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ONE-EQUATION SUBGRID SCALE MODEL FOR LARGE EDDY SIMULATION OF WEAKLY COMPRESSIBLE FLOW

Changhwa Han
Department of Mechanical Engineering,
Osaka University,
2-1 Yamadaoka, Suita, Osaka, 565-0871, Japan
ch.han@fluid.mech.eng.osaka-u.ac.jp

Takeo Kajishima
Department of Mechanical Engineering,
Osaka University
2-1 Yamadaoka, Suita, Osaka, 565-0871, Japan
kajisima@mech.eng.osaka-u.ac.jp

ABSTRACT

In this study, we are developing a large eddy simulation (LES) method suitable for the analysis of flow and aerodynamic noise source. Applications include low Mach number flows around a large-scale wind turbine blade, an aircraft at takeoff and landing, or high-speed trains. We improved a one-equation subgrid scale (SGS) model to take into account non-equilibrium and flow separation. It is incorporated into the computational scheme that modifies the pressure equation to deal with weakly compressible flows. Particularly, a coherent structure model proposed by Kobayashi (2005) was successfully incorporated into the energy production rate of SGS kinetic energy. A flow around the NACA0012 airfoil was simulated by this new method. The computational results such as the time averaged pressure coefficient and pressure fluctuation are in reasonable agreement with the experimental data by Miyazawa *et. al*. (2003). In addition, a characteristic density fluctuation, which might be related to the sound source, was captured in the separation near the leading edge.

INTRODUCTION

The clarification of sound sources around objects moving at high Reynolds number and low Mach number is one of the major topics of computational fluid dynamics. Since the pioneering paper of Lighthill (1952), computational techniques to deal with flow-induced noise have been classified into two categories: direct method and indirect method. In the direct method, sound sources are obtained as a result of the direct numerical simulation of Navier-Stokes equation of compressible fluid flows. Because it avoids any modeling approximations, the direct method is possible to reproduce the sound generation exactly. However, because of the different order of magnitude in pressure fluctuations concerning to flow and sound, the direct method is very difficult for the practical application in industry.

In the indirect method, unsteady flows are simulated by the incompressible scheme, usually with Reynolds averaged numerical simulation (RANS), LES/RANS hybrid model or LES. Then the acoustic field is predicted based on the theoretically estimated sound source: Curle (1955), Powell (1964) and so on. Although the indirect method is unable to consider the effect of feedback from sound to flow, it has been widely used in industrial applications.

The purpose of this study is to improve the method of capturing a sound source. To this end, we propose a computational technique which is able to treat flow-induced sound. Our computational technique is involved in the indirect method. LES, which may be one of an appropriate way for the prediction of unsteady turbulent flows, is used. As a SGS model for LES, we use the theoretically derived one-equation SGS model by Yoshizawa and Horiuti (1985), Horiuti (1985), Okamoto and Shima (1999) as basis. We improve the one-equation SGS model introducing the concept of coherent structure model. In addition, we derive an efficient computational scheme that modifies the pressure equation to deal with the density fluctuation in low-Mach number flows. It is incorporated into the LES to treat high Reynolds number flows. An advantage of our method is able to treat the density fluctuation even in low Mach number flows ranging from zero to approximately 0.3.

In this paper, our numerical results of turbulent flow around NACA0012 airfoil are compared with experimental results by Miyazawa *et. al*. (2003). The divergence of velocity field, which can be obtained from computation of flow field directly, is compared with theoretical sound source models to discuss the possibility of new methodology of aeroacoustics.

OUTLINE OF COMPUTATIONAL METHOD

Considering the analysis of sound source, it is essential to capture the weak fluctuation of the fluid density even in low Mach number flows. We modify the usual incompressible scheme, which is based on the elliptic equation for pressure, to improve the accuracy for turbulent flows which are considering weak compressibility.

Basic Equation of LES

We use compressible Navier-Stokes equations which are assumed to be the isothermal flow field and weak compressibility. All variables are non-dimensionalized by the chord length C and the mainstream velocity U_0. The filtered continuity, momentum conservation and ideal gas equations are represented by

$$\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial (\bar{\rho}\tilde{u}_j)}{\partial x_j} = 0, \tag{1}$$

$$\frac{\partial (\bar{\rho}\tilde{u}_i)}{\partial t} + \frac{\partial}{\partial x_j}(\bar{\rho}\tilde{u}_i\tilde{u}_j) = \frac{\partial}{\partial x_i}\left(\bar{p} + \frac{2}{3}\bar{\rho}k_{sgs}\right) + \frac{\partial \sigma_{ij}}{\partial x_j}, \tag{2}$$

$$\bar{p} = \bar{\rho}RT, \tag{3}$$

where $\bar{()}$, $\tilde{()}$ are filtering and Favre mean. ρ, μ and T denote the density, viscosity coefficient and absolute temperature. The stress tensor is represented by

$$\sigma_{ij} = 2\left(\frac{1}{Re} + \bar{\rho}\nu_{sgs}\right)\left(\bar{S}_{ij} - \frac{1}{3}\delta_{ij}\bar{S}_{kk}\right), \tag{4}$$

where $\bar{S}_{ij}$ is rate of strain tensor and $Re = \rho_0 U_0 C/\mu$ is Reynolds number (where, μ is constant).

Subgrid Scale Model

We use a one-equation SGS model because it can remove an assumption of local equilibrium. Referring the approach of previous research by Kajishima and Nomachi (2006), we focused on how to evaluate appropriately the production term of SGS kinetic energy k_{sgs} transport equation, corresponding to the energy transfer from grid scale to SGS portion of turbulence kinetic energy. We introduce the concept of coherent structure model by Kobayashi (2005), which needs no wall function and filtering operation, into the one-equation model of k_{sgs} equation.

The SGS eddy viscosity ν_{sgs} of Eq.(2) is represented as ν_{sgs}=$C_\nu\bar{\Delta}\sqrt{k_{sgs}}$ by the dimensional analysis. Since C_ν is positive constant, ν_{sgs} cannot have negative value. k_{sgs} is calculated by the transport equation

$$\frac{\partial k_{sgs}}{\partial t} + \tilde{u}_j\frac{\partial k_{sgs}}{\partial x_j} = -\tau_{ij}\bar{S}_{ij} - C_\varepsilon\frac{k_{sgs}^{3/2}}{\Delta} - \varepsilon_\omega + \frac{\partial}{\partial x_j}\left[\left(C_d\Delta_\nu\sqrt{k_{sgs}} + \nu\right)\frac{\partial k_{sgs}}{\partial x_j}\right]. \tag{5}$$

In order to meet the correct asymptotic behavior to the wall, the characteristic length Δ_ν and additional term ε_ω are defined by

$$\Delta_\nu = \frac{\bar{\Delta}}{1 + C_k\Delta^2|S|^2/k_{sgs}}, \tag{6}$$

$$\varepsilon_\omega = 2\nu\frac{\partial\sqrt{k_{sgs}}}{\partial x_j}\frac{\partial\sqrt{k_{sgs}}}{\partial x_j}. \tag{7}$$

The production term of k_{sgs} transport equation is constructed as follows:

$$-\tau_{ij}\bar{S}_{ij} = \left[2\mu_C\left(\bar{S}_{ij} - \frac{1}{3}\delta_{ij}\bar{S}_{kk}\right) - \frac{2}{3}\bar{\rho}\delta_{ij}k_{sgs}\right]\bar{S}_{ij}. \tag{8}$$

As a model of eddy viscosity μ_C, we propose the following SGS model which introduces the CS model function

$$\mu_C = C_1|Fcs|^{3/2}\bar{\Delta}\sqrt{k_{sgs}}, \tag{9}$$

$$Fcs = Q/E, \tag{10}$$

$$Q = \frac{1}{2}\left[\bar{S}_{ii}\bar{S}_{jj} - \bar{S}_{ij}\bar{S}_{ij} + \bar{W}_{ij}\bar{W}_{ij}\right], \tag{11}$$

$$E = \frac{1}{2}\left[\bar{S}_{ij}\bar{S}_{ij} + \bar{W}_{ij}\bar{W}_{ij}\right], \tag{12}$$

where Fcs, $\bar{W}_{ij}$, Q, E are the coherent structure function, vorticity tensor, the second invariant and the magnitude of a velocity gradient tensor. As shown above, our model does not need any procedure to avoid numerical instability.

Numerical Method

The time marching of the Navier-Stokes equation of motion is divided into 2 steps by the fractional step method:

$$\frac{(\bar{\rho}\tilde{u})^F - (\bar{\rho}\tilde{u})^n}{\Delta t} = \nabla\cdot[-(\bar{\rho}\tilde{u}\tilde{u}) + \tau] \tag{13}$$

$$\frac{(\bar{\rho}\tilde{u})^{n+1} - (\bar{\rho}\tilde{u})^F}{\Delta t} = -\nabla\bar{p}^{n+1} \tag{14}$$

where n is time step count, F indicates the fraction step, partially marched without the pressure gradient, Δt is the time increment and τ is the viscous stress. In our simulation, the second-order Adams-Bashforth is applied for the right-hand side of Eq.(13). For the spatial difference, the second-order central finite-difference is used, except for QUICK scheme for the convection term.

Coupling the mass conservation equation $(\bar{\rho}^{n+1} - \bar{\rho}^n)/\Delta t + \nabla\cdot(\bar{\rho}\tilde{u})^{n+1} = 0$ with Eq.(4) derives the elliptic equation for $\bar{p}^{n+1}$. Assuming the small change of $\Delta\bar{p}$ and $\Delta\bar{\rho}$ in one time step, we approximated the relation between the time evolutions of pressure and density by using the equation of state.

$$\bar{p}^{n+1} - \bar{p}^n = (\bar{\rho}^{n+1} - \bar{\rho}^n)RT \tag{15}$$

where T is assumed to be constant. Thus, the pressure equation is represented by

$$\nabla^2\bar{p}^{n+1} - \frac{\bar{p}^{n+1}}{(\Delta t)^2 RT^n} = \frac{\nabla\cdot(\bar{\rho}\tilde{u})^F}{\Delta t} - \frac{\bar{p}^n}{(\Delta t)^2 RT}. \tag{16}$$

The effect of compressibility is represented by the second term of each side.

Computational conditions

In order to validate our method, we used computational conditions corresponding to that in the experiment by Miyazawa *et. al*. (2003): the angle of attack, 9°; the Reynolds number based on the chord length and the mainstream velocity, 2×10^5; the Mach number, 5.76×10^{-2}.

The coordinate is selected as x in the mainstream direction, z in the spanwise direction and y in the direction perpendicular to x and z. The boundary-fitted grid of C-type is generated in the x-y plane. The size of domain is: the diameter of a half circle of C-type grid is $8C$; $8C$ in the wake side; $0.5C$ in the spanwise direction. The numbers of grid point are: 1600 in the circumferential direction, and 800 on the airfoil surface; 160 to the outward from the surface; 60 in the spanwise direction.

The uniform stream without disturbance is given at the inflow boundary, where the subgrid scale turbulence k_{sgs} is also zero. Thus, the turbulence develops in the boundary layer around the airfoil. The convective boundary condition is used at the exit. At the top and bottom boundaries, the normal components of the gradients of variables are assumed to be 0. The nonslip boundary condition is set up on the airfoil surface. For pressure, non-reflective boundary conditions by Okita and Kajishima (2002) are implemented in the inflow, outflow, top and bottom boundaries to prevent the reflection of pressure waves.

RESULTS AND DISCUSSIONS
Flow Field

Hereafter in this section, the ‘average’ denotes the temporally as well as spatially (in the spanwise direction) averaged values.

Fig.1 shows the profile of the averaged wall friction coefficient on the suction side of the airfoil near the leading edge. The reattachment point is estimated at x/C=0.050, and it is in good agreement with experimental data by Miyazawa *et. al*. (2003) whose value is x/C=0.054.

Fig.2(a) shows the time and the spanwise averaged pressure coefficient C_p on the airfoil surface. Fig.2(b) shows the pressure fluctuation coefficient $C_{P\ rms}$ on the suction side. The Smagorinsky model could not reproduce the laminar separation near the leading edge probably because it gives SGS eddy viscosity by the grid scale velocity gradient even in the non-turbulence region. On the other hand, the present model could reproduce the flow separation, which was measured by the experiment. Moreover, overall pressure profiles corresponded with experimental data. These computational results demonstrate reliability of our SGS model.

Acoustic Analogy

Fig.3(a) and Fig.3(b) show the instantaneous and cross-sectional view of the sound source term $\nabla \cdot (\nabla \cdot T)$ by Lighthill (1955) and $\nabla \cdot (\omega \times u)$ by Powell (1964). Fig.3(c) shows the profile of the velocity divergence $\nabla \cdot u$ for the same instance and cross-section to Fig.3(a) and Fig.3(b). Since $\nabla \cdot u$ is related to $D\rho/Dt$, it is interpreted as a phenomenon related to the sound source. Pairs of negative and positive $\nabla \cdot u$ are observed near the leading edge in the suction side in Fig.3(c), and the similar patterns are observed for $\nabla \cdot (\nabla \cdot T)$ and $\nabla \cdot (\omega \times u)$ in Fig.3(a) and Fig.3(b).

Fig.4 shows the time evolution of the velocity divergence $\nabla \cdot u$ near the leading edge. Quadruple patterns of $\nabla \cdot u$ are captured, and its moving period is 2.625×10^{-4} [s].

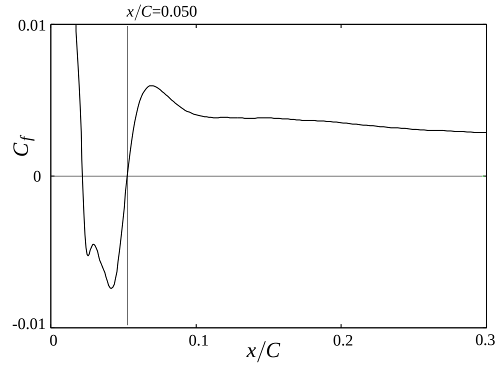
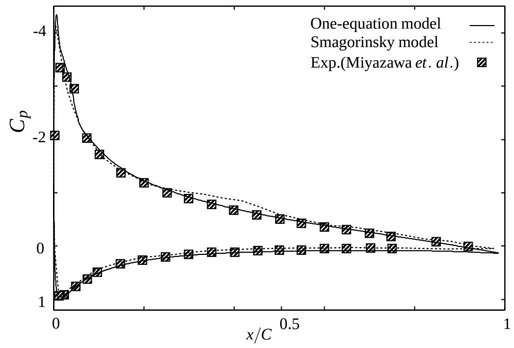
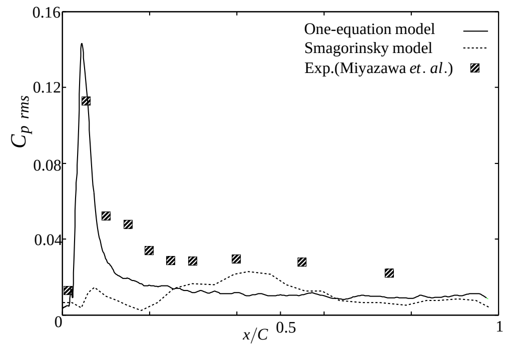


Figure 1. The friction coefficient on the suction side of the airfoil.



(a) pressure coefficient

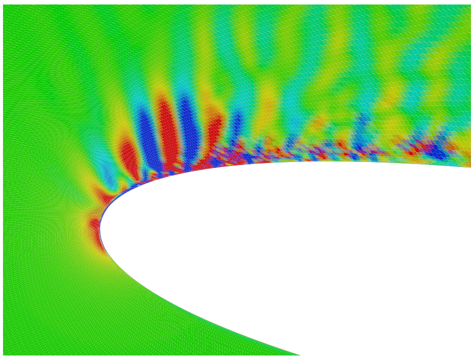


(b) pressure fluctuation on the suction side

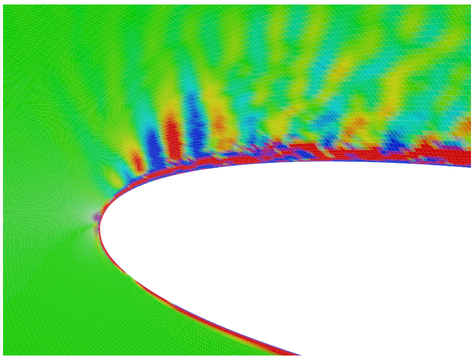
Figure 2. Time averaged pressure coefficients on the airfoil surface.

The distribution of $\nabla \cdot u$ is similar to the sound sources by Lighthill and Powell, and $\nabla \cdot u$ changes periodically. From these results, it is considered that $\nabla \cdot u$ has the possibility to be used as a sound source model.

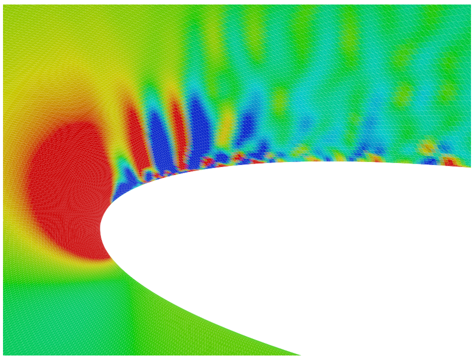
Fig.5 shows the sound pressure level (SPL) measured at point $6.7C$ from the leading edge in the upper direction normal to the mainstream velocity. As the method of acoustic analysis, the Curle’s equation (1955) assuming the acoustically compact sound source has been often used. In this time, we evaluated the sound using the Curle’s equation. In the high-frequency regions, the value of SPL is in agreement with experimental data, but it is overestimated in comparison with experimental data overall. From the study of Miyazawa *et. al*. (2004), it is considered that assumption of the compact sound source is a cause of overestimation.



(a) $\nabla \cdot (\nabla \cdot T)$



(b) $\nabla \cdot (\omega \times u)$



(c) $\nabla \cdot u$

Figure 3. Instantaneous and cross-sectional profiles of sound sources near the leading edge.

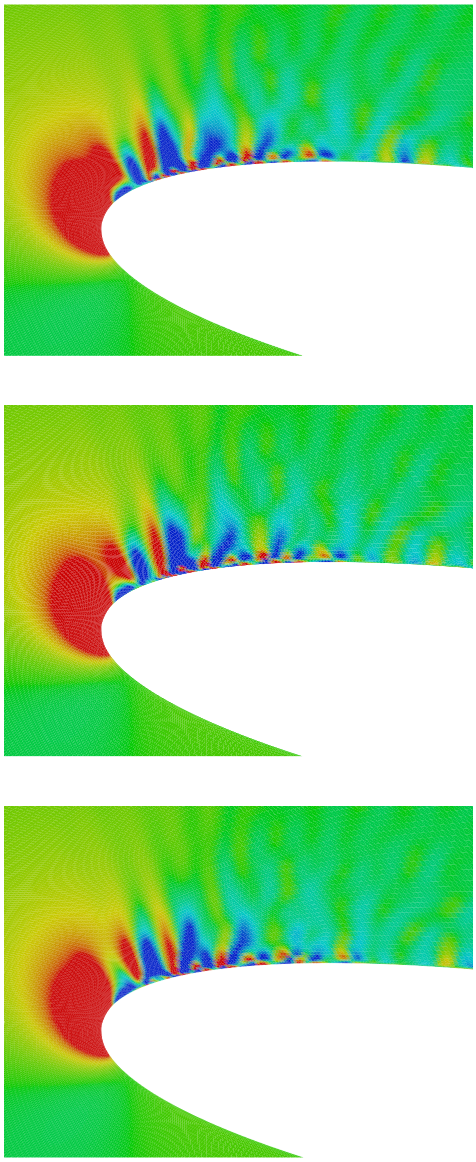


Figure 4. Some snapshots of $\nabla \cdot u$ profiles near the leading edge (The time interval of each snapshot is 1.313×10^{-4} [s].).

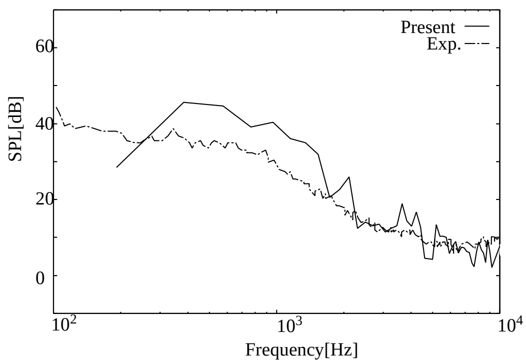


Figure 5. Sound pressure level.

CONCLUSION

In this paper, the numerical method, which consists of the one-equation SGS model and the weakly compressible scheme, was proposed to predict the sound source. This method was applied to the turbulent flow around NACA0012 airfoil at low Mach number. As for the flow field, our results were in good agreement with experimental results. The pressure profile and separation region were reproduced successfully. These results represent that our method is suitable for the prediction of turbulent flows around a object. Captured velocity divergence $\nabla \cdot u$ in the region of unsteady vortices near the leading edge has shown the similar distribution to sound source of the Lighthill's model and Powell's model. It suggested that our method has the possibility for representing the sound source from the flow at low Mach number. .

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