Three-dimensional Lagrangian Coherent Structures in the left ventricle model

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ABSTRACT

The left ventricle (LV) flow dynamics was reproduced in a laboratory model, and three dimensional velocity fields were reconstructed from two dimensional image analysis measurements on two sets of orthogonal planes. The flow features were investigated by means of Finite Time Lyapunov exponents (FTLE), a Lagrangian based methodology useful to assess Lagrangian Coherent Structure (LCS) time evolution and flow mixing properties.

INTRODUCTION

A key factor in determining a physiological healthy left ventricle (LV) condition is the proper development of the diastolic function, which is responsible for the left ventricular filling that highly affects the subsequent systolic ejection and, as a consequence, the heart pump efficiency (Mandinov et al, 2000; Vasan and Levy, 2000). Hence, the formation and evolution of the vortical structures during the diastolic inflow contains important information about the physiologic and pathophysiologic features of the flow in the human left ventricle.

Several studies have been devoted to investigate the LV flow dynamics (see Kheradvar and Pedrizzetti 2012 for a review), but the deep comprehension of this complex topic, which is scientifically relevant and has important implications in the diagnostic and surgical context, is yet to be achieved.

The most relevant features of the left ventricular filling were inferred using both experimental and numerical models, as well as clinical observation. From these studies, it emerged that ventricle filling is due to a jet propagating through the ventricle during the diastolic phase; once in the ventricle this jet organizes itself as a vortex ring that rapidly deforms: the posterior part of the vortex ring propagates with a lower velocity and tends to decrease in size and intensity due to the interaction with the ventricular wall, while the anterior side of the vortex ring during the diastasis evolves into a large vortex which at the end of the diastole occupies the whole ventricle, this flow structure appears to be favorable to ejection through the aortic valve during the systole (Pedrizzetti and Domenichini, 2005).

This process is highly complex due to the three-dimensionality of the flow, the asymmetry of the diastolic jet, and the interaction with ventricle walls.

In order to deepen these issues, we performed a series of experiments in a laboratory LV ventricle model, reproducing the main flow dynamics features observed in the human heart.

While most of the previous studies performed using similar apparatus, provided two dimensional velocity measurements on one (Cenedese et al, 2005; Querzoli et al, 2010; Brucker et al, 2002; Pierrakos et al, 2005) or two planes (Espa et al, 2012), in the present work three dimensional velocity fields were reconstructed from two dimensional image analysis measurements on two sets of orthogonal planes. This is huge advantage in the LV dynamics investigation, due to the aforementioned three dimensionality and asymmetry of the flow.

The computation of Finite Time Lyapunov Exponents (FTLE, Haller 2001; Shadden et al. 2005) allows the identification and tracking of Lagrangian Coherent Structures (LCS) of the flow. With respect to the traditional instantaneous analyses techniques (vorticity, Q-criterion, etc.) it has the advantage to reveal the vortical structures boundaries and encode the fundamental Lagrangian description into each single field, conveying key information about the underlying mechanism of fluid transport. This powerful technique has been seldom applied in literature to three-dimensional experimental fields (Vetel et al, 2006). Although important information can be inferred from 2D FTLE (Espa et al. 2012), it is apparent how the comprehension of the complex asymmetric dynamics highly benefits from the availability of 3D velocity fields.

DEFINITIONS AND METHODS

Investigations were performed in a laboratory model shown in Figure 1. The left ventricle is simulated by means of a transparent sack made of silicone rubber, housed in a rectangular tank (A) with transparent walls, allowing for the optical access. The ventricle sack is mounted on a circular plate, 56 mm in diameter, having one inlet and one outlet (simulating aortic and mitral orifice, respectively) that are connected to a constant head reservoir and provided with two one-way valves.

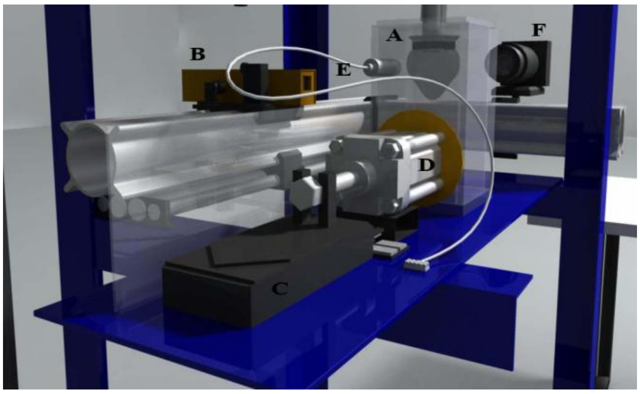

Figure 1. Experimental set-up. A: Ventricle chamber; B: Laser; C: Motor; D: Piston; E: pressure transducer; F: fast camera.

The motion of a piston, driven by a linear motor (C), produces the ventricular volume change, $\Delta V(t)$. Its derivative, Q(t), represents the flow rate through the mitral orifice during the diastole, and through the aortic one during the systole.

The dynamic flow similarity (i.e. matching the ratio of inertial to viscous effects) is assured by matching Reynolds (Re) and Womersley (Wo) numbers:

$$\mathrm{Re} = \frac{UD}{\upsilon} \quad (1)$$

$$Wo = \sqrt{\frac{D^2}{T\upsilon}} \quad (2)$$

between the real case and the model. Here, D is the maximum diameter of the ventricle, U the peak velocity through the mitral orifice, T the period of the cardiac cycle, ν the kinematic viscosity of the working fluid. The geometrical scale is 1:1. Working parameters used during the runs of the experiments are listed in Table 1, and have been chosen in order to obtain non-dimensional parameters within the physiological range. As witnessed by previous two-dimensional measurements performed on this laboratory model (Cenedese et al. 2005; Querzoli et al. 2010; Espa et al. 2012), the experimental set-up satisfyingly reproduces the observed left ventricular fluid dynamics observed in vivo.

Table 1. Experimental parameters.

Stroke Volume [ml]	T [s]	U [m/s]	Re [-]	Wo [-]
6	6	0.145	8322	22

The 3D ventricle flow is reconstructed using measurements on two sets of orthogonal planes. Specifically, for each set, 50 cycles of the cardiac flow on 24 parallel planes spaced 2.5 mm have been recorded (Figure 2). During each acquisition, the measurement plane is illuminated by a 12 W, infrared laser, while the working fluid inside the ventricle is seeded with neutrally buoyant particles, about of 30 μm in diameter. A high-speed digital camera (250 Hz, 1280×1024 pixel) is triggered by the motor to capture the time evolution of the phenomenon at known instants of the cycle. Images are then analyzed using a Feature Tracking algorithm (Cenedese et al. 2005), and velocity vectors are subsequently interpolated over a regular three-dimensional grid. Hence, the mean cycle is obtained by phase averaging.

LAGRANGIAN ANALYSIS

Finite Time Lyapunov Exponents (FTLE) are a measure of the maximum linearized rate of growth of the distance among initially adjacent particles advected by the flow over a finite time interval.

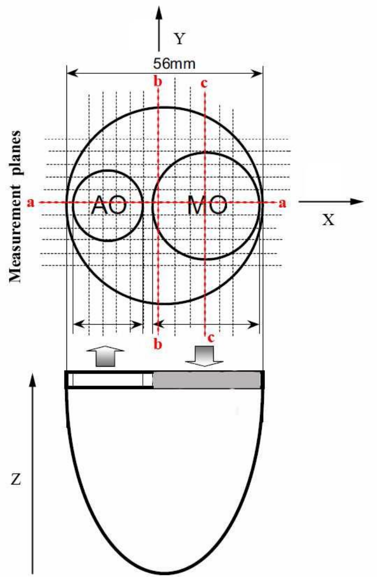

Figure 2. Position of all the measurement planes (red and black dashed lines). AO: aortic orifice. MO: mitral orifice.

FTLE computation is based on the definition of the flow:

$$\Phi_t^{t+T^*} : \boldsymbol{x}(t) \longrightarrow \boldsymbol{x}(t+T^*) \qquad (3)$$

that maps a material point x(t) at time t to its position at time t + T* along its trajectory. After a linearization, the amount of stretching about a trajectory can be defined in terms of the matrix:

$$\Delta = \left(\frac{d\Phi_t^{t+T^*}(\boldsymbol{x})}{d\boldsymbol{x}} \right)^2 \qquad (4)$$

where $\frac{d\Phi_t^{t+T^*}(\boldsymbol{x})}{d\boldsymbol{x}}$ is the Cauchy-Green deformation tensor and Δ represents its finite time representation. Since the maximum stretching occurs when the initial separation is aligned with the maximum eigenvalue of Δ, the FTLE is defined as:

$$\sigma(\boldsymbol{x},t,T^*) = \frac{1}{|T^*|} \ln \sqrt{\lambda_{\max}} \qquad (5)$$

where λ_{max} is the maximum eigenvalue of Δ, and $\sqrt{\lambda_{max}}$ corresponds to the maximum stretching factor.

Trajectories can be integrated forward or backward in time: when a positive T* is considered, the FTLE measure separation forward in time, thus identifying repelling structures, on the contrary, if a negative T* is considered, FTLE measure separation backward in time, thus highlighting attracting structures.

As shown by Shadden et al (2005), even though ridges tend to become sharper with increasing integration time T*, the structure of the FTLE fields is quite persistent with varying T*. However, the choice of an appropriate value for T* has to be related to the characteristic flow time scales.

FTLE computation was performed on experimental velocity fields of the mean heart cycle by means of a public domain code, NEWMAN (Du Toit and Marsden, 2010).

In order to analyse the different amount of information and the discrepancies between FTLE fields obtained from planar and volumetric measurements, the FTLE computation was performed on both 2D velocity fields (plane a-a in Figure 2) and 3D velocity fields.

For both the analyses, we adopted an integration time, T*, equal to the advection time scale, i.e. the ratio between the end-diastolic ventricle diameter and the peak mitral velocity, U (D/U ~ 0.40 s). FTLE were computed on a grid with finer space resolution than original velocity fields to better capture the FTLE ridges (Vetel et al, 2009).

2D FTLE

Despite the fact that the observed phenomenon is three-dimensional, the flow remains essentially two-dimensional on the a-a plane, because this is a symmetry plane. Hence, from the analysis of 2D FTLE computed from velocity measurements on this plane it is possible to infer important features on the vortical structures evolution.

In Figures 3 and 4, backward and forward FTLE fields obtained on the a-a plane are shown for different relevant phases of the cardiac cycle. Corresponding mean velocities are represented as white arrows in the plots. The small panel in the bottom-left corner of each plot shows the diagram of the flow rate as a function of the non-dimensional time (solid line), while a solid circle represents the initial time used for corresponding FTLE computation. We recall here that the first peak in the cardiac cycle, corresponding to the dilation of the ventricle, is called E-wave, while the second peak, called A-wave, is due to the contraction of the left atrium.

In all of these plots, high FTLE regions identify both LCSs as well as the ventricle boundary, since ventricle walls act as transport barriers for the flow.

The backward FTLE field plotted in Figure 3a shows the leading edge of the jet originating from the mitral valve during the E-wave and the rolling of the vortex ring, which is initially symmetrical. However, as it propagates through the ventricle, the jet posterior lobe tends to be smaller and slower than the anterior one due to the interaction with the ventricle wall (Figure 3b).

The spiralling pattern of the posterior lobe rapidly grows (Figure 3c) and turns into a well defined and delimited clock-wise rotating structure occupying a large portion of the ventricle (Figure 3d). The upstream front of this vortical region reaches the aortic orifice at the end of the diastasis, just before the A-wave (Figure 3e).

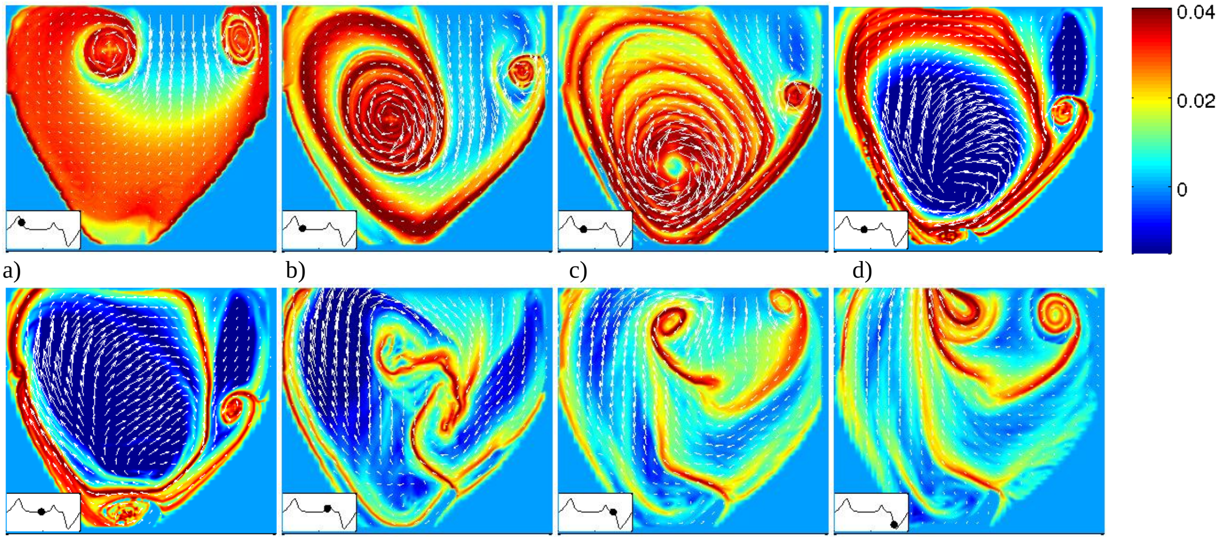

Figure 3. Backward 2D FTLE fields computed on the a-a plane (X-Z View). Initial times for FTLE computation are (a) t/T = 0.20, (b) t/T = 0.27, (c) t/T = 0.33; (d) t/T = 0.40; (e) t/T = 0.47; (f) t/T = 0.60; (g) t/T = 0.73; (h) t/T = 0.80. The velocity fields (white arrows) corresponding to the initial time are superimposed. Colormap units are 1/s.

At the beginning of the A-wave, the backward FTLE field in the a-a plane (Figure 3f) clearly shows the formation of the left side of the new vortex ring generated during the second ejection fed by the clock-wise rotation of the earlier one (Figure 3g). The simultaneous formation of this structure with the approach of the previous one to the aortic orifice helps fluid ejection during the systole (Pedrizzetti and Domenichini, 2005). During the last phase of the cardiac cycle, while the A-wave vortex is still developing in the LV, systolic ejection deforms and attracts its left bigger lobe towards the aortic orifice (Figure 3h).

Considering the forward FTLE fields at the same times (Figures 4a, 4b, 4c, 4d), one can identify the downstream front, i.e. the trailing edge of the entering jet, originated from the E-wave, its asymmetric evolution, and the redirection of the jet at the point of impingement on the posterior wall. Figure 4g shows the downstream front of the A-wave vortex ring, and its asymmetry enhanced from the dissolving E-wave structure feeding its left lobe.

That the joint analysis of backward and forward FTLE is a powerful tool for the localization of regions enclosing eddies (Shadden et al, 2005). From the conjunct analysis of backward and forward FTLE fields (Figure 3 and 4), it is evident how the superposition of attracting and repelling LCS clearly identify the elliptical region enclosing the vortex ring, and allows to following its deformation and evolution during the cardiac cycle (Espa et al, 2012).

3D FTLE

When analysing 3D FTLE, it has to be considered that, since the 2D velocity measurements could not be taken on planes almost tangent to the ventricle wall, 3D data were reconstructed on a grid which does not contain the whole ventricle. Hence, structures highlighted from 3D FTLE are partially cut next to the ventricle boundaries.

We will here focus on backward FTLE fields since they are easier to interpret. 3D backward fields are visualized in Figure 5, using FTLE isosurfaces, for some characteristic instants of the cardiac cycle, among those already examined in Figures 3 and 4. In Figure 6, the same fields of Figure 5 are sectioned by vertical planes in order to provide a clearer 3D picture The comments made for the 2D FTLE are still valid and we see how the vortex originated from the E-wave (Figure 5a-6a) soon becomes asymmetric (Figure 5b-6b) and is redirected towards the aortic orifice after the impingement on the ventricle wall (Figure 5c-6c). Unlike the 2D fields, the 3D analysis give a full picture of the asymmetrical LCS development: its horizontal section, whose shape initially follows the mitral orifice one (Figure 5a-6a), progressively stretches (Figure 5b-6b); the LCS moves following a diagonal propagation direction; its ridges create a virtual channel connected to the mitral orifice, that drives blood flow (Figure 5b-6b) and the structure folds onto the ventricle wall creating the recirculation path that facilitates subsequent ejection (Figure 5c -6c).

Similar conclusions emerge from the analysis of forward FTLE fields, here not reported.

Although important information can be inferred from 2D FTLE, it is apparent how the comprehension of the complex asymmetric vortical structure highly benefits from the availability of 3D velocity fields.

CONCLUSIONS

The role of vortical structure formation and development in the left ventricle has been acknowledged as an element of basic importance for the overall functionality of the heart, being responsible for efficient mixing and blood recirculation. Recent clinical studies based on Cardiac Magnetic Resonance (Charonko et al, 2013; Toger et al. 2012) have shown how FTLE can help in identifying LCS evolving in the left ventricle.

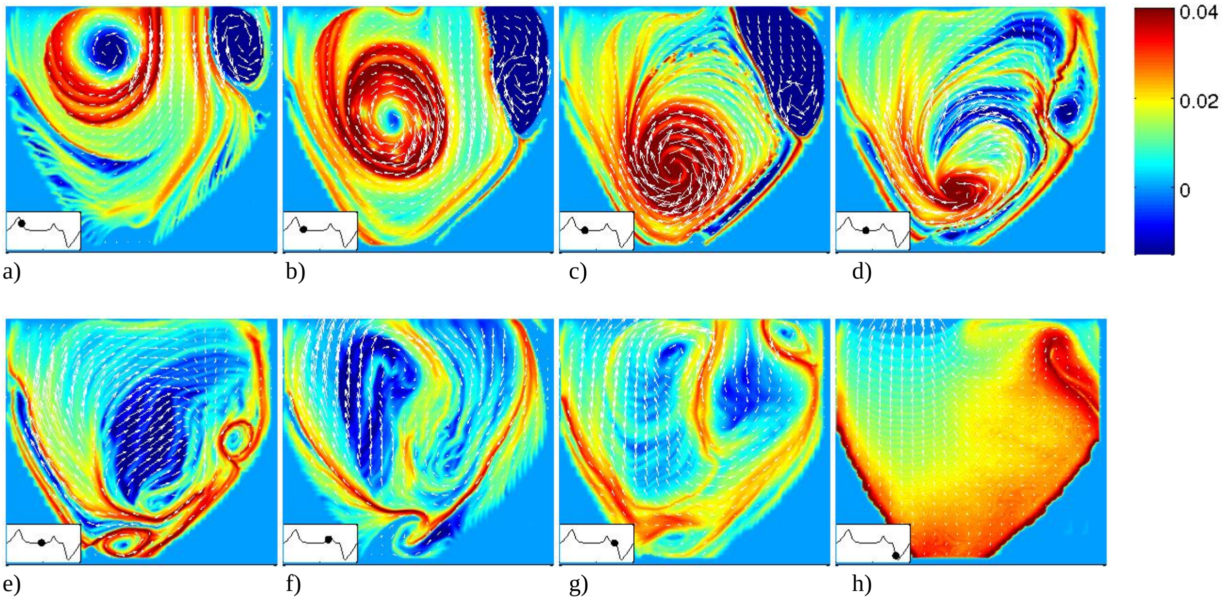

Figure 4. Forward 2D FTLE fields computed on the a-a plane (X-Z View). Initial times for FTLE computation are (a) t/T = 0.20, (b) t/T = 0.27, (c) t/T = 0.33; (d) t/T = 0.40; (e) t/T = 0.47; (f) t/T = 0.60; (g) t/T = 0.73; (h) t/T = 0.80. The velocity fields (white arrows) corresponding to the initial time are superimposed. Colormap units are 1/s.

Although these *in-vivo* results are very promising, they cannot be performed in controlled and repeatable conditions, thus *in-vitro* simulations represent a privileged method to investigate left ventricle complex LCS dynamics. In this perspective, the left ventricular flow was experimentally investigated in a laboratory model, where three dimensional velocity fields were reconstructed from two dimensional image analysis measurements on two sets of orthogonal planes. The analysis showed how FTLE fields (particularly 3D fields) help to elucidate the complex dynamics associated with the intraventricular flow, and to assess how the physiological flow is optimized for efficient ejection. Future work will be devoted to determine, in both physiological and pathological conditions, characteristic and synthetic LCS parameters that might provide a term of reference for future clinical investigations.

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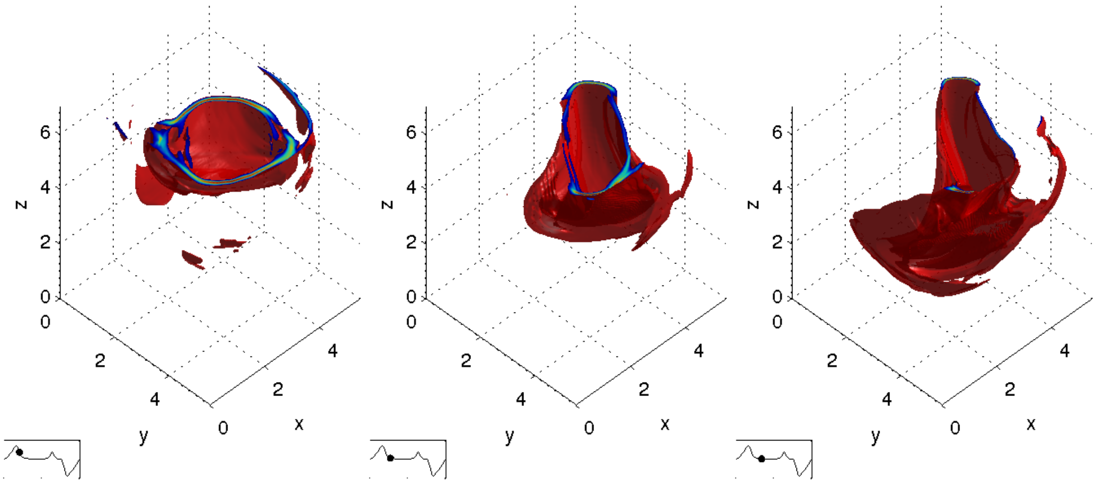

Figure 5. Backward 3D FTLE isosurfaces. Initial times for FTLE computation are (a) t/T = 0.20, (b) t/T = 0.27, (c) t/T = 0.33.

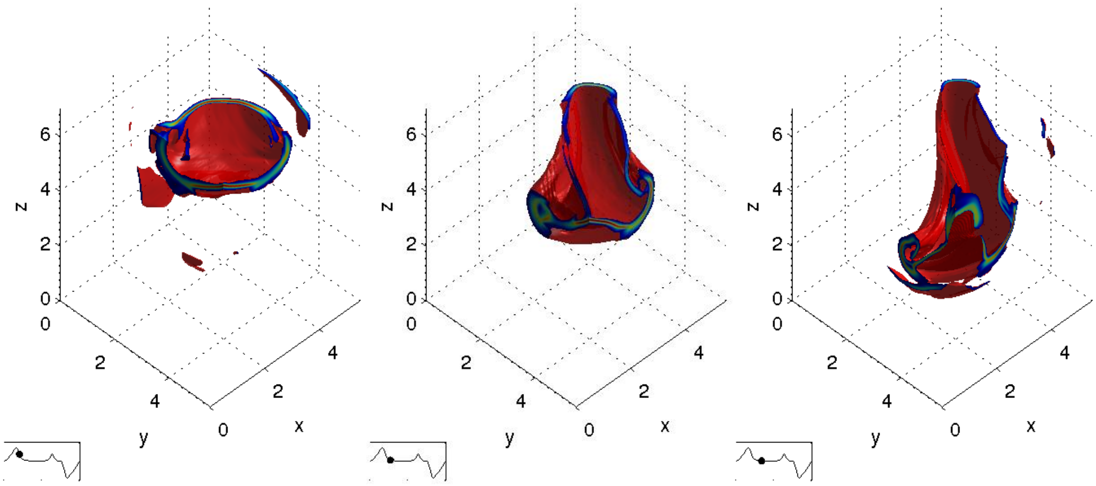

Figure 5. Backward 3D FTLE isosurfaces cut by vertical planes. Initial times for FTLE computation are (a) t/T = 0.20, (b) t/T = 0.27, (c) t/T = 0.33.