NEW SCALING LAWS FOR TURBULENT POISEUILLE FLOW WITH WALL TRANSPIRATION

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ABSTRACT

A fully developed, turbulent Poiseuille flow with wall transpiration (see figure 1), i.e. uniform blowing and suction on the lower and upper walls correspondingly, is investigated by both direct numerical simulation (DNS) of the three-dimensional, incompressible Navier-Stokes equations and Lie symmetry analysis. A new logarithmic mean velocity scaling law of wake type is found in the core region of the flow. The slope constant in the core region differs from the von Kármán constant and is equal to 0.3. Extended forms of the linear viscous sublayer law and the near-wall log-law are also derived, which, as a particular case, include these laws for the classical non-transpirating channel flow.

Introduction

Wall-bounded turbulent flows with transpiration may not only be a technologically important subject of investigation (Jiménez *et al.*, 2001; Kametani & Fukagata, 2011) but also important for theoretical reasons as we will subsequently show.

In comparison to the other wall-bounded flows with specific, non-standard boundary conditions, turbulent channel i.e. Poiseuille flow with wall transpiration is a relatively new subject of investigation. The only experimental study of this flow of an incompressible fluid known to the authors was conducted by Zhapbasbayev & Isakhanova (1998). They collected statistics for the mean velocity and turbulent stresses for different Reynolds numbers and a variety of small transpiration velocity numbers in the range $0 < v_0/u_\tau < 0.05$.

In the literature only a few DNS studies of the turbulent channel flow with blowing and suction were reported. Sumitani & Kasagi (1995) studied turbulent channel flow with uniform wall transpiration and heat transfer. The walls were kept isothermal, while their temperatures vary. The Reynolds number and the transpiration rate were held constant at Re_τ=150 and v_0/u_τ=0.05. Various statistical quantities including mean velocity and mean temperature, Reynolds stresses, and turbulent heat fluxes were obtained.

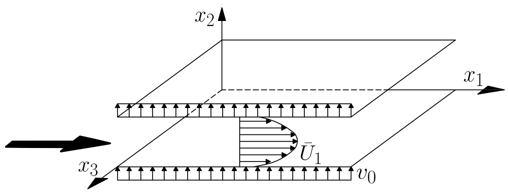

Figure 1. Sketch of the channel flow with wall transpiration. Fluid is uniformly blown through the lower wall and removed from the upper wall.

Energy budgets and temperature correlations were also calculated. One key overall result they have found was that blowing stimulates the near-wall turbulence and creates an excessive amount of small scale coherent streamwise vortical structures while suction suppresses turbulence and at the same time creates large scale near-wall coherent structures. Nikitin & Pavel'ev (1998) performed DNS computations at Re_τ=356, 681.2 for v_0/u_τ=0.112, 0.118, respectively. They investigated the near-wall logarithmic region of the mean velocity profile and found that the slope constant of the log-law at the blowing wall is not constant and increases with the increase of transpiration rate. Here it is important to mention that they used the local friction velocity at each wall as the velocity scale. This is rather natural to do so, however, presently we employ an averaged friction velocity from both walls, which is a measure of the pressure gradient, collapsing our DNS data to a considerably extended range. Chung & Sung (2001) investigated the initial relaxation of a turbulent channel flow after a sudden application of blowing and suction. Later, Chung *et al.* (2002), extended this by modulating the near-wall turbulence with uniform wall blowing and suction.

A purely analytical study of the turbulent channel flow with wall transpiration was performed by Vigdorovich & Oberlack (2008). Employment of the method of matched asymptotic expansions led them to the construction of the solutions for the near wall regions (both blowing/suction) as well as for the core region of the flow.

Summarizing all above mentioned studies we conclude that there is no comprehensive investigation of the mean velocity scaling laws of the Poiseuille flow with uniform wall transpiration based on first principles. This difficulty may be traced back to the problem of determining an appropriate velocity scale as there are multiple including v_0, u_τ on both walls and U_b being the bulk velocity, as well as the proper choice of equations on which the analysis should be based on.

Presently, the application of Lie symmetry method to the two-point correlation (TPC) equations is employed as the fundamental basis to find new mean velocity scaling laws as well as the proper scales on which it is based. DNS facilitate to evaluate the analytical results and finally allow to establish a clear connection between the different velocity scales.

In a series of papers Oberlack and co-authors (see Oberlack, 2000, 2001; Oberlack & Rosteck, 2010) studied the turbulent channel and other canonical wall-bounded flows using Lie symmetry theory by investigating the infinite series of multi-point correlation (MPC) equations. They showed that scaling laws are exact solutions of symmetry invariant type of the infinite dimensional series of MPC equations. They have shown that turbulent scaling laws may be generated from first principle and that most of the classical and new symmetry invariant solutions are based on one or several of the newly discovered statistical symmetry groups (Oberlack & Rosteck, 2010).

In this paper we propose new scaling laws for turbulent Poiseuille flows with wall transpiration. Besides, we obtain a new logarithmic scaling law in the center of the channel using Lie symmetry methods. The law is of defect type and covers up to 75% of the channel depending on the turbulent Reynolds number Re_τ and the transpiration velocity v_0. In order to validate the new scaling laws, various DNS of the channel flow at Re_τ=250, 480 and a wide range of the transpiration velocity v_0 are conducted.

Governing equations

The analysis presented below is based on the mean friction velocity defined as follows

$$u_\tau \equiv \sqrt{\frac{u_{\tau b}^2+u_{\tau s}^2}{2}} = \sqrt{\frac{1}{\rho}\frac{|\tau_{wb}|+|\tau_{ws}|}{2}} = \sqrt{\frac{h}{\rho}\left|\frac{\partial \bar{P}}{\partial x_1}\right|}, \quad (1)$$

which is a measure of the pressure gradient and the local friction velocities are defined as

$$u_{\tau b} = \sqrt{\nu \left|\frac{\partial \bar{U}_1}{\partial x_2}\right|_b}, \quad u_{\tau s} = \sqrt{\nu \left|\frac{\partial \bar{U}_1}{\partial x_2}\right|_s}. \quad (2)$$

Here, $\bar{U}_1$ and $\dfrac{\partial \bar{P}}{\partial x_1}$ are the mean velocity and mean pressure gradient in streamwise direction, ν is the kinematic viscosity and h is the channel half-width. Here and subsequently subscripts $()_b$ and $()_s$ correspond to variables taken on the blowing and the suction side respectively. For variables at the wall we use the subscript $()_w$ and variables without blowing and suction are denoted by $()_0$. Dimensionless variables in the near-wall scaling will have the superscript plus:

$$x_i^+ = \frac{x_i u_\tau}{\nu}, \quad \bar{U}_i^+ = \frac{\bar{U}_i}{u_\tau}, \quad \overline{u_i u_k}^+ = \frac{\overline{u_i u_k}}{u_\tau^2}, \quad v_0^+ = \frac{v_0}{u_\tau},$$
$$\tau^+ = \frac{\tau}{(|\tau_{wb}|+|\tau_{ws}|)/2}. \quad (3)$$

Note, that here u_τ is the mean friction velocity, which is a global parameter. We employ the channel half-width h as a core-region length scaling parameter. The bulk mean velocity is defined as

$$\bar{U}_b = \frac{1}{2h}\int_0^{2h} \bar{U}_1(x_2)dx_2. \quad (4)$$

Throughout this paper we use the following notations. The statistically averaged quantities are denoted by an overbar e.g. $\bar{U}_i, \bar{P}$ whereas fluctuating quantities are denoted by a lower case letters i.e. u_i and p. The governing equations for an incompressible turbulent flow, i.e. continuity and mean-momentum equations, are

$$\frac{\partial \bar{U}_k}{\partial x_k} = 0, \quad (5)$$

$$\frac{\partial \bar{U}_i}{\partial t} + \bar{U}_k\frac{\partial \bar{U}_i}{\partial x_k} = -\frac{\partial \bar{P}}{\partial x_i} + \nu\frac{\partial^2 \bar{U}_i}{\partial x_k \partial x_k} - \frac{\partial \overline{u_i u_k}}{\partial x_k}, \; i = 1,2,3, \quad (6)$$

where $\bar{U}_i(x_i,t)$ and $\bar{P}(x_i,t)$ are the mean velocity and mean pressure, and $\overline{u_i u_k}$ is the Reynolds stress tensor. For the incompressible flow investigated, pressure is normalized with the constant density.

We have the following boundary condition (BC) for the present flow

$$\bar{U}_i(x_1; x_2 = 0, 2h; x_3) = (0; v_0; 0)^T. \quad (7)$$

One of the key assumption, which has been confirmed by the DNS, is that of a constant mean wall-normal velocity across the channel height, i.e.

$$\bar{U}_2(x_2) = v_0. \quad (8)$$

With this, we obtain the streamwise component of mean momentum equation for the steady state

$$v_0\frac{d\bar{U}_1}{dx_2} = -\frac{d\bar{P}}{dx_1} - \frac{d\overline{u_1 u_2}}{dx_2} + \nu\frac{d^2\bar{U}_1}{dx_2^2}. \quad (9)$$

As the pressure gradient is specified as a constant, equation (9) may be integrated once and rearranged to obtain

$$\tau(x_2) - v_0\bar{U}_1 = -\overline{u_1 u_2} + \nu\frac{d\bar{U}_1}{dx_2} - v_0\bar{U}_1 = x_2\frac{d\bar{P}}{dx_1} + c_1. \quad (10)$$

Here c_1 is a constant that in the canonical channel flow is defined as ρu_τ^2 (Tennekes & Lumley, 1972). Due to different wall conditions of the channel flow with transpiration the wall shear stresses on the blowing and suction walls are different, which brought the necessity to use a local friction velocity in (10) rather than a global one.

The space and time correlation functions in the theory of turbulence was first introduced by Keller & Friedmann (1924). Various authors derived the complete system of two-point correlation equations (see e.g. Hinze, 1959; McComb, 1990), while Keller & Friedmann (1924) were also the first who closed it by writing the third moment via the second moment and the mean. Presently we only focus on the two-point correlation (TPC) equations in its most general form

$$\begin{aligned}\frac{\bar{\mathrm{D}}R_{ij}}{\mathrm{D}t} &+ R_{kj}\frac{\partial \bar{U}_i(x,t)}{\partial x_k} + R_{ik}\frac{\partial \bar{U}_j(x,t)}{\partial x_k}\bigg|_{x+r} \\ &+ \left[\bar{U}_k(x+r,t) - \bar{U}_k(x,t)\right]\frac{\partial R_{ij}}{\partial r_k} + \frac{\partial \overline{pu_j}}{\partial x_i} - \frac{\partial \overline{pu_j}}{\partial r_i} \\ &+ \frac{\partial \overline{u_i p}}{\partial r_j} - \nu\left[\frac{\partial^2 R_{ij}}{\partial x_k \partial x_k} - 2\frac{\partial^2 R_{ij}}{\partial x_k \partial r_k} + 2\frac{\partial^2 R_{ij}}{\partial r_k \partial r_k}\right] \\ &+ \frac{\partial R_{(ik)j}}{\partial x_k} - \frac{\partial}{\partial r_k}\left[R_{(ik)j} - R_{i(jk)}\right] = 0, \qquad (11)\end{aligned}$$

without introducing any closure, where the second and third order correlation tensors are defined as

$$\begin{aligned} R_{ij}(x,r;t) &= \overline{u_i(x,t)u_j(x+r,t)}, \\ \overline{pu}_j &= \overline{p(x,t)u_j(x+r,t)}, \quad \overline{u_i p} = \overline{u_i(x,t)p(x+r,t)}, \\ R_{(ik)j}(x,r;t) &= \overline{u_i(x,t)u_k(x,t)u_j(x+r,t)}, \\ R_{i(jk)}(x,r;t) &= \overline{u_i(x,t)u_j(x+r,t)u_k(x+r,t)}. \end{aligned}$$

Continuity equations for the TPC have the following form

$$\frac{\partial R_{ij}}{\partial x_i} - \frac{\partial R_{ij}}{\partial r_i} = 0, \quad \frac{\partial R_{ij}}{\partial r_j} = 0 \qquad (12)$$

and

$$\frac{\partial \overline{pu}_i}{\partial r_i} = 0, \quad \frac{\partial \overline{u_j p}}{\partial x_j} - \frac{\partial \overline{u_j p}}{\partial r_j} = 0. \qquad (13)$$

DNS

In order to verify the scaling laws to be obtained for the different regions of the flow in the sections to follow we conduct a number of DNS for different transpiration rates and Reynolds numbers.

For the present DNS we employ a numerical code developed at the School of Aeronautics, Technical University of Madrid (Hoyas & Jiménez, 2006). The code solves the Navier-Stokes equations for an incompressible fluid in velocity-vorticity formulation (see e.g. Kim *et al.*, 1987). In the streamwise and spanwise directions (x_1, x_3) Fourier discretization is used. In the wall-normal direction, (x_2), a seven-point compact finite difference scheme (Lele, 1992) is applied. The DNS data of Sumitani & Kasagi (1995) are used for the validation of our DNS results.

Production runs can be divided into two sets depending on the friction Reynolds number $Re_\tau = 250, 480$. Each simulation set consists of four cases for different transpiration rates $v_0^+ = 0.05, 0.1, 0.16, 0.26$. A complete summary of the flow and the numerical parameters are given in Avsarkisov *et al.* (2013).

Lie symmetry analysis

Application of the wall transpiration to the turbulent channel flow at moderate and high Reynolds numbers revealed a new logarithmic mean velocity scaling law in the core region. A starting point for this analysis was the observation, that transpiration velocity may be a symmetry breaking in a core region of the flow.

In order to derive a new turbulent scaling law for the present flow from the TPC equation we need to consider the appropriate symmetry transformations. For the present problem it is sufficient to focus on the three scaling groups $(\bar{T}_1, \bar{T}_2, \bar{T}_s')$, the translation group in space $(\bar{T}_{x_2})$ and the translation group of the averaged velocity $(\bar{T}_{\bar{U}_i})$. In global form these transformation groups are defined as:

$$\begin{aligned}
\bar{T}_1:\ & t^* = t,\ x^* = e^{k_1}x,\ r^*_{(l)} = e^{k_1}r_{(l)},\ \bar{U}^* = e^{k_1}\bar{U}, \\
& \bar{P}^* = e^{2k_1}\bar{P},\ R^*_{ij} = e^{2k_1}R_{ij},\ \overline{pu_j}^* = e^{3k_1}\overline{pu_j}, \\
& \overline{u_i p}^* = e^{3k_1}\overline{u_i p},\ \ldots\ , \qquad (14) \\
\bar{T}_2:\ & t^* = e^{k_2}t,\ x^* = x,\ r^*_{(l)} = r_{(l)},\ \bar{U}^* = e^{-k_2}\bar{U}, \\
& \bar{P}^* = e^{-2k_2}\bar{P},\ R^*_{ij} = e^{-2k_2}R_{ij},\ \overline{pu_j}^* = e^{-3k_2}\overline{pu_j}, \\
& \overline{u_i p}^* = e^{-3k_2}\overline{u_i p},\ \ldots\ , \qquad (15) \\
\bar{T}_s':\ & t^* = t,\ x^* = x,\ r^*_{(l)} = r_{(l)},\ \bar{U}_i^* = e^{k_s}\bar{U}_i, \\
& \bar{P}^* = e^{k_s}\bar{P},\ R^*_{ij} = e^{k_s}\left[R_{ij} + \left(1 - e^{k_s}\right)\bar{U}_i\bar{U}_j\right], \\
& \overline{pu_j}^* = e^{k_s}\overline{pu_j} + (e^{k_s} - e^{2k_s})\bar{P}\bar{U}_j, \\
& \overline{u_i p}^* = e^{k_s}\overline{u_i p} + (e^{k_s} - e^{2k_s})\bar{P}\bar{U}_i,\ \ldots\ , \qquad (16) \\
\bar{T}_{x_2}:\ & t^* = t,\ x^* = x + k_{x_2},\ r^*_{(l)} = r_{(l)},\ \bar{U}^* = \bar{U}, \\
& \bar{P}^* = \bar{P},\ R^*_{ij} = R_{ij},\ \overline{pu_j}^* = \overline{pu_j}, \\
& \overline{u_i p}^* = \overline{u_i p},\ \ldots\ , \qquad (17) \\
\bar{T}_{\bar{U}_i}:\ & t^* = t,\ x^* = x,\ r^*_{(l)} = r_{(l)},\ \bar{U}_i^* = \bar{U}_i + k_{\bar{U}_i}, \\
& \bar{P}^* = \bar{P},\ R^*_{ij} = R_{ij},\ \overline{pu_j}^* = \overline{pu_j}, \\
& \overline{u_i p}^* = \overline{u_i p},\ \ldots\ . \qquad (18)
\end{aligned}$$

The first two scaling symmetries are well-known from Euler and Navier-Stokes equations describing scaling of space and time. The third one is new and independent from them. It stands for scaling of all TPC tensors, and it is a purely statistical property of these equations (Oberlack & Rosteck, 2010). One of the most crucial symmetries for the results to follow and also a key ingredient of the classical log-law (Oberlack, 2001) is symmetry (18). It also is of purely statistical nature and was discovered in the context of an infinite set of statistical symmetries in Oberlack & Rosteck (2010).

From the symmetry transformations we may construct the invariance condition (see e.g. Bluman *et al.*, 2010) encompassing all groups given above

$$\frac{\mathrm{d}x_2}{k_1 x_2 + k_{x_2}} = \frac{\mathrm{d}r_{(k)}}{k_1 r_{(k)}} = \frac{\mathrm{d}\bar{U}_i}{(k_1 - k_2 + k_s)U_i + k_{\bar{U}_i}} = \cdots\ , \qquad (19)$$

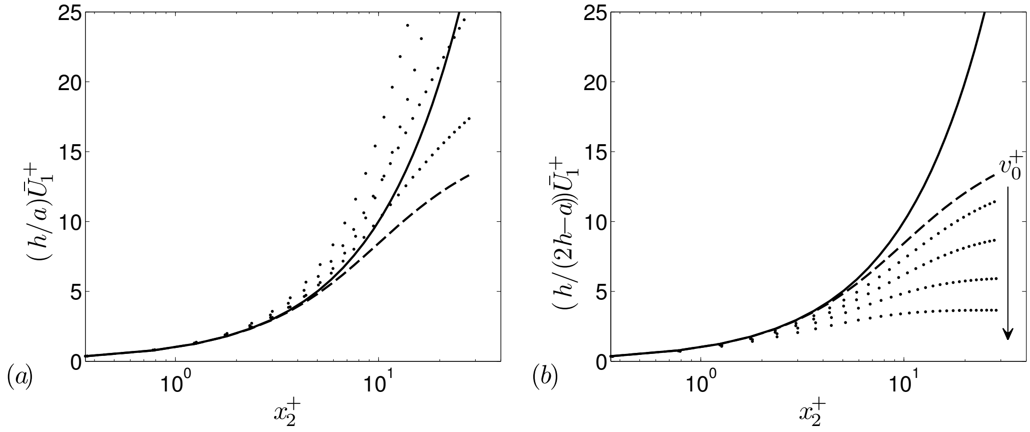

Figure 2. Mean velocity profiles and linear law. (a) at the blowing wall, (b) at the suction wall. Solid lines correspond to the linear laws (26), (27), dashed lines to the velocity profile without transpiration and dotted lines to the velocity profiles with transpiration. In direction of arrow: $v_0/u_\tau = 0.0, 0.05, 0.1, 0.16, 0.26$, $Re_\tau = 250$.

where in the present paper any further invariance conditions for higher correlations are omitted. In order to determine group parameters k_i we may invoke a boundary condition $\bar{U}_2 = v_0$, as this is the key influencing factor for altering the flow. As it acts primarily on the velocity we consider the concatenated global transformations for the mean velocity

$$\bar{U}_i^* = e^{k_1 - k_2 + k_s} \bar{U}_i, \tag{20}$$

taken from (14 - 16). Invariance, and in turn similarity reduction, requires at the first place the symmetries admitted by the underlying equation, like the TPC equation (11). In a second step, however, for the construction of a concrete solution, symmetries have to be constructed with the imposed boundary conditions, that leads to

$$e^{-(k_1 - k_2 + k_s)} \bar{U}_2^* = v_0. \tag{21}$$

As symmetry by definition implies form invariance this provides the constraints

$$k_1 - k_2 + k_s = 0. \tag{22}$$

That leads to the conclusion that transpiration velocity (v_0) is symmetry breaking in the core region of the flow. Imposing the latter constraint onto (19) and integrating the first with the third term leads to a new logarithmic scaling law for the streamwise mean velocity in the core region

$$\bar{U}_1 = A_1 \ln\left(\frac{x_2}{h} + B_1\right) + C_1, \tag{23}$$

where A_1, B_1 and C_1 are simple functions of the group parameters k_i. If it may be presumed that v_0 is sufficiently large ($0.05 \leq v_0^+$) we will subsequently show that the latter new log-law is valid in the core region of a turbulent channel flow with wall transpiration.

For the wall-normal component of the mean velocity $\bar{U}_2$ a result similar to (23) is obtained from (19). Taking into account the additional constraint $k_{\bar{U}_2} = 0$, we obtain $\bar{U}_2(x_2) = C_2$, which nicely confirms the assumption (8) that the wall-normal component of mean velocity is a constant and is equal to the transpiration velocity v_0.

New mean velocity scaling laws

The occurrence of the convective momentum transport term on the left hand side of the mean momentum equation

$$v_0 \frac{d\bar{U}_1}{dx_2} = -\frac{d\bar{P}}{dx_1} - \frac{d\overline{u_1 u_2}}{dx_2} + \nu \frac{d^2 \bar{U}_1}{dx_2^2} \tag{24}$$

modifies classical scaling laws (viscous sublayer and law of the wall) which were regarded as universal for all non-transpiring wall-bounded flows. While for moderate blowing/suction rates $0.04 < v_0/u_\tau < 0.1$ (Tennekes, 1965; Vigdorovich & Oberlack, 2008) the viscous sublayer appears on both walls, near-wall log-law of the wall is observed only on the blowing side, where local friction velocity is considerably lower then on the suction side.

An extended form of the linear viscous sublayer law and the near-wall log-law have been derived, which include the laws derived for the canonical flows without transpiration as a particular case.

In order to reformulate the local friction velocities by the averaged friction velocity u_τ which is related to the streamwise pressure gradient we will use the following transformations

$$u_{\tau b}^2 = \frac{a}{h} u_\tau^2, \; u_{\tau s}^2 = \frac{2h - a}{h} u_\tau^2. \tag{25}$$

The coefficients a/h and $(2h-a)/h$ represent the relations τ_{wb}/τ_w and τ_{ws}/τ_w respectively, and a is a parameter that depends on the transpiration velocity. This facilitates a normalization of the terms of the momentum equation with u_τ rather than with local ones, which allows us to directly compare the scaling regions of the blowing and the suction wall based on the same scaling parameter.

For the viscous sublayers on blowing and suction sides respectively the following velocity scaling laws are obtained:

$$\bar{U}_1^+ = \frac{a}{h} x_2^+, \tag{26}$$

$$\bar{U}_1^+ = \frac{2h - a}{h} x_2^+. \tag{27}$$

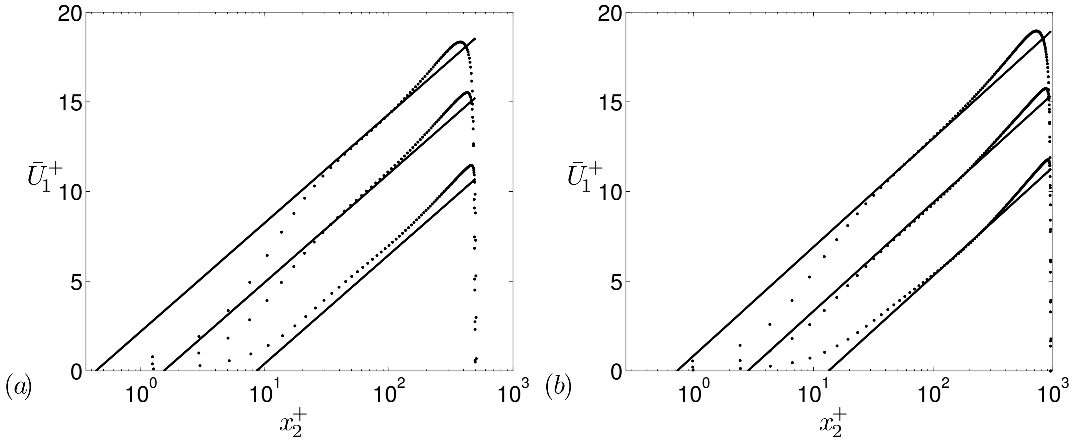

Figure 3. Mean velocity profiles of a turbulent channel flow with wall transpiration at the blowing wall at (a) $Re_\tau = 250$ and (b) $Re_\tau = 480$. Solid lines correspond to the log-law (28), where $A^+ = 0$. Dotted lines correspond to the mean velocity profiles. From top to the bottom $v_0/u_\tau = 0.05, 0.1, 0.16$.

Since the viscous stress at the blowing wall is smaller than in a channel with impermeable boundaries, the viscous sublayer at the blowing side is thinner then for the classical flow, as can be taken from 2(a).

On the suction side in the presence of a strong viscous stress and weak Reynolds shear stress the region of linear scaling appears to be longer than in the canonical channel flow, as can be taken from figure 2(b). However, suction alters the linear scaling coefficient emphasizing that it is not a purely viscosity induced effect.

The near-blowing-wall log-law has the following form

$$\bar{U}_1^+ = \frac{1}{\kappa}\ln\left(x_2^+ + A^+\right) + C + C_1\left(\frac{v_0}{u_\tau}\right), \tag{28}$$

here κ and C are independent of v_0 and hence are universal constants obtained for the case without transpiration and based on the global u_τ. The function C_1 vanishes for vanishing v_0.

As no first principle idea is known to determine $C_1(v_0/u_\tau)$ we employ a simple curve fitting procedure to fit the new additive function C_1 which comes down to the following power-law

$$C_1(v_0^+) = \alpha\left(\frac{v_0}{u_\tau}\right)^\beta, \tag{29}$$

where $\alpha = -90.62$ and $\beta = 1.188$.

The results from the DNS and the modified log-law close to the blowing wall calculated from equation (28) with $A^+ = 0$ are compared in figure 3. We observe that for moderate transpiration rates ($0.05 < v_0/u_\tau < 0.1$) the log region does not change its size and location. However, for higher transpiration rates the validity of the scaling region becomes thinner and is shifted towards the core region of the channel. At high transpiration rates ($0.16 < v_0/u_\tau < 0.26$) which results are not shown here, the near-wall log-region almost vanishes, and cannot be validated with the expression (28).

The scaling law (23) obtained using Lie symmetry method contains constants A_1, B_1 and C_1, that cannot be obtained using Lie group analysis alone. For this reason one of the main aim of the present study is to determine the constants employing the DNS results.

The best fit to all DNS data is obtained if instead of v_0 we invoke u_τ as the appropriate velocity scale. We recall that u_τ is a measure of the pressure gradient as the local $u_{\tau b}$ and $u_{\tau s}$ on each wall are very different. The analysis of the DNS results together with the employment of u_τ as a scaling velocity for A_1 leads to the fact that the overall scaling appears rather insensitive to the Reynolds numbers and the relative transpiration rates. The latter rescaling leads to $A_1 = u_\tau/\gamma$, where $\gamma = 0.3$ has been taken from the DNS data. Note that this is not the usual von Kármán constant κ.

An analysis of the present DNS data disclosed C_1 to be the bulk velocity U_b (4) without an additional non-dimensional pre-factor.

In its final form the new logarithmic scaling law for the core region of the channel flow with wall transpiration is found in deficit form

$$\frac{\bar{U}_1 - U_b}{u_\tau} = \frac{1}{\gamma}\ln\left(\frac{x_2}{h}\right). \tag{30}$$

The new Lie symmetry induced scaling law (30) represents the velocity defect law that scales the data in the whole core region of the flow as may be taken from figure 4.

Conclusion

In the present paper we combined Lie symmetry analysis of the TPC equations and DNS to investigate the statistical characteristics of the turbulent channel flow with wall transpiration. Lie symmetry analysis revealed a new mean velocity logarithmic type of scaling law that, afterwards, has been confirmed in the center of the channel and studied in detail by DNS. By using the new results from the DNS data it was found, that the slope constant (γ) of the new log-law is different from the von Kármán constant and that its value is $\gamma = 0.3$. Presence of the transpiration makes the log-region much longer than that of the velocity defect law for the classical channel flow. The new scaling law covers from 65% to 80% of the channel height depending on the transpiration rate.

The classical near-wall scaling laws, i.e. the linear law in the viscous sublayer and logarithmic law of the wall, were validated though in slightly modified form. That in-

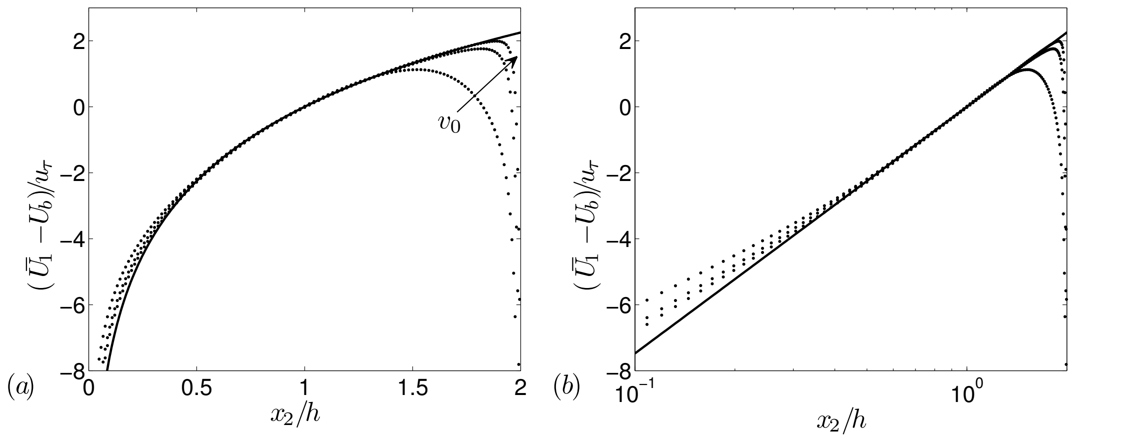

Figure 4. Mean velocity profiles in (a) linear and (b) semi-log scaling. Solid lines correspond to the new log-law ($\gamma = 0.3$). Dotted lines correspond to the mean velocity profiles. In the direction of the arrow: $v_0/u_\tau = 0.05, 0.1, 0.16,\ Re_\tau = 480$.

dicates that the permeability of the channel walls affects the near-wall region and in particular the wall shear stress.

Further, it has been shown that the von Kármán constant is universal in the near-wall region, while the additive constant C had to be modified by the transpiration rate. A near-wall log region persisted only on the blowing side and only for moderate transpiration rates $0.05 \leq v_0/u_\tau \leq 0.1$. An interesting conclusion that can be made from the figure 3 is that high Reynolds number and high transpiration rate effects counter balance each other in the log-region. The logarithmic region grows as the Reynolds number increase and decreases at high transpiration rates. The most plausible explanation for this effect is that transpiration shifts the buffer layer deep into the channel where the latter at high transpiration rates directly meets the core region of the flow and hence eliminating the logarithmic region.

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