ESTIMATING THE FRICTION VELOCITY IN A TURBULENT PLANE WALL JET OVER A TRANSITIONALLY ROUGH SURFACE

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ABSTRACT

In this paper, a power law velocity profile is used to estimate the friction velocity in a plane turbulent wall jet on a rough surface based on experimental data collected by laser Doppler anemometry (LDA). The Reynolds number based on the slot height and exit velocity of the jet was approximately Re = 7500. A 36-grit sheet was used as the rough surface, creating a transitionally rough flow regime ($44 < k_s^+ < 70$). Power law velocity profiles were fitted using both inner and outer scales. The power-law coefficients were obtained by applying the expressions proposed by Seo and Castillo (2004) and Brzek et al. (2009) for rough boundary layer flows. According to the present results, the power-law velocity profiles are in good agreement with the conventional logarithmic profile in the overlap region. The power law coefficients can also be used to evaluate a theoretical expression for the skin friction coefficient. The skin friction coefficient for the rough surface turns out to be substantially greater than that for a wall jet on a smooth surface.

INTRODUCTION

Estimating the friction velocity, u_τ, and the skin friction coefficient, C_f, in turbulent near-wall flows such as boundary layers and wall jets over rough surfaces remains a significant challenge. Presently, there is no comprehensive theory for determining the friction velocity in flows over rough surfaces based on a generic description of the roughness geometry. Part of the inherent difficulty in developing an appropriate correlation relates to the challenge of making experimental measurements in the surface region of a rough wall. Furthermore, there is still some diversity in the scaling laws used to characterize the velocity profile in near-wall turbulent flows.

Conventional logarithmic profile fitting is one of the most common indirect methods for obtaining the friction velocity from velocity measurements in the overlap region of smooth wall-bounded flows. For a rough surface, the mean velocity profile on a semi-log plot using inner coordinates is shifted downward and to the right, so that a so-called roughness shift ($-\Delta U^+$) is added to the logarithmic law. Typically, the roughness shift is assumed to be a function of the friction velocity. George and Castillo (1997) proposed a power-law velocity profile for a smooth-wall turbulent boundary layer, with power-law coefficients that depend on the Reynolds number. They showed a good collapse of the data to the power-law profiles in the overlap region of a turbulent boundary layer. Brzek et al. (2009) proposed a composite velocity profile and skin friction law for a transitionally rough boundary layer using the theoretical approach of George and Castillo (1997) and the power-law coefficients obtained by Seo and Castillo (2004) for rough-wall boundary layers.

The turbulent wall jet is a flow of specific interest to turbulence researchers because of its special two-scale character, i.e. an outer region with the characteristics of a free jet and an inner region with the characteristics of a boundary layer. A definition sketch of this flow is shown in Figure 1. In this figure, x and y denote the streamwise and wall-normal distances, respectively; U and V are the streamwise and wall-normal components of the mean velocity, respectively; U_o is the jet exit velocity; H is the slot height; U_m is the maximum velocity; y_m and $y_{1/2}$ are the wall-normal locations where $U = U_m$ and $U_m/2$, respectively.

Researchers are still trying to measure the effects of surface roughness on the flow characteristics of a turbulent wall jet. There have been relatively few experiments of wall jets on a rough surface compared to the smooth-wall case. The experiments of Tachie et al. (2001) and Smith (2008), as well as Rostamy et al. (2011), appear to be the only studies to report measurements of the friction velocity, u_τ, for a wall jet on a rough surface.

In this paper, the expressions developed by Seo and Castillo (2004) for a turbulent boundary layer on a rough surface have been modified and applied to a transitionally rough wall jet. More specifically, power-law expressions for the velocity profiles are fitted to the experimental data in the overlap region of the wall jet in order to estimate the friction velocity and skin-friction coefficient.

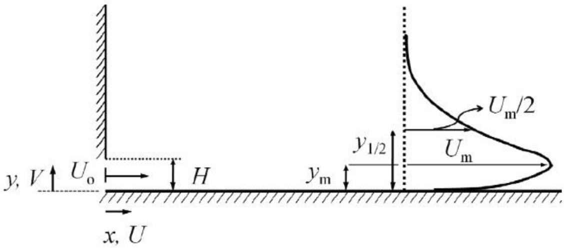

Figure 1: Schematic of a wall jet flow.

EXPERIMENTAL APPARATUS

The experiments were carried out in a water tank with a length, width, and height of 4.16 m, 1.28 m, and 1.7 m, respectively. The water flow was supplied by a pump which discharged through a rectangular slot at a jet exit velocity of approximately U_o = 1.21 m/s. The slot had a width of 750 mm and height of H = 6 mm, so that the width-to-height ratio was large enough to make the jet two-dimensional. Velocity measurements were carried out at different streamwise positions measured from the jet exit up to $x = 80H$. The Reynolds number of the wall jet, based on the jet exit velocity (obtained from the integral of the velocity across the slot at the exit) and the slot height, was approximately Re = 7500. All measurements were made at a water temperature of 22°C. A glass plate was used for the smooth surface, while the rough surface consisted of a 36-grit sheet glued to the glass plate using contact cement. The grit sheet did not cover the entire length of the plate, but instead began at a distance of approximately 10H from the jet exit. This sheet was manufactured by Gator Grit® and had a nominal grain size of k_g = 0.53 mm, creating a transitionally rough flow with a roughness Reynolds number ($k_s^+ = k_s u_\tau/\nu$) in the range of $44 < k_s^+ < 70$. The velocity measurements were made using a two-component laser Doppler anemometry (LDA) system with a burst mode processor supplied by Dantec Inc. The LDA system was powered by a 750 mW argon ion laser. The measurement volume size was 0.184 × 3.88 mm and 0.194 × 4.09 mm for the streamwise and wall-normal velocity components, respectively. Version 4.10 of the BSA Flow software was used for data collection and reduction. Hollow glass beads with an average diameter of 10 μm were used to uniformly seed the flow. The number of samples used to obtain mean velocity components at each measurement point was 5000. To reduce the velocity bias in the LDA measurements due to the turbulence intensity levels, the raw data were corrected using the analytical techniques of McLaughlin and Tiederman (1973) and Zhang (2002) for different turbulence intensity levels. The uncertainty of the mean velocity measurements taken using LDA was estimated as 0.5 - 1.5% depending on the wall-normal location. It should be noted that one of the characteristics of the present wall jet apparatus was the use of a special nozzle configuration which produced a uniform velocity profile over most of the slot exit except near the lower and upper walls. The turbulence intensity in the central region of the jet at the exit plane was less than 1 %.

RESULTS AND DISCUSSION

An analysis of the experimental velocity fields measured for wall jets on both smooth and rough surfaces is presented in this section. In particular, the present experimental data are used to demonstrate the application of power laws for estimating the friction velocities in rough wall-bounded flows. George et al. (2000) identified inner and outer regions for a turbulent plane wall jet on a smooth surface. The inner layer is defined as the region from the wall up to $y^+ = 0.1\, y_{1/2}^+$, while the outer layer is the region above $y^+ = 30$. Hence, the upper bound of the overlap between the inner layer and the outer layer in a wall jet is a function of the local Reynolds number based on half-width, i.e. $30 < y^+ < 0.1\, y_{1/2}^+$.

According to George et al. (2000), the velocity profiles in the overlap region of a wall jet on a smooth surface in outer and inner coordinates are given by,

$$\frac{U}{U_m} = C_o\left(\frac{y}{y_{1/2}} + \bar{a}\right)^\gamma \qquad (1)$$

$$u^+ = \frac{U}{u_\tau} = C_i(y^+ + a^+)^\gamma \qquad (2)$$

where the power law coefficients C_o, C_i and γ are functions of the local Reynolds number $y_{1/2}^+ = y_{1/2}\, u_\tau/\nu$ and can be calculated as follows:

$$C_o = C_{o\infty}\left[1 + 0.283\exp\left(-0.00598 y_{1/2}{}^+\right)\right] \quad (3)$$

$$C_i = C_{i\infty}\left[1 + 0.283\exp\left(-0.00598 y_{1/2}{}^+\right)\right] \times \exp\left[\frac{-(1+\alpha)A}{(\ln y_{1/2}{}^+)^\alpha}\right] \qquad (4)$$

$$\gamma = \gamma_\infty + \frac{\alpha A}{(\ln y_{1/2}{}^+)^{1+\alpha}} \qquad (5)$$

Using the suggested values of $C_{o\infty} = 1.30$, $C_{i\infty} = 55$, $A = 2.90$, $\alpha = 0.47$, and $\gamma_\infty = 0.0362$ of George et al. (2000), the power law coefficients reduce to:

$$C_o = 1.3 + 0.3679\exp\left(-0.00598\, y_{1/2}{}^+\right) \quad (6)$$

$$C_i = 55 + 15.565\exp\left(-0.00598 y_{1/2}{}^+\right) \times \exp\left[\frac{-4.263}{(\ln y_{1/2}{}^+)^{0.47}}\right] \quad (7)$$

$$\gamma = 0.0362 + \frac{1.363}{(\ln y_{1/2}{}^+)^{1.47}} \quad (8)$$

As can be seen, the coefficients C_o, C_i, and γ are functions of $y_{1/2}{}^+$ only and hence depend on u_τ.

Following George and Castillo (1997), Seo and Castillo (2004) proposed the following power-law velocity profiles for the overlap region of the flow in outer and inner coordinates for a rough-wall turbulent boundary layer:

$$\frac{U}{U_\infty} = \tilde{C}_o\left(\frac{y}{\delta} + \bar{a}\right)^{\tilde{\gamma}} \quad (9)$$

$$\frac{U}{u_\tau} = \tilde{C}_\iota\left(y^+ + a^+\right)^{\tilde{\gamma}} \quad (10)$$

where the coefficients $\tilde{C}_o, \tilde{C}_i$ and $\tilde{\gamma}$ are functions of δ^+ and k^+, and k is the roughness height of the rough surface. Here, U_∞ and δ are the free stream velocity and boundary layer thickness, respectively, and the coefficient $a^+ \approx -16$. According to Seo and Castillo (2004), the power-law coefficients in Equations (9) and (10) can be modeled using the following correlations:

$$\widetilde{C_o} = C_o(1 + C_{ok}) \quad (11)$$

$$\tilde{C}_\iota = C_i/(1 + C_{ik}) \quad (12)$$

$$\tilde{\gamma} = \gamma + \gamma_k \quad (13)$$

where the coefficients C_o, C_i, and γ are the power-law coefficients for a smooth-wall boundary layer obtained from Equations (6), (7) and (8), respectively. In Equations (11), (12) and (13), C_{ok}, C_{ik}, and γ_k are roughness functions determined by the following relations proposed by Seo and Castillo (2004),

$$C_{ok} = 0.00576(k^+)^{0.517} \quad (14)$$

$$C_{ik} = 0.03551(k^+)^{0.88647} \quad (15)$$

$$\gamma_k = 0.0065(k^+)^{0.60126} \quad (16)$$

As seen from the above equations, the coefficients C_{ok}, C_{ik}, and γ_k are functions of k^+ only.

For a wall jet on a rough surface, by substituting the wall jet velocity and length scales for those of a boundary layer, Equations (9) and (10) become

$$\frac{U}{U_m} = \tilde{C}_o\left(\frac{y}{y_{1/2}} + \bar{a}\right)^{\tilde{\gamma}} \quad (17)$$

$$\frac{U}{u_\tau} = \tilde{C}_\iota\left(y^+ + a^+\right)^{\tilde{\gamma}} \quad (18)$$

Here, the power-law coefficients can be obtained using Equations (11) to (16). By combining Equations (17) and (18) in the overlap region, a new friction law for the wall jet on a rough surface can be formulated as follows:

$$\frac{u_\tau}{U_m} = \frac{\widetilde{C_o}}{\tilde{C}_\iota}\left(y_{1/2}{}^+\right)^{-\tilde{\gamma}} \quad (19)$$

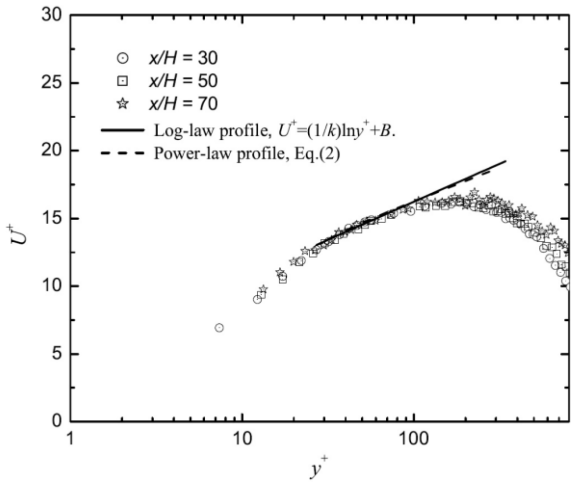

Figure 2: Mean velocity profiles for a smooth wall jet in inner coordinates.

Using Equation (19), the friction velocity for a rough wall jet can be determined directly since for a given velocity profile, all parameters on the right-hand side of the equation are functions of the friction velocity. Table 1 below summarizes the values of the power law coefficients and corresponding values of the friction velocity obtained from the present wall jet experiment.

The mean velocity profiles for the wall jet on the smooth surface at three different downstream distances are shown using inner coordinates in Figure 2. In this figure, the power law profiles are plotted using the power law coefficients obtained from Equations (3), (4), and (5) (shown in Table 1). According to Figure 2, the present experimental data collapse well with the power law profiles, as well as the conventional log law in the narrow overlap region of the wall jet. This suggests that both profiles can be used (even simultaneously) to estimate the friction velocities for a smooth wall jet. Figure 2 also offers support for the power law coefficients proposed by George et al. (2000) for a wall jet on a smooth surface.

For rough-wall boundary layers and wall jets, most researchers have used the classical law of the wall modified as follows for roughness:

$$\frac{U}{u_\tau} = \frac{1}{\kappa}\ln y^+ + B - \Delta U^+ \quad (20)$$

where κ and B are the log-law constants which are assumed to be universal and independent of Reynolds number. In the present study the values of κ and B are assumed to be 0.41 and 5.0, respectively. The so called roughness shift ΔU^+ represents the vertical displacement between the smooth-wall and rough-wall velocity profiles on a semi-logarithmic plot. For a fully rough flow, the roughness shift ΔU^+ for sand-grain roughness is often modeled as

$$\Delta U^+ = \frac{1}{\kappa}\ln k_s{}^+ - 3.5 \quad (21)$$

where k_s^+ is the equivalent sand grain roughness. The modified log-law velocity profile can be considered an alternative method to determine the friction velocity for a wall jet on a rough surface.

Table 1: Power law coefficients for the wall jet on smooth and rough surfaces.

x/H	C_o	C_i	γ	C_{ok}	C_{ik}	γ_k	$\tilde{C}_o$	$\tilde{C}_i$	$\tilde{\gamma}$	u_τ (m/s)
30	1.3010	9.86	0.113	0.052	1.532	0.084	1.368	3.893	0.196	0.0607
40	1.3007	9.91	0.112	0.048	1.343	0.076	1.363	4.229	0.189	0.0526
50	1.3004	10.03	0.110	0.045	1.183	0.070	1.358	4.594	0.181	0.0456
60	1.3002	10.12	0.109	0.042	1.085	0.066	1.355	4.855	0.175	0.0413
70	1.3002	10.17	0.108	0.040	0.975	0.062	1.352	5.146	0.170	0.0366
80	1.3001	10.25	0.107	0.038	0.900	0.058	1.349	5.396	0.166	0.0335

The friction velocity of a wall jet on a rough surface can be determined by fitting the experimental data to the power law velocity profiles (Equations (17) and (18)) in the overlap region of the wall jet and simultaneously evaluating the values of $\tilde{C}_o$, $\tilde{C}_i, \tilde{\gamma}$ and u_τ, for a given roughness height, k. The value of the friction velocity can also be determined by fitting the experimental data to the modified logarithmic law profile (Equation (20)) in the overlap region of the wall jet and determining the values of u_τ and ΔU^+ at the same time. In comparison to these two methods, Equation (19) provides a third more expeditious approach which typically requires less iteration. In this equation, the power law coefficients are obtained using the formulations proposed by Seo and Castillo (2004), and the friction velocity is obtained by iteration of Equation (19), without fitting experimental data or modifying additional parameters. It should be noted that in all of these methods, the ambiguity of specifying the roughness height (k) for rough surfaces remains a challenge.

The present wall jet data were fitted to both the power-law profiles being proposed by the present authors, Equations (17) and (18), and the modified log-law profile, Equation (20), in the inner region of the rough wall jet as shown in Figure 3. Figure 3 shows the mean velocity profiles in the overlap region of a rough wall jet at four different downstream distances of x/H = 30, 40, 50, and 60. As seen in this figure, the proposal of Brzek et al. (2009) as modified by the present authors for a wall jet on a rough surface, results in a good collapse of the data to the power law profiles. This indicates that the modified power law can be considered a valid velocity profile for scaling wall jets on rough surfaces. In Figure 3, the power law and the log law profiles extend up to the upper bound of the overlap region of the wall jet, i.e. $0.1y_{1/2}^+$. For the present rough-wall data, the virtual origin was located a distance of ε below the nominal top of the roughness elements.

The Reynolds shear stress profile can be used to check the validity of the estimate for u_τ. The value of the friction velocity, u_τ, should be such as to ensure that the near-wall peak of the Reynolds shear stress normalized by the friction velocity, i.e. $(-\langle uv\rangle/u_\tau^2)$, does not exceed unity in magnitude. Figure 4 shows the Reynolds shear stress in the inner layer of a wall jet on a rough surface normalized by the square of the friction velocity at four different streamwise locations: x/H = 30, 40, 50 and 60. The magnitude of the peak value is close to unity.

The skin friction coefficient defined as $C_f = 2(u_\tau/U_m)^2$ can be estimated from the value of the friction velocity based on either the proposed power law formulation Equation (19) or the classical logarithmic formulation Equation (20).

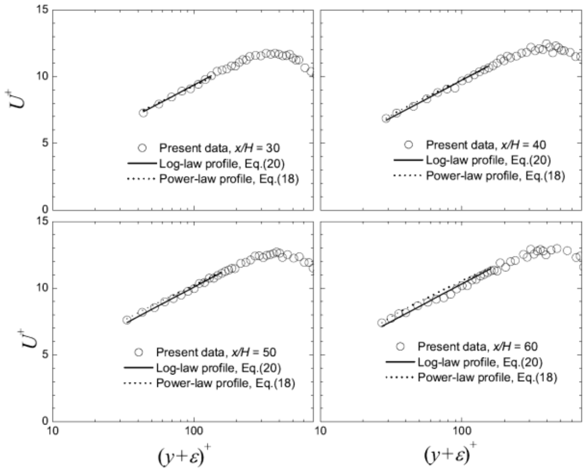

Figure 3: Mean velocity profiles for a wall jet on a rough surface using inner scales.

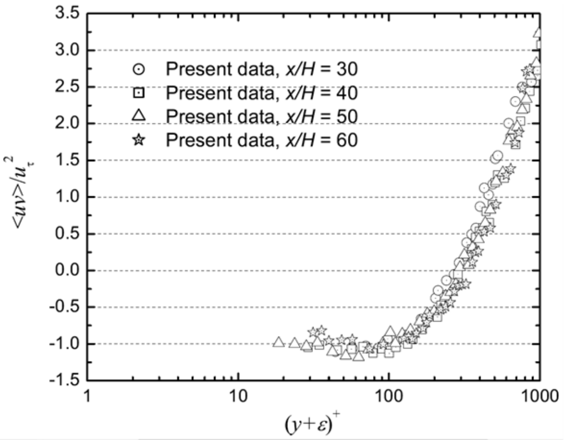

Figure 4: Normalized Reynolds shear stress in the inner layer of a rough wall jet.

Figure 5 compares the present C_f values estimated using both power-law and log-law formulations. The LDA data of George et al. (2000) for smooth wall jet are also shown in Figure 5. As seen in Figure 5, there is good agreement between the C_f values estimated from the power law and those obtained from the modified logarithmic law. This figure also indicates that the new proposed friction law for the rough wall jet can be considered a direct and convenient method for estimating the friction velocity (and the corresponding skin friction coefficient) in rough wall jets. According to this figure, the value of the skin friction coefficient for a wall jet on a rough surface is substantially higher than that for a wall jet on a smooth surface.

CONCLUSIONS

A new friction law for a plane turbulent wall jet over a rough surface was proposed in this paper based on the proposal by Seo and Castillo (2004) for a rough-wall turbulent boundary layer. LDA measurements of the streamwise velocity component for wall jets on smooth and rough surfaces were used to verify the new proposed friction law. LDA measurements were carried out at different streamwise locations from the jet exit up to x/H = 80. The Reynolds number of the wall jet, based on the jet exit velocity and the slot height, was approximately Re = 7500. A glass plate was used for the smooth surface, while the rough surface consisted of a 36-grit sheet glued to the glass plate creating a transitionally rough flow with a roughness Reynolds number ($k_s^+ = k_s u_\tau / \nu$) in the range of $44 < k_s^+ < 70$. The present smooth wall measurements support the power law profile proposed by George et al. (2000) in the overlap region of the wall jet. The present data also collapsed well to the log law profile in the overlap region, showing that both power law and log law profiles can be used to estimate the friction velocity of a wall jet on a smooth surface. According to the present study, good agreement was also obtained between the rough wall data and the new power law profile proposed for the overlap region of a wall jet on a rough surface. The present data also collapsed to the modified log law profile in this region. The experimental results verify the usefulness of the power law velocity profile and the friction law proposed in this paper. As a check on the estimated value of the friction velocity for the rough surface, the Reynolds shear stress measurements were used to demonstrate that the near-wall peak magnitude of the Reynolds shear stress normalized by the friction velocity, $(-\langle uv \rangle / u_\tau^2)$, did not exceed unity. The friction velocities obtained from the power law coefficients were also used to calculate the skin friction coefficient in a wall jet on a rough surface. The skin friction coefficients for the present rough surface were significantly higher than those for a smooth surface. Finally, the ambiguity of specifying the roughness height (k) still remains a challenge for wall jets on rough surfaces.

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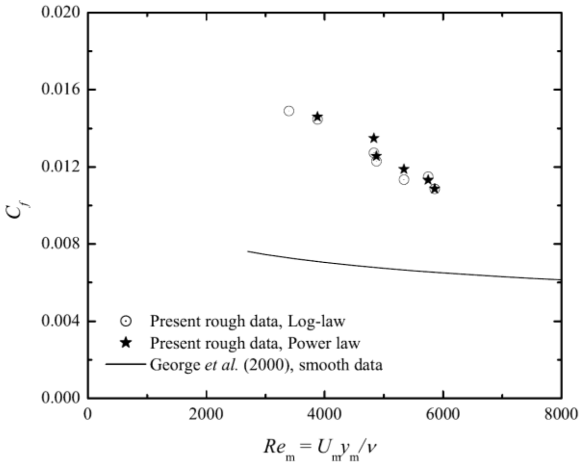

Figure 5: Variation of the skin friction coefficient for a wall jet on rough surface.

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