A "FREE" HIGH-REYNOLDS-NUMBER TURBULENCE: CONSISTENCY OF MEASUREMENTS WITH THE KOLMOGOROV-OBOUKHOV PREDICTIONS AND LES/SSAM APPROACH

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ABSTRACT

This work is motivated by recent experimental observations of intermittent dynamics of Lagrangian acceleration in a high-Reynolds-number "free" turbulence. First, we have shown that those observations are consistent with the Kolmogorov-Oboukhov theory. Second, in lines with Kolmogorov-Oboukhov's predictions, we proposed a new sub-grid scale (SGS) model of residual acceleration, which was introduced in the framework of an approach here referred to as LES-SSAM (Subgrid Stochastic Acceleration Model). The coarse-grid computation of a high-Reynolds-number stationary homogeneous turbulence provided: (i) non-Gaussianity in the acceleration distribution with stretched tails; (ii) rapid decorrelation of acceleration vector components; (iii) "long memory" in correlation of its norm.

Introduction

The experimental studies of Lagrangian statistics in turbulence at a high-Reynolds-number [1, 2] showed the strong intermittency in the fluid-particle (Lagrangian) acceleration. It was reported that Lagrangian acceleration has distribution with stretched tails, depending on the Reynolds number. The intermittency was manifested by long range correlation in time of the magnitude of acceleration, much longer than its direction. The latter was correlated on the Kolmogorov's timescale. Hereafter we shall present the new numerical approach targeting on prediction of those effects.

Filtering of the Navier Stokes equation leads to two unclosed sets of equations. The first one governs the filtered (resolved) velocity field:

$$\overline{a}_i \equiv \overline{\left(\frac{d\,u_i}{d\,t}\right)} = -\frac{1}{\rho}\frac{\partial \overline{P}}{\partial x_i} + \nu \frac{\partial^2 \overline{u}_i}{\partial x_k \partial x_k} \qquad (1)$$

$$\frac{\partial \overline{u}_k}{\partial x_k} = 0$$

where $\overline{a}_i$ is the filtered total acceleration:

$$\overline{a}_i = \frac{\partial \overline{u}_i}{\partial t} + \frac{\partial \overline{u_k u_i}}{\partial x_k} = \frac{d_\Delta \overline{u}_i}{d\,t} + \frac{\partial \tau_{ik}}{\partial x_k} \qquad (2)$$

$$\frac{d_\Delta}{d\,t} = \frac{\partial}{\partial t} + \overline{u}_k \frac{\partial}{\partial x_k}$$

The second set of equations governs the subgrid-scale (SGS) motion:

$$a_i' \equiv \left(\frac{d\,u_i}{d\,t}\right)' = -\frac{1}{\rho}\frac{\partial P'}{\partial x_i} + \nu \frac{\partial^2 u_i'}{\partial x_k \partial x_k} \qquad (3)$$

$$\frac{\partial u_k'}{\partial x_k} = 0$$

where a_i' is the residual total acceleration (on non-resolved scales):

$$a_i' = \frac{\partial u_i'}{\partial t} + \frac{\partial \left(u_k u_i - \overline{u_k u_i}\right)}{\partial x_k} \qquad (4)$$

Both filtered $\overline{a}_i$ and residual a_i' accelerations are not closed. The classical LES approaches are based on the first set of equations, (1)-(2), with closure models for the residual-stress tensor $\tau_{ik} = \overline{u_i u_k} - \overline{u}_i \overline{u}_k$. Essential is that such models are customary invariant on the Reynolds number; they disregard the intermittency effects on subgrid-scales. In our paper, we propose to address the main issue to the second set of equations (3) - (4), i.e. to the residual total acceleration a_i'. To this end, we introduce a new approach, referred to as LES-SSAM, which gives an approximation to the instantaneous non-filtered velocity field by simulation of both terms in $a_i = \overline{\left(\frac{d\,u_i}{d\,t}\right)} + \left(\frac{d\,u_i}{d\,t}\right)^{/}$. Discussion on this approach can be found in [3].

Model-equation in the LES-SSAM approach

There are three assumptions in LES-SSAM approach. First, we replace the exact non-closed equation (3) by the following expression:

$$\left(a_i'\right)_{mod} = \left(\left(\frac{d\,u_i}{d\,t}\right)^{/}\right)_{mod} = -\frac{1}{\rho}\frac{\partial\,p^*}{\partial\,x_i} + a_i^* \tag{5}$$

where a_i^* is the stochastic term, which emulates the non-resolved total acceleration. Since, in general, the modelled acceleration a_i^* is not solenoidal, the "pseudo pressure" p^* is introduced in (5) in order to maintain the incompressibility of the velocity field. Index mod denotes modelling. The second assumption is to use the simple eddy-viscosity closure in the exact non-closed equation (2) for filtered total acceleration $\overline{a}_i$:

$$\left(\overline{a}_i\right)_{mod} = \left(\overline{\left(\frac{d\,u_i}{d\,t}\right)}\right)_{mod} = \frac{d_\Delta \overline{u}_i}{d\,t} - \frac{\partial\left(2\nu_t \overline{S}_{ik}\right)}{\partial x_k} = -\frac{1}{\rho}\frac{\partial\,\overline{P}}{\partial\,x_i} + \frac{\partial\left(2\nu \overline{S}_{ik}\right)}{\partial x_k} \tag{6}$$

where the eddy-viscosity ν_t is given by the Smagorinsky model: $\nu_t = \left(C_S\,\Delta\right)^2 \overline{S}$, and $\overline{S} = \left(2\overline{S}_{ij}\overline{S}_{ij}\right)^{1/2}$ is a characteristic filtered strain rate. Summing (5) and (6) gives the following expression

$$\frac{d_\Delta \overline{u}_i}{d\,t} + \left(a_i'\right)_{mod} = -\frac{1}{\rho}\frac{\partial\left(\overline{P} + p^*\right)}{\partial\,x_i} + \frac{\partial\left(2(\nu_t + \nu)\overline{S}_{ik}\right)}{\partial x_k} + a_i^* \tag{7}$$

Our third assumption concerns (7) and consists of two points. First, we consider the sum $\left(\frac{d_\Delta\,\overline{u}_i}{d\,t} + \left(a_i'\right)_{mod}\right)$ as the total acceleration $\hat{a}_i$ of a surrogate velocity field $\hat{u}_i$ (denoted hereafter by the carat ^). Second, we assume that $\overline{S}_{ik} \approx \hat{S}_{ik}$. Then, for such a surrogate incompressible velocity field, the third assumption leads to the following SNSE and continuity equation:

$$\hat{a}_i = \frac{d\,\hat{u}_i}{d\,t} = \frac{\partial\,\hat{u}_i}{\partial\,t} + \hat{u}_k\frac{\partial\,\hat{u}_i}{\partial\,x_k} = -\frac{1}{\rho}\frac{\partial\,\hat{P}}{\partial\,x_i} + \frac{\partial}{\partial\,x_k}\left[\left(\nu + \nu_t\right)\left(\frac{\partial\,\hat{u}_i}{\partial\,x_k} + \frac{\partial\,\hat{u}_k}{\partial\,x_i}\right)\right] + a_i^* \tag{8}$$

$$\frac{\partial \hat{u}_k}{\partial x_k} = 0 \tag{9}$$

The pressure $\hat{P}$ maintains the incompressibility of such a surrogate velocity field.

Consistency of experimental observations of intermittency with predictions of the Kolmogorov-Oboukhov's theory

As in [4], and according to the mentioned above experimental results in [1, 2], let us represent the Lagrangian acceleration vector

$$\mathbf{A}(t) = \frac{d\ \mathbf{v}(t)}{d\,t} = \frac{d\,\Delta\,\mathbf{v}(t)}{d\,t}$$

(here $\mathbf{v}(t)$ is the Lagrangian velocity of fluid particle) in the form of a product of two independent stochastic processes:

$$\mathrm{A_i}(t) = \mathrm{A}(t)e_i(t) \tag{10}$$

The first one is for modulus $\mathrm{A}(t)$, with the timescale assumed to be comparable to T_L. The second stochastic process is for unit vector of orientation $\mathbf{e}(t)$; the components of this vector $e_i(t)$ are assumed to be correlated on the Kolmogorov's timescale τ_η, and $e_k e_k = 1$. One can show that such assumptions in (10) are consistent with the Kolmogorov-Oboukhov's prediction [5]. Indeed, considering for simplicity the motion of particle in one direction, say x_1, we have with (10):

$$\frac{d\left\langle(\Delta\mathrm{v}_1)^2\right\rangle}{d\,t} = 2\int_{t_0}^{t}\left\langle\mathrm{A}(t)\mathrm{A}(\tau)e_1(t)e_1(\tau)\right\rangle d\tau \tag{11}$$

According to assumptions in (10), the acceleration modulus does not practically change in the inertial range $\tau_\eta << t - t_0 << T_L$; thereby correlation in (11) may be split:

$$\frac{d\left\langle(\Delta\mathrm{v}_1)^2\right\rangle}{d\,t} = 2\left\langle\mathrm{A}^2(t)\right\rangle\int_{t_0}^{t}\left\langle e_1(t)e_1(\tau)\right\rangle d\tau \tag{12}$$

Since

$$\int_{t_0}^{t}\left\langle e_1(t)e_1(\tau)\right\rangle d\tau \approx \tau_\eta,\ \tau_\eta << t - t_0 << T_L, \tag{13}$$

and using $\left\langle A^2(t)\right\rangle \approx \langle\varepsilon\rangle^{3/2}\nu^{-1/2}$; $\tau_\eta = \sqrt{\nu/\langle\varepsilon\rangle}$, it is seen that (12) gives the Oboukhov's diffusion equation [5]:

$$\frac{d\left\langle(\Delta v_1)^2\right\rangle}{dt} \sim 2\langle\varepsilon\rangle$$

Stochastic model of non-resolved total acceleration

The subgrid total acceleration a_i^* in (8) may be represented by two independent stochastic processes, one for its norm $a^*(t)$, another for components of its direction $e_i(t)$:

$$a_i^* = a^*(t)\, e_i(t) \qquad (14)$$

with the Reynolds number, as parameter in both stochastic processes; dependence of $a^*(t)$ and $e_i(t)$ on the spatial point $\mathbf{x}$ is omitted for the notation simplicity. The stochastic process for $a^*(t)$ is presumed in accordance with the Kolmogorov-Oboukhov's 62 theory [4] in which the acceleration, conditionally averaged on the kinetic energy dissipation rate ε, is stated as: $\left\langle a_i a_j \middle| \varepsilon\right\rangle \sim \varepsilon^{3/2}/\nu^{1/2}\delta_{ij}$, where $\left\langle \cdot \middle| \varepsilon\right\rangle$ denotes the conditional mean. Using the Oboukhov's log-normality conjecture on the stochastic field of ε [4], and applying the Ito transformation to the Pope & Chen's stochastic equation for ε [5], one can obtain the following stochastic equation for the norm of acceleration:

$$da^* = -a^*\left(\ln\frac{a^*}{a_\eta} - \frac{3}{16}\sigma_\chi^2\right)\frac{dt}{T_\chi} + \frac{3}{4}a^*\sqrt{\frac{2\sigma_\chi^2}{T_\chi}}\, dW(t) \qquad (15)$$

where $dW(t)$ is the increment of standard Brownian process, i.e. $\langle dW\rangle = 0$, $\left\langle (dW)^2\right\rangle = dt$, and $a_\eta = \frac{\langle\varepsilon\rangle^{3/4}}{\nu^{1/4}}$ is the Kolmogorov's acceleration. The parameters of this equation are introduced in the following form:

$$T_\chi = \frac{\nu_t}{\Delta^2};\quad \sigma_\chi^2 = A + \mu \ln \mathrm{Re}_\Delta^{3/4};\quad \mathrm{Re}_\Delta = \frac{\nu_t}{\nu} \qquad (16)$$

where according to DNS in [6]: $A \approx -0.863$; $\mu \approx 0.25$. It should be noted that the stochastic process for the acceleration norm $a^*(t)$ is correlated in space, since the coefficients in (16) depend on the Smagorinsky eddy-viscosity ν_t calculated by LES velocity field.

As to the stochastic model for a unit vector of the acceleration direction, the main emphasis is on its correlation on the Kolmogorov's timescale. Using the Kolmogorov's assumption of local isotropy in a high Reynolds number turbulent flow, the orientation of residual acceleration at each spatial point can be emulated by Brownian random walk on the surface of a unit radius sphere, where the diffusion coefficient is inversely proportional to the Kolmogorov's time scale. In the Cartesian coordinate system, the Langevin equation for direction components $e_i(t)$ can be derived in this case in the following form:

$$de_i = -2\,\tau_\eta^{-1} e_i\, dt + \left(\delta_{ij} - e_i e_j\right)\sqrt{2\tau_\eta^{-1}}\, dW_{\mathbf{x}j} \qquad (17)$$

$$\left\langle dW_{\mathbf{x}i}\, dW_{\mathbf{x}'j}\right\rangle = \delta_{ij} F\left(\frac{|\mathbf{x}-\mathbf{x}'|}{\Delta}\right)dt;\ F(0)=1 \qquad (18)$$

where $W_{\mathbf{x}j}$ represent independent components of Brownian vector process $\mathbf{W}_\mathbf{x}$ at spatial point $\mathbf{x}$. Function $F(\xi)$ is the spatial correlation of Brownian process $\mathbf{W}_\mathbf{x}$. It follows from (17) that norm of the vector $e_i(t)$ is conserved: $e_i e_i = 1$, and the temporal correlation of e_i is an exponential function:

$$\left\langle e_i(t) e_j(t_0)\right\rangle = \delta_{ij}\, e^{-\frac{2|t-t_0|}{\tau_\eta}} .$$

The numerical solution of (17) requires the time step much less that the Kolmogorov's timescale, $\Delta t << \tau_\eta$, which is inconsistent with LES. Therefore the procedure of simulation is simplified: on each time-step of order τ_η, the direction is generated randomly, providing its correlation on the Kolmogorov's timescale.

The set of equations (8), (9), (14) - (18) formulates LES-SSAM approach. The stochastic process for the residual total acceleration, described by (14) - (18), is correlated in time and not in space. This approach was applied here to the 3D stationary box turbulence, with parameters taken from [1, 2]. The code from [7] was used, in which the forcing procedure from [8] was incorporated.

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Examples of coarse-grid (32^3) computation of isotropic 3D box turbulence

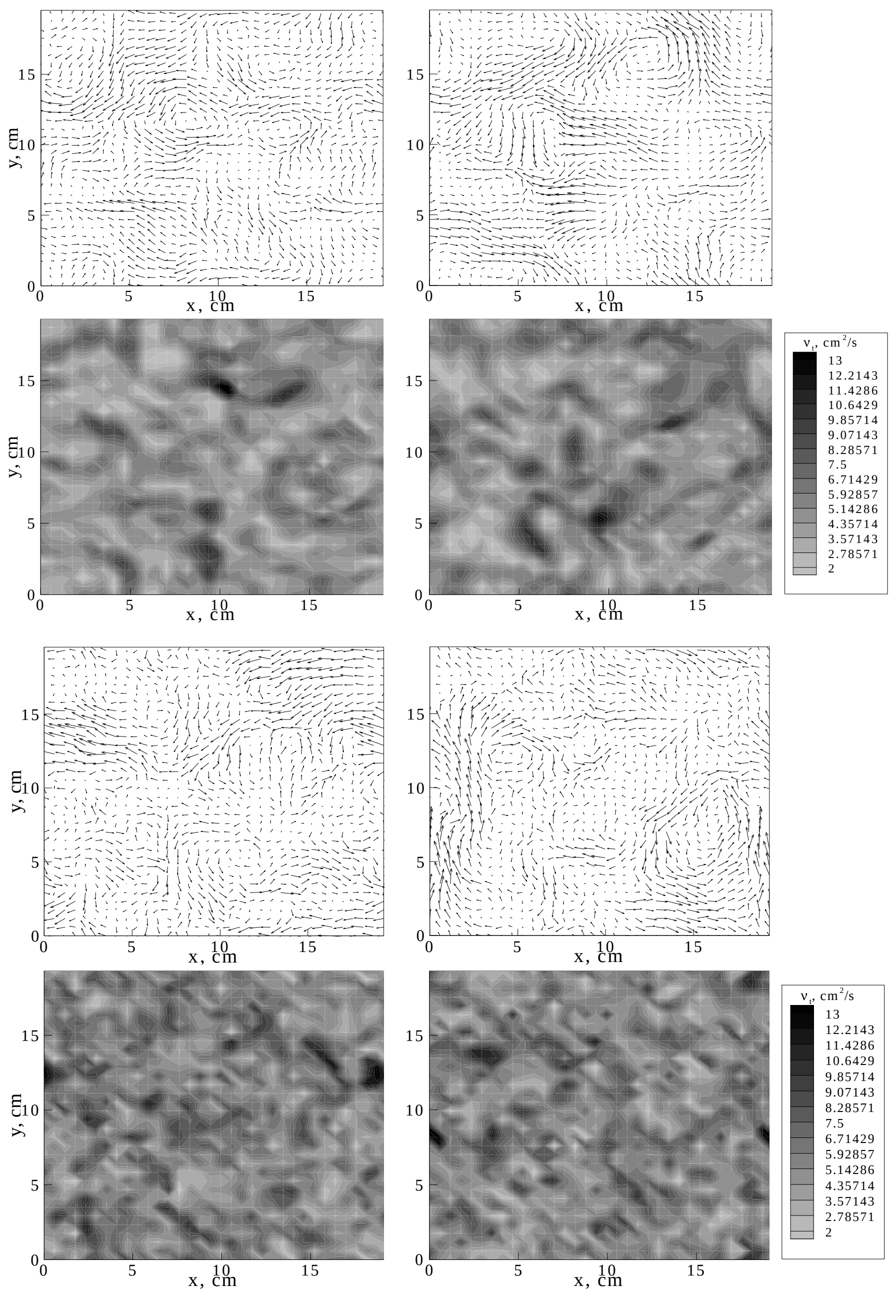

Fig.1 Velocity field and sub-grid viscosity distribution on center-plane in stationary isotropic 3D box turbulence at two times, $t\langle\varepsilon\rangle/\langle\sigma_u^2\rangle = 16.8; 35.4$; upper part: standard LES with the Smagorinsky closure; bottom part: LES-SSAM.

Although LES-SSAM simulates intermittency in time and not in space, it is seen that LES-SSAM model produces more intermittent spatial structures at small-scales than standard LES with the Smagorinsky closure

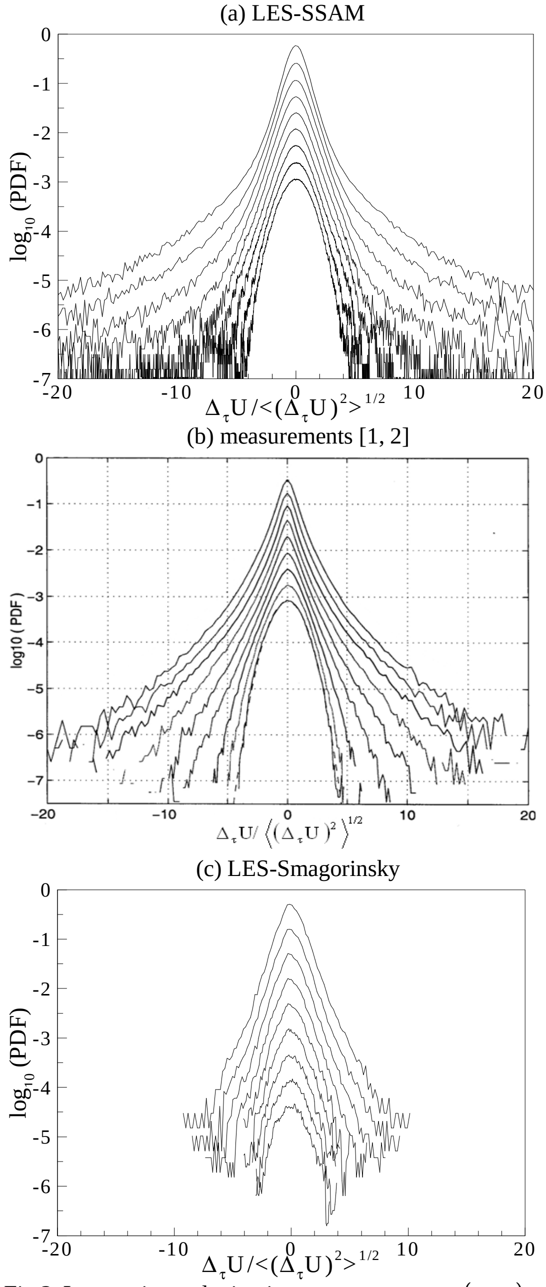

Fig.2 Lagrangian velocity increment $\Delta_\tau V_i = V_i(t+\tau) - V_i(t)$ probability density function at different time lag $\tau = 0; 0.15;\ 0.3;\ 0.6; 1.2;\ 2.5; 4.9;\ 9.8;\ 20$ et 39 ms ; on the top: LES-SSAM; in the middle: measurements in [1,2]; on the bottom: standard LES with the Smagorinsky closure. At time lag of order of integral time, the velocity increment is normally distributed. However, at smaller time lags, closely to measurements, the velocity increment probability density function displays a growing central peak with stretched tails.

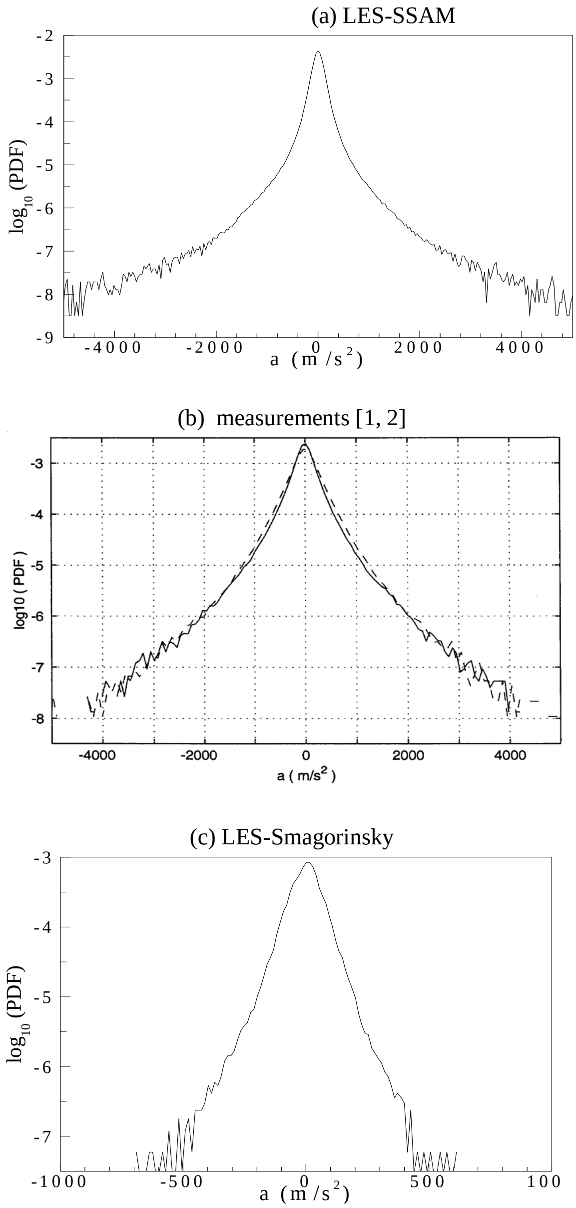

Fig.3 The Lagrangian acceleration probability density function; on the top: LES-SSAM; in the middle: measurements in [1,2]; on the bottom: standard LES with the Smagorinsky closure. The acceleration of fluid particle may attain very large values.

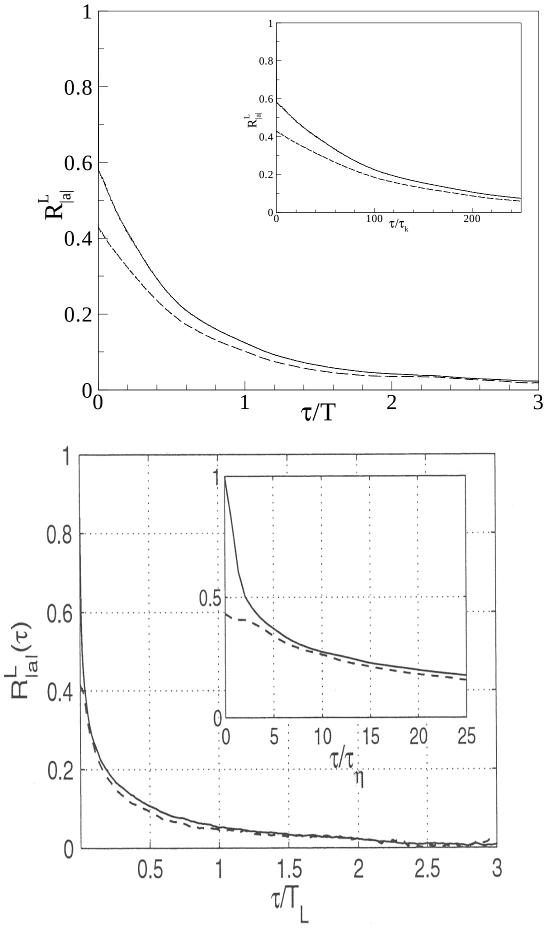

Fig.5: Autocorrelation of the norm of acceleration $R_{|a|}(\tau)$; on the top part: LES-SSAM, on the bottom part: measurements in [1,2]. It is seen that although the computed correlation are somewhat longer than in measurements, the tendency is similar: $R^L_{|a|}(\tau)$ persists during a significant time.

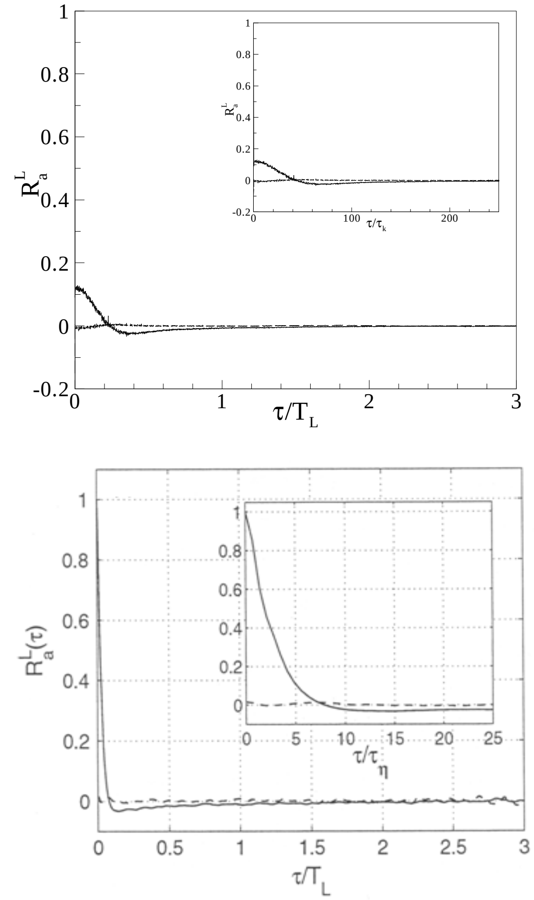

Fig.6 Autocorrelation of acceleration $R_{a_i}(\tau)$; on the top part: LES-SSAM, on the bottom part: measurements in [1,2]. It is seen that autocorrelation of acceleration decreases rapidly with progressing of time-lag τ