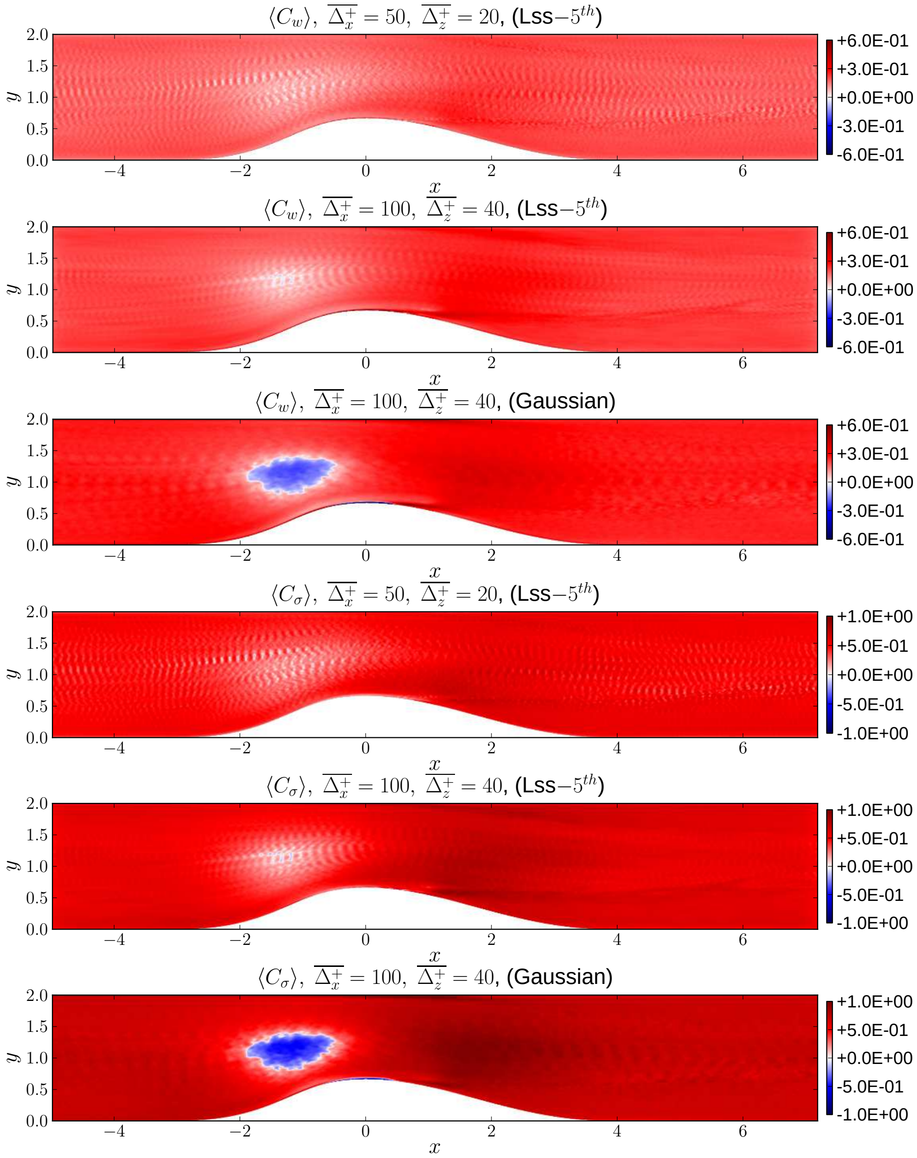

Figure 6. Coefficients of WALE model C_w (top 3 plots)and σ model (C_σ) computed with a Gaussian and LSS-5^{th} filters and two different filter scales

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REFERENCES

[1] M. Breuer, N. Peller, Ch. Rapp, and M. Manhart. Flow overperiodic hills - numerical and experimental study in a wide range of reynolds numbers. *Computers & Fluids*, 38:433–457, 2009.

[2] P. Diercks. An algorithm for smoothing, differenciation and integration of experimental data using spline functions. *J. Comp. Appl. Math.*, 1(3):165–184, 1975.

[3] L. Kuban, J.-P. Laval, and W. Elsner. Les modeling of converging diverging turbulent channel flow. *Journal of Turbulence*, 13(11):1–19, 2012.

[4] J.-P. Laval, M. Marquillie, and U. Ehrenstein. On the relation between kinetic energy production in adverse-pressure gradient wall turbulence and streak instability. *Submitted to Journal of Turbulence*, 2012.

[5] D. K. Lilly. A proposed modification of the Germano subgrid-scale closure method. *Phys. Fluids A*, 4:633–635, 1992.

[6] S. Liu, C. Meneveau, and J. Katz. On the properties of similarity subgrid-scale models as deduced from measurments in a turbulent jet. *J. Fluid Mech.*, 275:83–119, 1994.

[7] C. Meneveau. Statistics of turbulence subgrid scale stresses: necessary conditions and experimental tests. *Phys. Fluids*, 6:815–833, 1994.

[8] C. Meneveau and J. Katz. Scale-inveriance and turbulence models for large eddy simulation. *Annu. Rev. Fluid Mech.*, 32:1–32, 2000.

[9] F. Nicoud, H. B. Toda, O. Cabrit, S. Bose, and J. Lee. Using singular values to build a subgrid-scale model for large eddy simulations. *Phys. Fluids*, 23, 2011.