APPLICATION OF KALMAN FILTERING AND PARTIAL LEAST SQUARE REGRESSION TO LOW ORDER MODELING OF UNSTEADY FLOWS

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ABSTRACT

This study takes place in the framework of the enhancement of the stability of POD-Galerkin Low Order Dynamical System (LODS) identified from Time-Resolved particle Image Velocimetry (TR-PIV) measurements.

As proved by Noack et. al. (2005), the dynamics of incompressible flows can be efficiently represented by empirical LODS containing constant, linear and quadratic terms. However, POD-Galerkin LODS are known to diverge, or damp, rapidly in time if left uncontrolled, which led a number of authors to introduce regularization terms in order to stabilize the models via constrained optimization problems (Bergmann et al. 2005).

From a stochastic point of view, the Kalman Filter (Kalman, 1960) and its derivatives can be used to stabilize the evolution of dynamical systems, provided that state laws and observable data are available. In this formalism, LODS are modelised as discrete time hidden Markov processes in which the data of interest are represented by hidden variables that are only accessible from the state law and their relationships with observable variables. The purpose of Kalman filtering is to operate from both a state law and measured data in order to generate the best possible estimate of the hidden variables. In this study the observations are provided by hot-film measurements and linked to the Kalman filter using the partial Least Square Regression (PLSR) (Wold, 1966) as an alternative to the Linear Stochastic Estimation (LSE) (Taylor and Glauser, 2004) for over-determined data. Thus, applying a combination of PLSR and Kalman filters to POD-Galerkin LODS can be seen as a solution to reconstruct and analyse unsteady flows while preserving the physics governing their dynamics and without inserting adjustment or regularisation terms.

MOTIVATION AND OBJECTIVES

For a dynamical system and a series of observations of this system in time, sequential data assimilation is a method which combines the observations with prior knowledge of the current state of the system to obtain updated and improved estimates of the distribution of true model states or parameters. However, a complete observation of the state of a dynamical system is usually impossible. Furthermore, in a sequential data assimilation framework, the observation space can consist of physical quantities of a different nature to those of the state of the dynamical system under consideration. A dynamical system may therefore be controlled via independent experimental data.

In fluid mechanics, the linear stochastic estimation (LSE) is widely used to reconstruct the velocity field (or other quantities) from measurements carried out at one or more points in the flow field (Adrian 1977). This estimate can also be made using the extended POD which serves to estimate the rate of correlation between a signal (parietal pressure, velocity, etc.) and each POD mode of the flow. The estimation of the correlated part of the signal is then identical to a LSE decomposition (Boree 2003). Examples of the use of LSE can be found in the works of L.Hudy (2007), Murray (2003) and Taylor (2004), which combine LSE with pressure measurements in order to obtain a complete description of velocity fields for various geometries such as axi-symmetric jets or cavities. Several extensions of LSE have also been studied; on this subject the works of Hudy (2007) and Tinney (2006) can be consulted. However, and despite being extensively, and successfully, used in fluids mechanics, the LSE can suffer from "over-fitting" phenomena in configurations where the input data are highly correlated in time (Gapentine 1997, Mason 1991, Lipovetsky 2001). In such cases, the lack of statistical independance in the observations makes the linear model highly sensitive to noise or small disturbances (Cornillon and Matzner-Lober 2011). Generalized regression techniques like the Partial Least Squares Regression (PLSR) can be envisioned to overcome these LSE issues. The PLSR is a data analysis method which was initially proposed by Wold (1966) as an alternative to multiple linear regression when there is strong multicollinearity among the descriptors or when the number of descriptors is much higher than the number of individuals. It is mainly used to predict a set of dependent variables from a set of explanatory or descriptive variables.

Besides, LSE is considered as a static mapping since it uses only one sample of the unconditional source to estimate the flow state (Cattafesta 2008). A more accurate reproduction of the dynamics of an unsteady flow can be obtained using dynamic estimators like the Kalman filter (Rowley 2005). A POD complementary technique called Multi-time-delay LSE was proposed by Durgesh (2010) as an alternative to LSE. Given a number of past and future measurements of the pressure signal, this method consists in estimating the time evolution of the first POD mode coefficients and providing a low dimensional and time-resolved reconstruction of the flow field at a given instant (Cattafesta 2008; Tu 2012), In this work, the construction of the dynamical system is based on a Galerkin projection of the Navier-Stokes equations on a reduced-dimension basis determined by Proper Orthogonal Decomposition (POD). Thanks to its optimality property, the POD allows the definition, via a set of flow solutions originating from a numerical or experimental database, of the best approximation of the flow field database from an energy perspective, using a limited number of proper functions. A model reduction methodology, called POD-Galerkin, is then used to define a reduced order flow model by Galerkin projection of the Navier-Stokes equations on a POD basis (see for instance Holmes 1996 for a review). The corresponding reduced order model (ROM) is a system of ordinary differential equations (ODE) of small dimensions, which governs the evolution of the temporal coefficients associated with the POD modes (Galletti 2004; Buffoni 2006). In order to use a reduced order model to reconstruct the flow, its accuracy and robustness must be guaranteed. Indeed, it has to reproduce as faithfully as possible the complex dynamics contained in the experimental database while being robust to variations of the input parameters or to changes in the flow configuration. However, the POD-Galerkin reduced order model is non-robust and structurally unstable. It determines in general a dynamical system that in many cases may lead to erroneous states after a relative short time of integration (Rempfer 2000; Noack 2003; Leroux et al. 2012). The origins of the lack of robustness of the POD-Galerkin ROM are evidenced at various levels (Holmes 1996; Rempfer 2000; Noack 2003).

In this paper a calibration procedure is considered, that is based on stochastic methods which includes forecast and observation models. The forecast model propagates the systems dynamics forward in time and the observation model maps the observations to model states and a probability density function, (pdf) of model and measurements errors. These techniques serve to reconstruct the state of a dynamical system by combining the information contained in the spatial and temporal evolution equations of the dynamical system under consideration with the physical information contained in the observations of this system over time. These methods are commonly used in fields such as oceanography and meteorology where it is necessary to introduce observation data into the model in order to take account of the spatial and temporal specificity of the physical phenomenon being studied, and thus be able to make an historical analysis of this phenomenon or of the forecast (Cao 2007; Fang 2009; Ledimet 1986).

This paper concerns the use of the ensemble Kalman filter (EnKF) to reconstruct the temporal evolution of a velocity flow field around a NACA0012 airfoil at an angle of attack α=20° and a Reynolds number Re=1000 from a hot-wire probe signal (for more details on the application of Kalman filtering on the POD-Galerkin ROM see Leroux et al. 2012). In order to compare the observations with the state of the system, we propose to modify the observation process in order to link the POD coefficients with scalar measures from an external voltage signal obtained by hot film anemometry. The observation operator is determined by PLSR and then associated with the equations-of-state process provided by POD-Galerkin reduced model (POD ROM). The ensemble Kalman filter is then applied to the thus-defined state space model in order to provide an estimate of the time coefficients of the POD from indirect observations.

EVOLUTIONMODEL

The aim of this study is to obtain a dynamic estimation of the temporal prediction coefficients of the Galerkin POD ROM over the entire time domain using measurements of a voltage signal obtained by a hot film sensor. In order to assimilate the observations sequentially and to dynamically estimate the prediction error variances and covariances over time, we apply the EnKF filter on the POD-Galerkin reduced order model (Leroux et al. 2012) . The state space model used then writes :

$$\begin{cases} x_k = f_{k-1}(x_{k-1}, w_{k-1}) \\ \mathbf{y}_k = h_k(x_k) + v_k = G(s) + \varepsilon_k \end{cases}$$

Where x_k denote the state vector of the dynamical system at time k with initial pdf $p(x_0)$. The mappings functions f and h represent the system and observation models respectively. Here f_k is a non linear operator describing the state propagation between two consecutive time steps k-1 and k. The true state at time k is assumed to be related to h_k , the observation vector which describes what observation would be measured given the state x_k . The observation y_k is conditionaly independant given x_k and the observation is represented by the pdf $p(y_k|x_k)$ which is often named as *likelihood*.

In this paper, f_k correspond to the POD-Galerkin ROM and the observation y_k are the temporal POD modes. We assume that the stochastic process w_k and v_k are i.i.d (*independant identically distributed*) additive temporal white gaussian process with zero mean and covariance matrices Q_k and R_k respectively. The three following processes are considered :

$X_{1:k}=\{x_1, x_2 ..., x_k\}$ the process of the prediction modes $a_i(t_k)$ of the POD-Galerkin ROM.

$Y_{1:k}=\{y_1, y_2 ..., y_k\}$ the process of the projection modes $a_i(t_k)$ from the POD.

$S_{1:k}=\{s_1, s_2 ..., s_k\}$ the process of the observations based on the voltage signal from the hot film anemometry.

At each time t in the assimilation process, the observation process $Y_{1:k}$ is associated with an observation process $S_{1:k}$ constructed from samples of signal s taken in a time interval I defined as follow :

$$I_k = \{t_k + \tau_1, \ldots, t_k + \tau_n\}$$

where τ_n are time lags seating time t from the samples of S chosen for the observation. Here the objective is to estimate the process X from the sole process of observations S. The operator G is determined using a multiple nonlinear regression between the temporal projection coefficients of the POD and the hot film signal. The determination of G is based on King's law (King 1914) that establishes a relationship between the norm of the incident flow on the sensor and the voltages delivered by the hot film anemometer. We thus get the following relation :

$$V^2 = A + BU^n$$

where A,B and n are generally obtained by calibration. Here, no calibration is performed and the initial value n=0.5 given by King is retained. We can then write the following relation :

$$U \simeq \frac{1}{B^2}V^4 - \frac{2A}{B^2}V^2 + \frac{A^2}{B^2}$$

Since the hot wire is in the wake of the airfoil, the flow velocity on the hot wire is partially correlated with incident flow speed U on the hot film sensor. In this way, time series a_i and s can be correlated, allowing us to attempt to identify the following relation :

$$a_i(t_k) = \alpha_n + \sum_j \beta_{ij} s^2(t_k + \tau_j) + \sum_j \gamma_{ij} s^4(t_k + \tau_j)$$

where the series of coefficients are obtained by multiple linear regression. In the framework of this study, operator G is determined by PLSR and directly links signal s to

$$R_k^N = \frac{1}{N-1}\sum_{i=1}^{N}(\varepsilon_k^i)(\varepsilon_k^i)^T \quad \text{with} \quad \varepsilon_k^i = y_k^{POD} - y_k^{PLSR}$$

coefficients a_i, therefore the processes X and Y are directly linked by the identity operator. In this formalism, the observed process S is thus a vector comprising the values taken by s^2 and s^4 in the time interval I_k. The assumption is made that observation errors are uncorrelated to each other, i.e. the error in a given observation has no statistical link with the errors in other observations. The observation errors are assumed to be Gaussian (Kalman 1961). The empirical covariance marix R_k^N of the EnKF is thus :

where ε_k is the deviation between the observations estimated by regressions and the observations, $y_k^{(POD)}$ are the observations obtained by the POD and $y_k^{(PLSR)}$ those obtained by PLSR. An approximation by the EnKF of the POD temporal prediction coefficients is then given by :

$$x_k^{a,i} = x_k^{f,i} + K_k^N[y_k^{PLSR} - x_k^{f,i}]$$

where the coefficients $x_k^{(f,i)}$ are the predicted modes obtained with the EnKF applied to the Galerkin POD ROM.

EXPERIMENT

The experimental configuration settled for this work consists of the flow around a NACA0012 airfoil of chord c=60mm at a Reynolds number Re of 1000 and an angle of attack α of 20°. The velocity measurements have been done in a square (160mm×160mm) section water tunnel. Time-resolved 2D-2C PIV measurements have been carried out using a Nd-YAG laser (Quantel, 2*120mj) and a Pulnix Dual tap Accupixel camera (2048×2048px image size), seeding was comprised of 15 µm mean diameter polyamid particles. The series of 2048 PIV records have been analyzed through a cross-correlation technique implemented with a Fast-Fourier-Transform algorithm in a multi-grid process with 3 iterations (1 at 64×64 and 2 at 32×32) with 75% and 50% overlapping respectively, using window shifting with iterative deformations and Gaussian sub-pixel peak localization (Lavision Davis 7). Spurious velocity vectors have been identified by a median filter and replaced by using secondary cross-correlation peaks. The full series of experiments comprised 2048 samples of velocity fields obtained at a sampling frequency $f^{(PIV)}$ =6,4Hz.

Voltage signal measurements using a TSI hot film probe 1269W of diameter 5mm connected to a constant-temperature hot-wire anemometer system (DISA55M01) is synchronized with the PIV apparatus. This probe is placed at 7 chords downstream of the trailing edge of the NACA0012 profile, as seen in Figure 1. The sampling frequency of the hot-wire measurements is f_s =2,5kHz. Thus a large number of samples of voltage signal is avalaible between each PIV measurements of the flow.

The low Reynolds number of the flow makes it possible to assume a swirling, turbulence-free flow which remains mainly two-dimensional. Different kinds of vortex shedding for a NACA0012 profile according to the angle of attack and Reynolds number were determined by Huang (2001). Following their classification for α=20° the vortex shedding is of type “leading edge vortex“. The full sequence covers 23 vortex shedding cycles with a Strouhal number =0.26 S_t he sampling frequency enables to obtain approximatively 89 snapshots of velocity field per by vortex shedding period

$$T_{S_t} = \frac{1}{S_t}\frac{c}{U_0}$$

Figure 2 represents snaphots of the flow field estimated from PIV for non dimensional times $t^* = t/T_{S_t}$

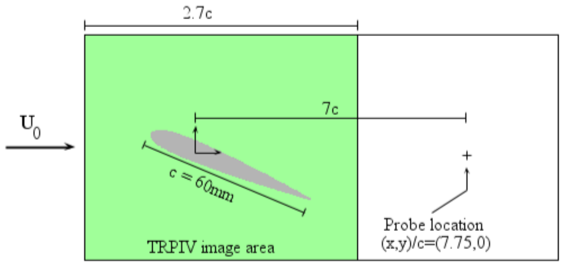

Figure 1. Schematic of experimental set-up with TRPIV and hot-wire signal.

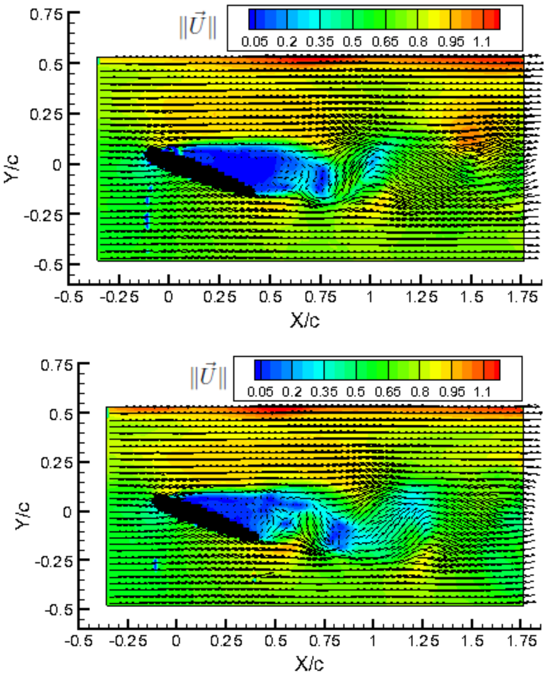

Figure 2 Snaphots of U(x,y,t) from PIV - t*=5.74 and t*=23

ESTIMATIONOFTHEOPERATORG BY MULTILINEAR PLSR

We consider the regression model defined in section 3 and used for the PLSR. Response variables Y correspond to the temporal projection coefficients of the POD that we are seeking to approximate from signal s. Explanatory variables X correspond to signal s^2 and s^4. The EnKF filter is sensitive to the noise contained in the observations. This is why, according to a given window, the largest possible percentage of variance explained in Y will be sought by PLSR with cross-validation. The intervals $I_k^{(i)}$ (i=1...5) are defined as :

$$I_k^{(1)} = \{t_k - 100, t_k - 99, .., t_k + 99, t_k + 100\}$$
$$I_k^{(2)} = \{t_k - 600, t_k - 599, .., t_k + 599, t_k + 600\}$$
$$I_k^{(3)} = \{t_k - 1000, t_k - 999, .., t_k + 999, t_k + 1000\}$$
$$I_k^{(4)} = \{t_k, t_k + 1, .., t_k + 1499, t_k + 1500\}$$
$$I_k^{(5)} = \{t_k - 1500, t_k - 1499, .., t_k - 1, t_k\}$$

The intervals selected are those that provide a maximum percentage of explanatory variance and a reconstruction error between the temporal projection coefficients and those estimated by PLSR. As the system for acquisition of signals does not allow us to use a multiple frequency f_s of $f^{(PIV)}$, there is a variability in the positioning of these intervals in relation to t. Intervals $I_k^{(i)}$ constructed from the intervals in the neighbourhood of t thus undergo this variation.

The PLS regression is applied to these intervals in order to determine operator G. For each interval, we increase the time lag with a step of 100 until the PLS regression captures the maximum explanatory variance whilst providing a good estimation of the POD temporal projection coefficients. For the PLS regression, we therefore select the number of principal components by cross-validation with a percentage of explanatory variance higher than 99.9%, using cross-validation in order to avoid problems of overestimation of the model. The quality of the regression is evaluated while using the RMSE *(Root Mean Square Error)* between the coefficients $a_i^{(PLSR)}(t_k)$ predicted by regression and the observed coefficients $a_i^{(POD)}(t_k)$, i.e. temporal coefficients of projection resulting from the POD and the multiple coefficient R^2 where

$$R^2 = Cor^2(a_i^{(POD)}, a_i^{(PLSR)})$$

in [0,1]. Considering the minimal value of RMSE and the maximal values of R^2 obtained for each intervals $I_k^{(i)}$, the intervals $I_k^{(i)}$ with i=2,4,5 are selected.

So as to validate the EnKF results a correction of the empirical covariance of the EnKF is provided thanks to a $\chi^2(K)$ distribution with K degrees of freedom, where K is the observation vector dimension.

Once the EnKF filter has been correctly set, it is used to reconstruct the temporal prediction coefficients. The temporal prediction coefficients obtained are then compared with the coefficients from the POD.

RESULTS

The Kalman filter and Ensemble Kalman Filter (EnKF; Evensen, 1994) have been applied to the empirically defined LODS. First, the principle of Kalman filtering POD-Galerkin LODS is validated on dynamical systems reduced to their linear part. Then, the Ensemble Kalman filters are applied to the non-linear LODS using different sets of observable data.

Finally, The PLSR is applied to provide a regression model between the hot-film signal and the POD temporal coefficients. This model is then injected as the observation term in the Kalman filters to stabilize the LODS.

Figure 3 shows a comparison of a temporal mode reconstruction between POD reconstruction and EnKF LODS with PLSR for the 10th mode of a 10 mode LODS. Figure 4 shows RMSE between projection and predicted coefficient for the linear and quadratic POD-ROM. Figure 5 shows a comparison of the corresponding reconstruction of velocity fields between the raw experimental data and the EnKF with PLSR applied to a 10-mode quadratic LODS.

On this test case, the EnKF filter faithfully reports the temporal dynamics of the coefficients. The reconstruction error on each coefficient is low, both for the first coefficients and for the higher-order coefficients, the temporal dynamic of which is more difficult to approximate. The amplitude of the coefficients is respected over time, and the POD-Galerkin reduced model remains stable over the entire data assimilation process. The phase relation of the coefficients is also reproduced. However, no definite trend in the reconstruction can be deduced from error between the temporal projection and temporal prediction coefficients as same orders of magnitude are obtained for all intervals and modes.

Indeed, this error remains the same for the predicted coefficients $a_2(t), a_7(t), a_9(t)$ for the three intervals.

For interval $I_k^{(2)}$ the reconstruction error is maximal for the predicted coefficients $a_1(t), a_3(t)$, for interval $I_k^{(4)}$ its maximal for $a_6(t)$ and for interval $I_k^{(5)}$ its maximal for $a_4(t), a_{10}(t)$.

With the linear model and the $\chi^2(K)$ test we achieve a reconstruction with an error between 2.10^{-3} and 8.10^{-3}, which is relatively homogenous according to the selected interval.

CONCLUSION

Kalman Filtering can be used as an efficient strategy for stabilizing the evolution of experimentally identified LODS which naturally tend to vanish or diverge. Compared to regularization procedures which result in complex implementations and tuning, the Kalman filters are parameter-independent and, despite their complex underlying theoretical basis, easy to operate. The injection of external data conditionned using PLSR provides reliable observation terms which can be efficiently processed by the Kalman filters. It can be concluded that Kalman filters can be applied to stabilize POD-Galerkin LODS using observable data of different nature and quality.

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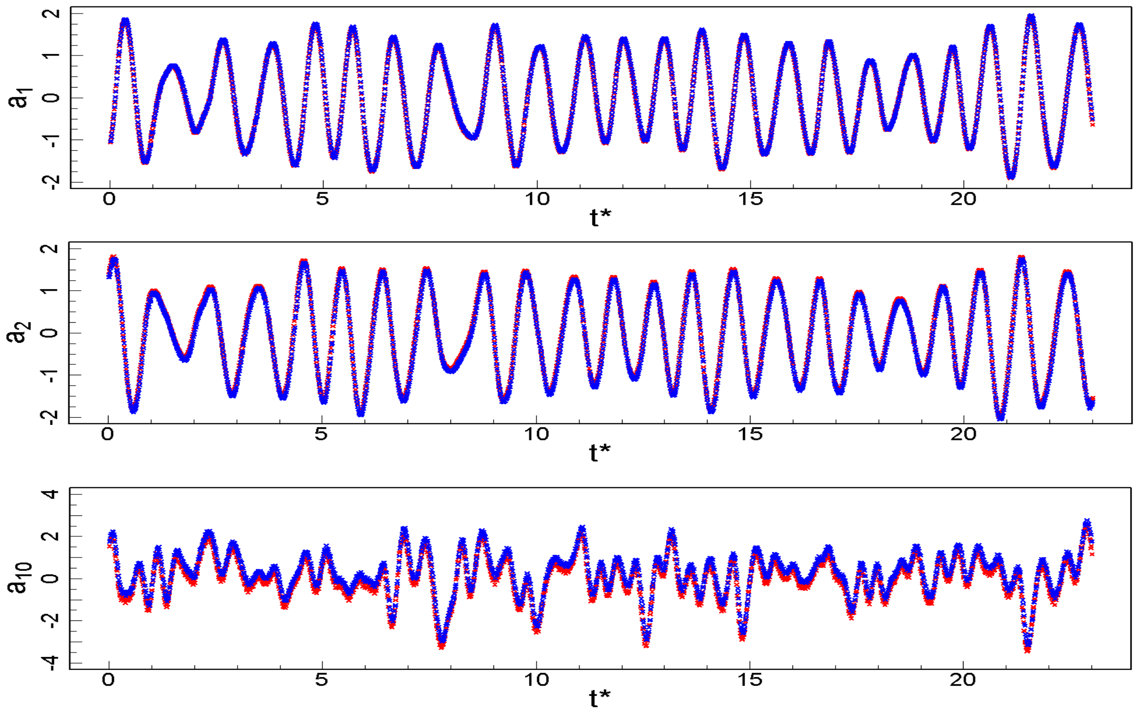

Figure 3. Reconstructed temporal modes a1(t), a2(t), a10(t) - Linear ROM : red dashed line, the temporal POD modes ; blue dashed line, result of the EnKF filter for interval $I_k^{(2)}$

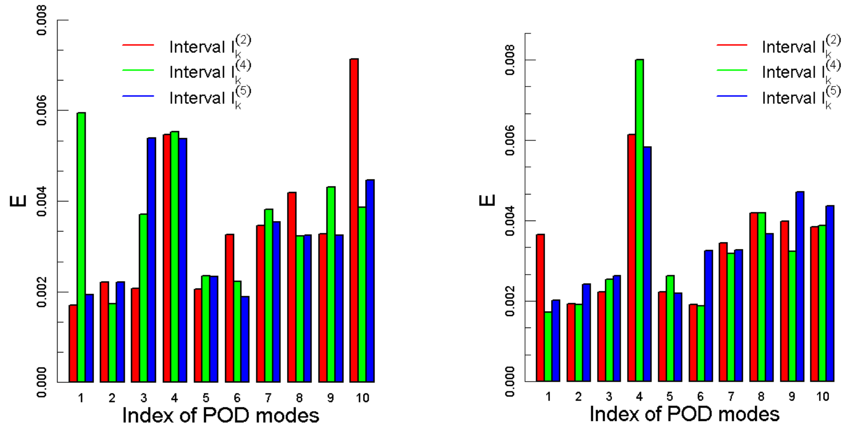

Figure 4. RMSE of the temporal modes for interval $I_k^{(2)}, I_k^{(4)}, I_k^{(5)}$ (Left) Linear POD ROM - (Right) Quadratic POD ROM

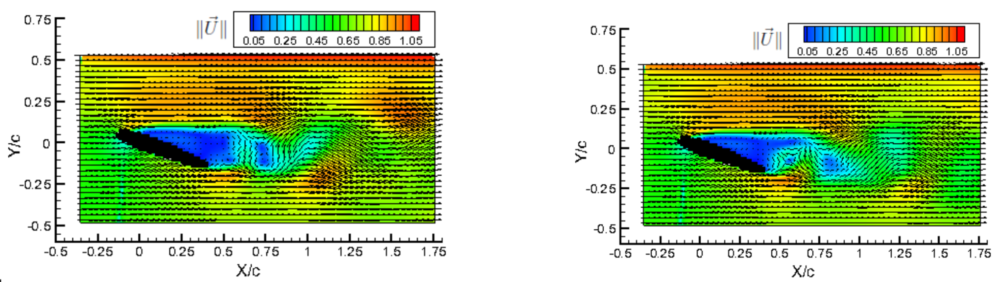

Figure 5. U(x,y,t) reconstructed flow field with the EnKF on the quadratic POD-ROM with 10 temporals modes - t*=5.74 and t*=23 - Interval $I_k^{(2)}$