# COMPARATIVE ANALYSIS OF RELATIONSHIP BETWEEN INSTANTANEOUS AND STATISTICAL PROPERTIES OF THE DETERMINISTIC TURBULENCE

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## ABSTRACT

As known from previous studies (Borodulin et al., 2007, 2011) the deterministic turbulence represents a post-transitional wall-shear flow that is turbulent according to the generally accepted statistical characteristics but possesses, meanwhile, a significant degree of determinism, i.e. reproducibility of its instantaneous structure. It is found that the deterministic turbulence can occur in those cases when transition is caused by convective-type instabilities.

However, there was a question remaining: "Whether the deterministic turbulence corresponds exactly to the ordinary random turbulent flow or not?" If the answer is affirmative, the stochastic properties of such turbulence (such as mean-velocity profiles, disturbance amplitude profiles, spectra of turbulent fluctuations, etc.) must be: (*i*) independent of particular realization of the deterministic turbulent flow and (*ii*) identical to those of the ordinary (stochastic) turbulence. *The present study concentrates on investigation of the answer to point (i) indicated above.*

## INTRODUCTION

There was a long-term discussion about the scenarios of the transitional flow 'randomization' which has to occur, as believed, at late stages of the boundary-layer laminar-turbulent transition. It was usually assumed by the term 'randomization' that some broadband, uncontrolled disturbances have to grow rapidly and lead to appearance of stochastic turbulent motions inherent for fully turbulent flow. Meanwhile, some detailed experimental and numerical studies of late stages of transition performed during past decade (see e.g. papers by Borodulin et al. 2002 and Borodulin et al. 2006) have not revealed any definite mechanisms of the final flow randomization. Therefore, the question appeared: "Is it possible that the instantaneous structure of transitional flow would remain deterministic, reproducible and repeatable in the main (at repetition of the same initial conditions) at super-late, final stages of transition and even in the post-transitional fully turbulent boundary layer?" The clear answer to this question was obtained recently by Borodulin et al. (2007, 2011). This answer was: "Yes, it is possible".

One has to note that the investigation of the deterministic turbulence is very important because, in particular, it represents a powerful tool for both applied and fundamental investigations. Indeed, the first practical application of the deterministic turbulence method performed by Borodulin et al. (2008, 2009) has been demonstrated its great efficiency. The instantaneous flow field of a deterministic turbulence was documented in detail in those experiments in a boundary layer. Then a Large-Eddy-Break-Up (LEBU) device was installed at a position, where the boundary layer was already practically turbulent, and the same instantaneous flow field was documented again. Detailed comparison of two flow fields allowed the authors to see very clearly the physical mechanism of the LEBU device affect on the turbulent boundary layer.

However, it was necessary to answer a question about the exact correspondence in statistical sense of the deterministic turbulence to the ordinary random turbulent flow. First of all, it was necessary to study the independence of the statistical characteristics of the deterministic turbulence on its instantaneous structure, which depends on the spectrum of initial disturbances initiated the turbulent

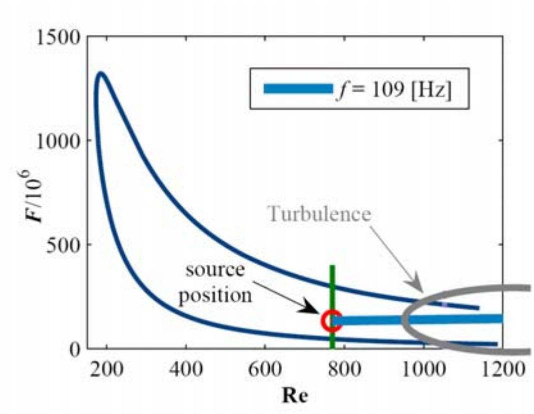


Figure 1. Region of measurements with respect to the neutral stability curve.

flow. The main goal of the present work is to investigate this problem.

## EXPERIMENTAL PROCEDURE

The experiments were performed in a self-similar adverse-pressure-gradient (APG) boundary layer at fully controlled disturbance conditions at free-stream speed of about 9 m/s. The boundary layer developed over a flat plate surface and the APG was induced by a contoured wall bump mounted above the plate on the wind-tunnel test-section ceiling. The base-flow properties were similar to those studied by Borodulin et al. (2006, 2011) with the laminar boundary layer (present in absence of disturbance excitation) having Hartree parameter $\beta_H = -0.115$.

The laminar-turbulent transition and the deterministic turbulence were produced due to a natural development of Tollmien-Schlichting (TS) waves. The initial disturbances represented a superposition of a 2D TS-wave at frequency of 109 Hz and a broadband spectrum of 3D TS-waves (random in time and space). Fig. 1 demonstrates a region of measurements with respect to the neutral stability curve for the studied base flow. The amplitude of uncontrolled background velocity disturbances was less than 0.03% of the incident flow velocity (in the frequency range above 1 Hz). The controlled initial disturbances were produced by a generator of TS-waves described in Borodulin et al. (2011). The streamwise evolution of them started from their linear amplification, through nonlinear stages, and ending with the fully turbulent flow.

In fact, we carried out several experiments representing different realizations of the deterministic turbulent boundary layer. They have been named the experiments: *A*, *B*, *C*, *AL1*, *BL1*, *CL1*, *AL2*, *BL2*, and *CL2*. The characters *A*, *B*, and *C* designate three different particular realizations of initial signals excited by the disturbance source. The 2D TS-wave component of the signal was the same in all these three cases while the broadband 3D NS-wave components were different. Symbols *L1* or *L2* designate presence in the flow of either LEBU-device #1 of LEBU-device #2. The two devices represented thin metal plates located parallel to the surface at a distance of 4.2 mm from the wall and having chord lengths of 4 and 8 mm respectively. The LEBU-device trailing edge always located at $x = 500$ mm. Every particular realization of the instantaneous flow structure was produced by a particular set of signals (either *A* or *B* or *C*) used for the

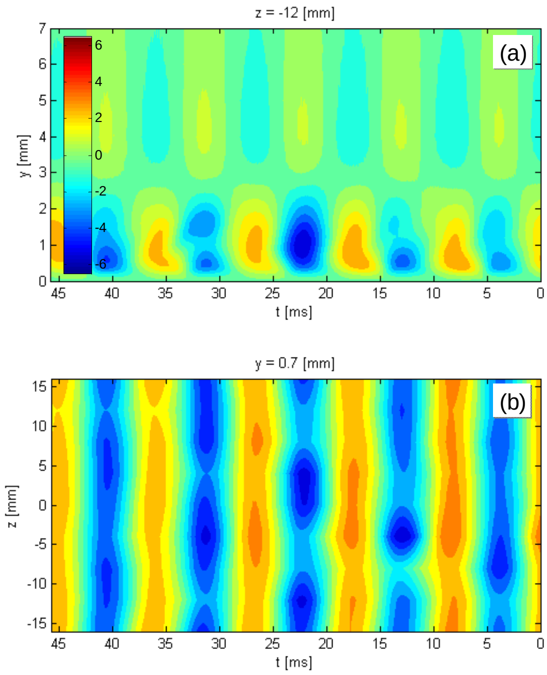


Figure 2. Side (*a*) and plan (*b*) views of instantaneous streamwise-velocity field measured at “initial” streamwise position $x = 350$ mm in experiment ***A***.

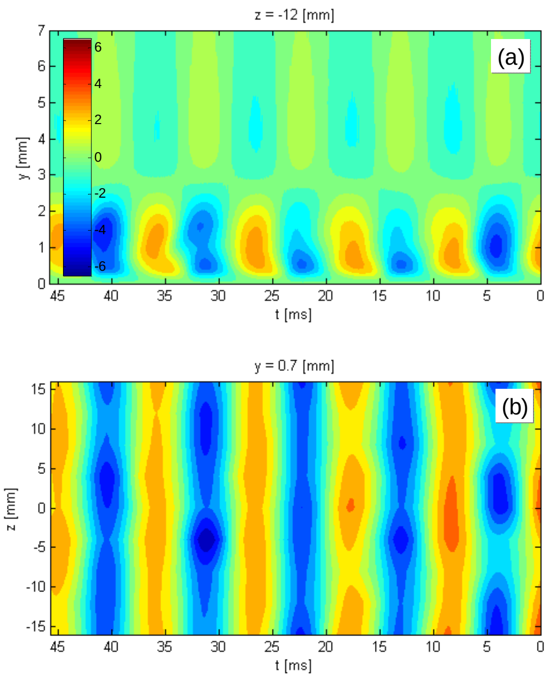


Figure 3. Side (*a*) and plan (*b*) views of instantaneous streamwise-velocity field measured at “initial” streamwise position $x = 350$ mm in experiment ***B***.

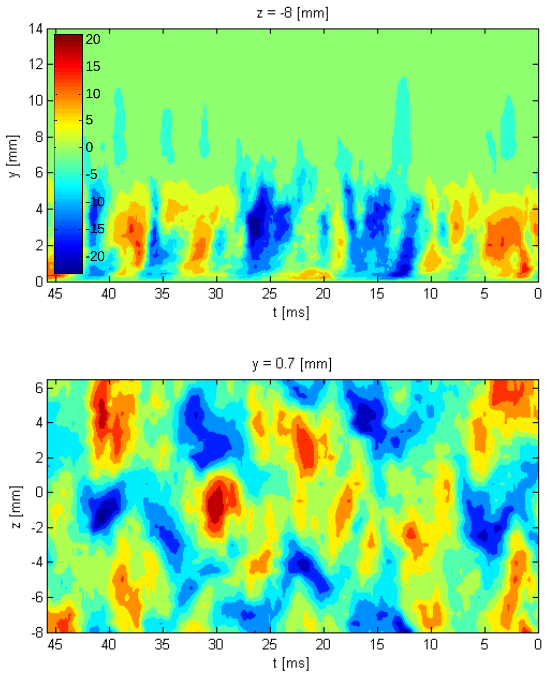


Figure 4. Side (*a*) and plan (*b*) views of instantaneous streamwise-velocity field measured at streamwise position $x$ = 520 mm in experiment ***A***.

boundary-layer excitation. This instantaneous structure was reproduced many times (up to three hundred thousand times in one set of measurements) by means of a precise reproduction of initial disturbances and maintenance of the mean flow characteristics. Then we have analysed the

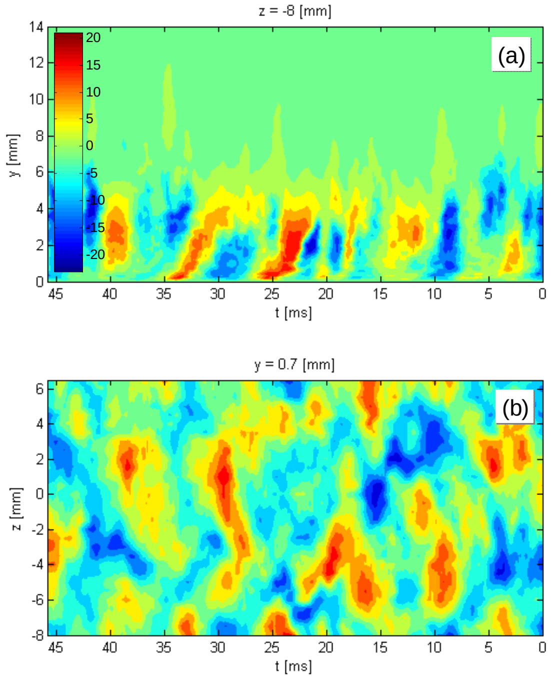


Figure 5. Side (*a*) and plan (*b*) views of instantaneous streamwise-velocity field measured at streamwise position $x$ = 520 mm in experiment ***B***.

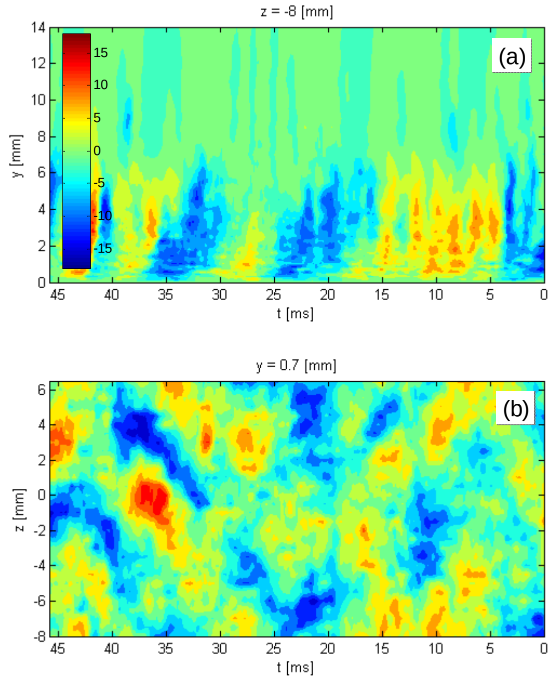


Figure 6. Side (*a*) and plan (*b*) views of instantaneous streamwise-velocity field measured at streamwise position $x$ = 550 mm in experiment ***A***.

constancy of statistical flow characteristics in different experiments, i.e. we have check independence of these characteristics from the particular instantaneous-flow structure.

## INSTANTANEOUS DISTURBANCE FIELDS

Two examples of results of measurements of the "initial" instantaneous structure of the perturbed boundary-layer flow, carried out at $x$ = 350 mm (i.e. 50 mm downstream the disturbance source location) are shown in Figs. 2 and 3 for regimes *A* and *B*, respectively. Every figure is obtained by a single hot-wire probe after scanning (point by point) the flow field in the ($y$, $z$, $t$)-space. Thus, every plots represent just a cross-section of one of three arrays of experimental data related to the particular streamwise position $x$ = 350 mm, which corresponds approximately to the initial nonlinear stage of the laminar-turbulent transition processes. The figures display contours of constant values of the instantaneous streamwise velocity component.

It is seen that the difference of the particular realizations of the broadband components excited in regimes *A* and *B* (as well as in regime *C*, which is not shown) results initially in a significantly different 3D perturbations of the same 2D component of the excited TS-waves.

Further downstream the whole disturbance fields become significantly different. For the same two regimes *A* and *B* this point is illustrated in Figs. 4 and 5 ($x$ = 520 mm) and Figs. 6 and 7 ($x$ = 550 mm). These sections correspond practically to the fully developed deterministic turbulent boundary layer.

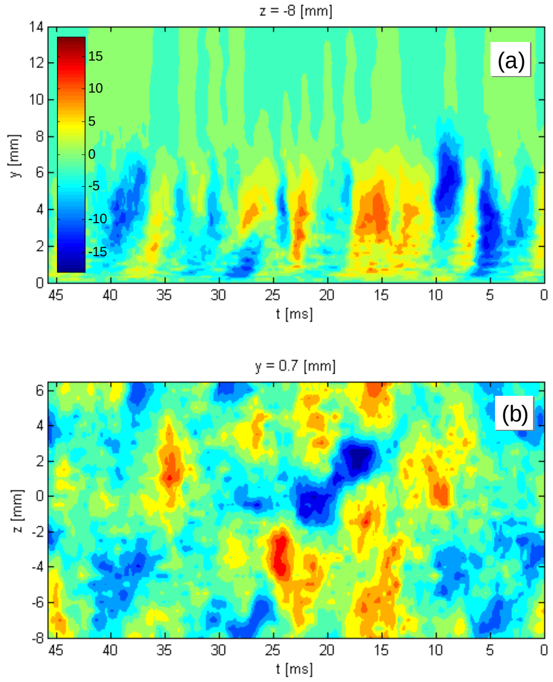


Figure 7. Side (*a*) and plan (*b*) views of instantaneous streamwise-velocity field measured at streamwise position $x$ = 550 mm in experiment ***B***.

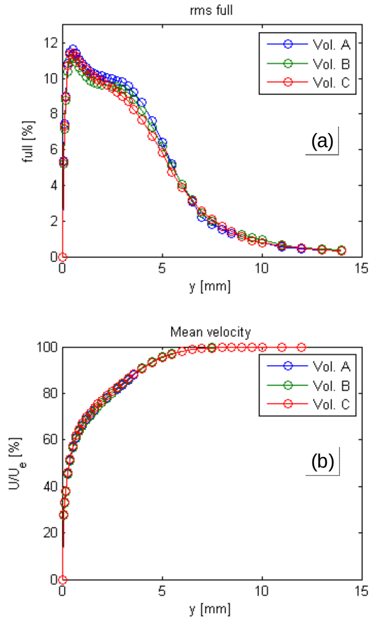


Figure 9. Profiles of rms-intensity of velocity fluctuations (*a*) and mean velocity (*b*) measured in experiments ***A***, ***B***, and ***C*** for streamwise-velocity component at stage of just formed deterministic turbulent boundary layer ($x$ = 520 mm).

Thus, it has been found that in the discussed three regimes of excitation (*A*, *B*, and *C*), the instantaneous flow fields are very much different from each other at these 'late' streamwise locations, although a general character of the disturbance field remains the same.

Qualitatively similar results are observed in triplets of regimes *AL1*, *BL1*, and *CL1*, as well as in *AL2*, *BL2*, and *CL2* (not shown in figures).

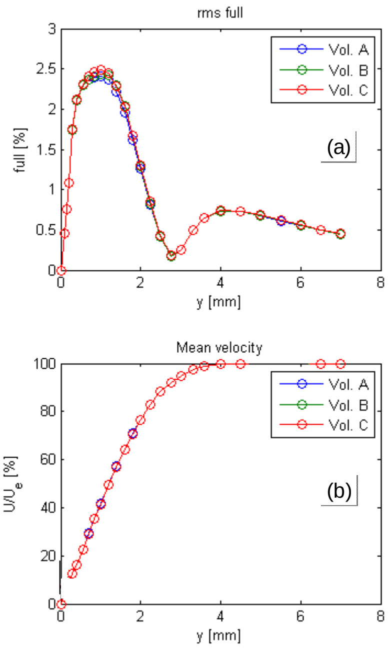


Figure 8. Profiles of rms-intensity of velocity fluctuations (*a*) and mean velocity (*b*) measured in experiments ***A***, ***B***, and ***C*** for streamwise-velocity component at initial nonlinear stage of laminar-turbulent transition ($x$ = 350 mm).

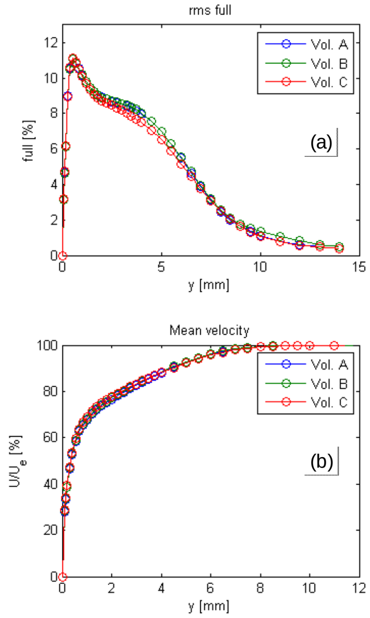


Figure 10. Profiles of rms-intensity of velocity fluctuations (*a*) and mean velocity (*b*) measured in experiments ***A***, ***B***, and ***C*** for streamwise-velocity component at stage of developed deterministic turbulent boundary layer ($x$ = 550 mm).

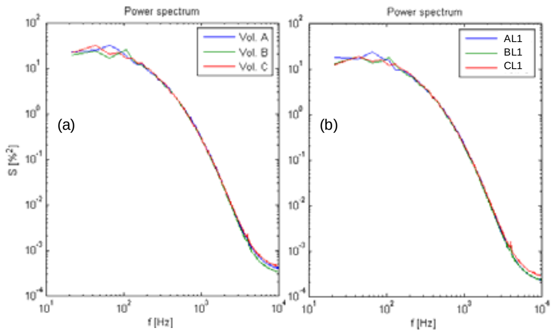


Figure 11. Frequency spectra of velocity fluctuations measured in experiments ***A***, ***B***, and ***C*** (*a*) and ***AL1***, ***BL1***, and ***CL1*** (*b*) for streamwise-velocity component at stage of just formed deterministic turbulent boundary layer ($x$ = 520 mm). Wall-distance $y$ = 1.0 mm.

## AVERAGED CHARACTERISTICS

The mean velocity profiles and the profiles of the rms-intensity of velocity fluctuations are the most important statistical characteristics of velocity fields in turbulent boundary layers. These profiles have been measured for all studied regimes and compared with each other. Shown in Figs. 8, 9, 10 are the profiles obtained at $x$ = 350, 520, and 550 mm, respectively, in three regimes of excitation: *A*, *B*, and *C*. All profiles are averaged in time (during a rather short interval of 46 ms) and in the spanwise direction (in order to increase the accuracy of the measurements). It is seen that both the mean velocity profiles and the profiles of velocity fluctuations coincide practically with each other within a certain scattering of the experimental points (associated basically with a rather short time interval of averaging used).

Similar results are observed for frequency spectra of streamwise-velocity disturbances presented in Figs. 11 to 14 for wall-normal distances $y$ = 1 mm (Figs. 11 and 12) and $y$ = 4.5 mm (Figs. 13 and 14), two groups of regimes: *A*, *B*, *C* (plots *a*) and *AL1*, *BL1*, *CL1* (plots *b*), and two streamwise locations: $x$ = 520 (Figs. 11 and 13) and 550 mm (Figs. 12 and 14).

The spectral shapes depends on wall-normal location and presence of LEBU-device but are independent of the particular signal realization (*A*, *B*, or *C*), i.e. of the particular instantaneous structure of the deterministic turbulence.

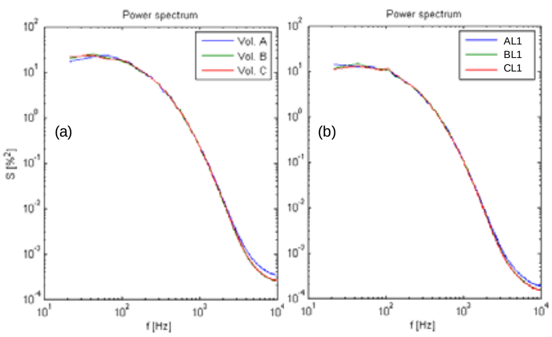


Figure 12. Frequency spectra of velocity fluctuations measured in experiments ***A***, ***B***, and ***C*** (*a*) and ***AL1***, ***BL1***, and ***CL1*** (*b*) for streamwise-velocity component at stage of developed deterministic turbulent boundary layer ($x$ = 550 mm). Wall-distance $y$ = 1.0 mm.

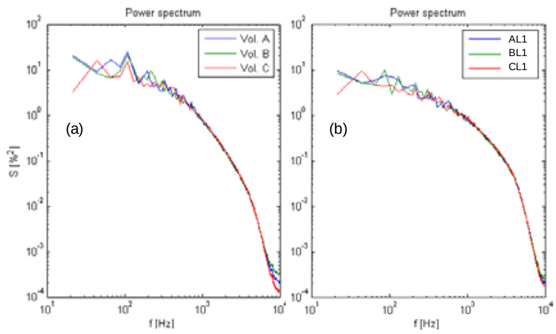


Figure 13. Frequency spectra of velocity fluctuations measured in experiments ***A***, ***B***, and ***C*** (*a*) and ***AL1***, ***BL1***, and ***CL1*** (*b*) for streamwise-velocity component at stage of just formed deterministic turbulent boundary layer ($x$ = 520 mm). Wall-distance $y$ = 4.5 mm.

## INFLUENCE OF LEBU-DEVICES

Note that in presence of LEBU-devices the flow reproducibility remains almost the same as in absence of LEBUs. Moreover, close to the LEBU-device (at $x$ = 520 mm) the instantaneous flow structure remains almost the same as that measured without LEBU (excluding the wall-normal distance of the LEBU location at $y$ = 4.2 mm) if the same signals are used for the boundary-layer excitation. This fact is illustrated by comparison of the instantaneous velocity-field cross-sections obtained in regime *AL1* (Fig. 15), which look very similar to those measured in the corresponding regime *A* (Fig. 4) in absence of LEBU. Similar agreement is observed for the instantaneous fields measured in pairs of regimes *B*, *BL1* and *C*, *CL1*. Again, the mean-velocity profiles and profiles of the velocity fluctuations display their independence practically from the particular instantaneous structure produced by every particular set of signals used for the disturbance source excitation (Figs. 16, 11*b*, and 13*b*).

Similar result is observed further downstream, at $x$ = 550 mm in presence of both LEBU #1 (Figs. 17, 12*b*, and 14*b*) and LEBU #2 devices (not shown in plots).

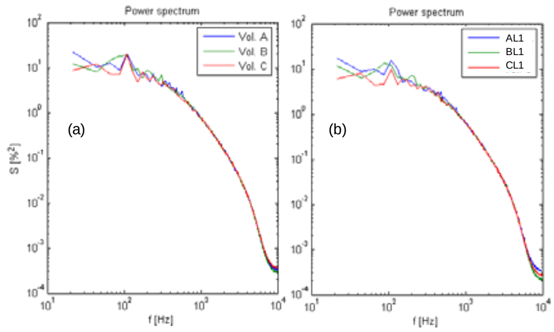


Figure 14. Frequency spectra of velocity fluctuations measured in experiments ***A***, ***B***, and ***C*** (*a*) and ***AL1***, ***BL1***, and ***CL1*** (*b*) for streamwise-velocity component at stage of developed deterministic turbulent boundary layer ($x$ = 550 mm). Wall-distance $y$ = 4.5 mm.

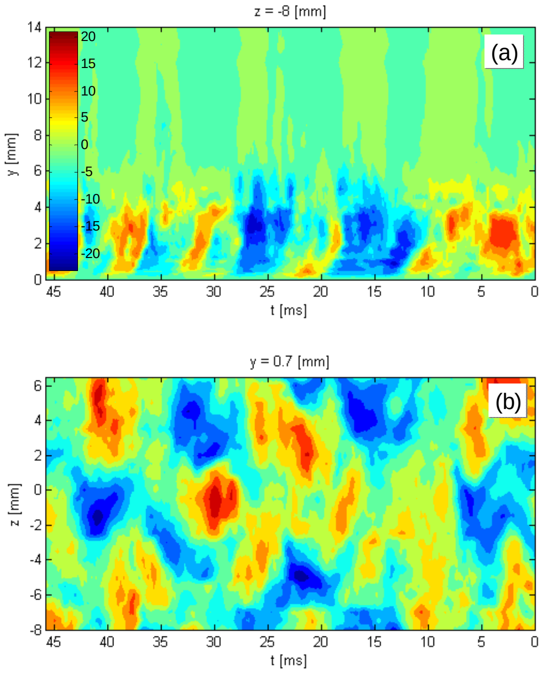


Figure 15. Side (*a*) and plan (*b*) views of instantaneous streamwise-velocity field measured at streamwise position $x$ = 520 mm in experiment ***AL1***.

Thus, it is found that variation of initial disturbance conditions changes significantly the instantaneous structure of the post-transitional deterministic turbulent flow. However, the most important statistical characteristics of all studied flow realizations turned out to remain practically the same and are typical to those of the regular (stochastic) turbulent boundary layer.

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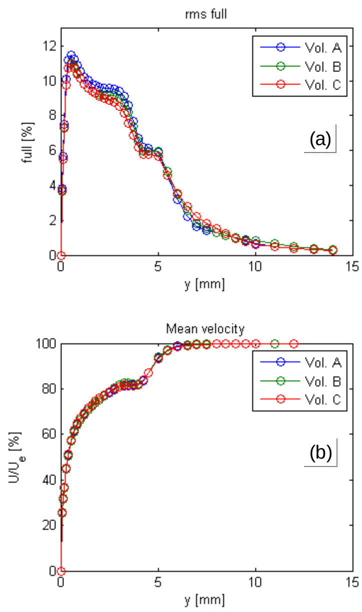


Figure 16. Profiles of rms-intensity of velocity fluctuations (*a*) and mean velocity (*b*) measured in experiments ***AL1***, ***BL1***, and ***CL1*** for streamwise-velocity component at stage of deterministic turbulent boundary layer just downstream LEBU#1-device ($x$ = 520 mm).

## REFERENCES

Borodulin, V.I., Gaponenko, V.R., Kachanov, Y.S., Meyer, D.G.W., Rist, U., Lian, Q.X., and Lee, C.B., 2002, “Late-stage transitional boundary-layer structures. Direct numerical simulation and experiment”, *Theoret. Comput. Fluid Dynamics,* Vol. 15, pp. 317-337.

Borodulin, V.I., Kachanov, Y.S., and Roschektayev, A.P., 2006, “Turbulence production in an APG-boundary-layer transition induced by randomized perturbations”, *Journal of Turbulence,* Vol. 7, N 8, pp. 1-30.

Borodulin, V.I., Kachanov, Y.S., and Roschektayev, A.P. 2007, “The deterministic wall turbulence is possible”, *Advances in Turbulence XI. Proceedings of 11th European Turbulence Conference, June 25–28, 2007, Porto, Portugal*, J.M.L.M. Palma and A. Silva Lopes, eds., Springer, Heidelberg, 2007, pp. 176–178.

Borodulin, V.I., Kachanov, Y.S., and Roschektayev A.P., 2008, “Investigation of LEBU-device effect on turbulent boundary layer structure by means of ‘deterministic turbulence method’,” *XIV International Conference on Methods of Aerophysical Research. June 30 – July 6, 2008. Proceedings*, V.M. Fomin ed., ITAM SB RAS, Novosibirsk, 10 pp.

Borodulin, V.I., Kachanov, Y.S., and Roschektayev, A.P., 2009, “Application of the deterministic turbulence method to study of LEBU-device mechanism”, *Advances in Turbulence XII. Proceedings of the 12th EUROMECH European Turbulence Conference*, *Springer Proceedings in Physics*., B. Eckhardt ed., Springer**,** Berlin, Heidelberg, Vol. 132, pp. 313–316.

Borodulin, V.I., Kachanov, Y.S., and Roschektayev, A.P., 2011, “Experimental detection of deterministic turbulence”, *J. Turbulence*, Vol. 12, No 23, pp. 1-34.

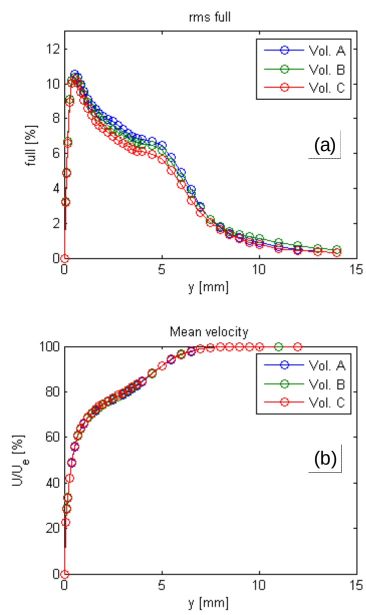


Figure 17. Profiles of rms-intensity of velocity fluctuations (*a*) and mean velocity (*b*) measured in experiments ***AL1***, ***BL1***, and ***CL1*** for streamwise-velocity component at stage of deterministic turbulent boundary layer father downstream LEBU#1-device ($x$ = 550 mm).