SCALE DEPENDENT NATURAL COORDINATES FOR ANISOTROPIC ATMOSPHERIC TURBULENT VARIANCES

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ABSTRACT

Current theories of atmospheric turbulence focus on isotropic turbulence, which applies only to the smaller spatial scale/higher frequency temporal scale motions. The larger spatial scale/lower frequency eddies carry more energy and are more active in the transport of scalars and momentum. These larger eddies are anisotropic – the variances at these scales are not the same in all directions.

Although it is natural to use streamwise coordinates or other surface-based coordinate systems in models and field measurements, the turbulent variances and covariances that comprise the Reynolds stress tensor have their own natural coordinate system, which is not usually aligned with commonly-used coordinate systems. This means that the direction of maximum variance, and therefore maximum turbulent transport, is not usually the stream-wise direction, nor is the direction of minimum variance and turbulent transport exactly vertical. Dispersion models often base the degree of spreading on estimates of turbulent variance, so improved knowledge of the variances and covariances will improve the performance of such models.

It is often found in laboratory flows that the Reynolds stress coordinates are rotated 17° around the cross-stream axis with respect to a streamwise, cross-stream, wall normal coordinate system (Hanjalic and Launder, 1972). In the case studied here, the Reynolds stress coordinate angles differ for different scales of motion, but are similar to the 17° value.

DATA

This paper looks at the degree and nature of the anisotropy at a wide range of spatial scales and at elevations of 1.5 m, 5.0 m, 30 m, and 50 m from the surface at a relatively flat and open location near Leon, Kansas, using sonic anemometer data from the Cooperative Atmosphere–Surface Exchange Study, CASES99, a major field campaign which took place in October 1999 (Poulos et al, 2002). October 17, a relatively windy night (9 ms^{-1} at 10m above ground level (agl)), was chosen for near neutral thermal stratification.

Only data from the Campbell CSAT3 sonic anemometers at 1.5 m, 5.0 m, 30 m, and 50 m agl are being used here because the three transducer pair paths coincide within roughly the same $(10\ cm)^3$ volume, while the model (ATI-K) used at the other elevations (10m, 20m, 40m, 55m) has a separation between transducer pairs of 25-40 cm although the separation path between transducers is 10 cm. This separation of sonic paths makes calculating covariances between the paths problematic for smaller multiresolution scales.

For reference with laboratory flows, the outer scaling for the four elevations are about 0.003, 0.01, 0.06, and 0.1. The corresponding inner scaling values are on the order of 10^5 – 10^6. The boundary layer Reynolds number is on the order of 10^8 for this time.

ANALYSIS TOOLS

This analysis will use two primary tools, one to express the variances and covarinaces as a sum of values, each representing the amount of the variance or covariance due to motion at discrete scales of 2^n data points where n ranges from 1 to 16 for this data. Anisotropy has two degrees of freedom and therefore requires two parameters to fully descibe it (Choi and Lumley, 2001). The barycentric plots of Banerjee et al. (2007) will be used to plot the two parameters on a plane.

Multiresolution Decomposition

Each of the six variances and covariances of the Reynolds stress tensor are broken down into the amount of the variance and covariance due to motion at different time scales using the simple dyadic multiscale decomposition used in Vickers and Mahrt (2003), mathematically equivalent to a Haar wavelet transform.

This method satisfies the requirements of Reynolds averaging allowing the total variance or covariance to be expressed as a sum of sub-values, each for a different scale of motion. Since the analysis is based on time series data, the scales are fundamentally temporal. At each elevation, this is converted to a spatial scale by multiplying by the mean wind at that elevation. For the smaller scales, it can be interpreted as an eddy scale, but for the larger scales, the conditions for Taylor's hypothesis do not hold, and therefore the larger scales do not correspond to a physical eddy dimension.

Anisotropy Barycentric Plots

Once each variance and covariance is decomposed into a sum of contributions of different scales of motion, the full Reynolds stress tensor can now be written as a sum of sub-tensors, one for each scale of motion (Klipp, 2010). Each sub-tensor, like the full tensor, is real and symmetric, therefore Hermitian, allowing one to interpret the eigenvalues as fundamental variances of the flow at each scale. The eigenvectors form an orthogonal coordinate system, which are the fundamental directions for the variances. The eigenvalues for the Reynolds stress tensor differ from the eigenvalues for the anisotropy tensor by the addition of 1/3 to each eigenvalue of the anisotropy tensor. The Reynolds stress tensor and the anisotropy tensor have the same eigen vectors (Banerjee, et al, 2007).

The eigenvalues are used to evaluate the degree and nature of the anisotropy using the method of Banerjee, et al (2007). In this method, the Reynolds stress tensor, non-dimensionalized by the trace, is decomposed into one dimensional, two dimensional and three dimensional components, with coefficients C_1, C_2, and C_3. Although there are three coefficients, they are constrained to sum to one, therefore there are only two independent values of the coefficients, sufficient to fully describe the anisotropy. The values of the three coefficients can be plotted in two dimensions through the use of a barycentric plot. The vertices of the triangular barycentric plots (Figure 1) represent motion that is either three dimensional (fully isotropic), two dimensional (one eigenvalue vanishes), or one dimensional (two eigenvalues vanish). The line between 3D and 2D represents pancake-like axisymmetry (two large identical eigenvalues, one small) and the line between 3D and 1D represents cigar-like axisymmetry (two small identical eigenvalues, one large). Purely isotropic turbulence will plot at the top vertex with $C_3 = 1$ and $C_1 = C_2 = 0$. Turbulence with two identical eigenvalues and one smaller eigenvalue will plot along the line from C_3 to C_2. Turbulence with two identical eigenvalues and one larger eigenvalue will plot along the line from C_3 to C_1. If the smallest eigenvalue vanishes then $C_3 = 0$, the turbulence is two dimensional, and plots along the bottom line. Since this barycentic plot is linear in the eigenvalues, it is more practical to use than the more elegant turbulence triangle of Lumley (Choi and Lumley, 2001).

Note that the variances and the eddies do not necessarily share the same axisymmetry (Choi and Lumley, 2001; Simonsen and Krogstad, 2005). Although it is possible to draw an inverse relationship between eddy axisymmetry and variance axisymmetry when anisotropy is produced in a wind tunnel, it is not clear if such a relationship exists in the atmosphere.

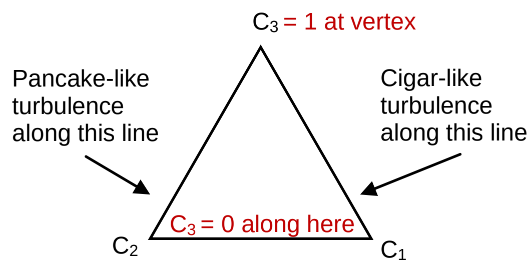

Figure 1: Diagram of the barycentric plot for turbulence parameters. See text for description.

RESULTS

Reynolds Stress Components

In Figure 2, the scales with maximum turbulent kinetic energy (TKE), TKE = (u'u' + v'v' + w'w') / 2, range from a broad peak centered about the 20 meter length scale at the 1.5m elevation (Fig. 2a) to a more defined peak about the 300 meter length scale at the 50 m elevation (Fig. 2d). Due to the nature of the multiresolution decomposition, these length scales are subject to a factor of two uncertainty. Also, the smallest scales are subject to aliasing effects, which seem to be more prominent at the lower elevations where there is a higher percentage of the total TKE in the smaller scales.

The multiresolution spectra at lower elevations (Figs. 2a and 2b) show very distinct spectral gaps at the 5000 and 6500 meter scale (both correspond to 13.6 minutes) where the energy at that length scale becomes quite small compared to the energy in the other scales. This gap scale delineates the smaller scale turbulence motion from the larger meso-scale motion (Stull, 1997; Vickers and Mahrt, 2003). The origins of the meso-scale motions are not well known, but may in part be due to boundary layer superstructures (Smits et al, 2011).

At the higher elevations, the gap is less well defined, although a scale of reduced energy is evident at 5000 – 9000 meter scales (6.8 – 13.6 minutes), the amount of energy in these gap scales is quite significant. One possibility is that since the scale of peak TKE energy is increasing with increased elevation, the turbulence scales are overlapping the meso-scales which are not changing scale as dramatically with elevation resulting in a range of scales with motion due to both turbulence and meso-scale motion.

Note that at a given elevation, the peak energy scale for the vertical component is much smaller than the scales for the transverse and streamwise components. The streamwise peak scale is also larger than the TKE peak scale. The values at the 50 m level are consistent with values reported by Krogstad and Kaspersen (2002) for a wind tunnel zero pressure gradient case. The peak TKE scale also coincides with the scale of maximum contribution to the u'w' covariance.

In the plots in Figure 2, the total variances and covariances are proportional to the area under the spectrum. The total area under the TKE line is comparable to the area under the u'u' line, but the length scales of their peak values are quite different. So although the total u'u' variance can be a good approximation for the total TKE, u'u' does not reproduce the scales of TKE.

Although the vertical flux of the transverse component, v'w', is nearly zero for all scales for this hour, the u'v' covariance is quite significant at larger scales, especially at the higher elevations. This term is usually assumed to be insignificant by argument of horizontal homogeneity. Although the terrain around the CASES99 main tower seemed to be homogeneous, there may be significant inhomogeneities affecting the covariances. If this is true, then it is most likely that all real-world locations have comparable levels of inhomogeneity.

Anisotropy

In the plots of Figure 3, for all four elevations, the smallest scales are nearly isotropic (C_3 = 1), the largest scales are nearly two dimensional (one eigenvalue nearly vanishes). The scales of peak TKE lie along the C_2 = 0.5 line, close to the 2D line (small C_3 values). The major difference between the elevations is in the scales at which these distinctions occur. At 1.5 m, the transition from $C_3 > 0.5$ to $C_3 < 0.5$ occurs at a scale of about 2.5 m, while the same transition at the 50 m elevation occurs at a length scale of about 80 meters. Due to the nature of the multiresolution decomposition, the largest scale analyzed is always exactly 1D and the next largest scales is always exactly 2D so both have been left off of the barycentric plots.

The largest values of C_3 at the lower elevations are smaller than the largest C_3 values at the higher elevations indicating that the surface has a larger influence on the isotropy at the scales resolvable by the sonic anemometers.

Eigen Vector Directions

For the laboratory flow case (Hanjalic and Launder, 1972) where the eigen coordinates are rotated 17° around the cross-stream axis with respect to a streamwise, cross-stream, wall normal coordinate system, the angles in Figure 4 would be 17° for the large eigenvector direction with respect to (WRT) streamwise (red lines), 17° for the small eigenvector direction WRT vertical (blue lines), and 0° for the middle eigenvector direction WRT the lateral cross-stream direction (green lines). The laboratory flow case is computed using the total variances and covariances, not the multiresolution decomposition values.

For the multiresolution decomposition sub-tensors, the angles between the eigenvectors and the associated nearest streamwise coordinate are not always well defined. In the case of near-isotropy, any small change in a measured value can produce significantly different directions chosen by the software package. In the case of near pancake axisymmetry, the direction of the smallest eigenvalue is well defined, but the two nearly identical larger values become indistinguishable. As the Reynolds stress matrix becomes less isotropic, the condition number increases, thus the calculations at the largest scales are more sensitive to small measurement errors. This makes only the mid-range of scales reliable for eigen-coordinate direction information.

At the 1.5m elevation, the mid-range angles are close to the ideal rotation about the lateral cross-stream axis, but the angles vary from 26° at the smaller scales to only 4° at the larger scales. The angle between the middle valued eigenvalue’s eigenvector and the cross-steam axis is small, but not zero, ranging from 4° to 1°. At 5m elevation, the angles for the largest and smallest eigen-directions are 23° at the smaller scales to 6° at the larger scales, and for the middle valued eigenvector, the angles are 2-8° but do not vary uniformly with scale.

At the 30m elevation, the number of near isotropic scales is increasing compared to the lower elevations, and at the other scales, the three angles are close to equal in value, meaning that the difference in direction between the streamwise coordinates and the eigen coordinates are no longer described by a simple rotation around one axis. The angles vary from about 30° at the 36 meter scale to about 10° at the 1.15 kilometer scale. The 50m elevation is similar to the 30m.

CONCLUSIONS

Turbulence anisotropy is a little studied area of fluid dynamics and the behavior of anisotropy at different scales is even less known. Anisotropy varies significantly from one scale to another as well as from one elevation above the ground to another. This behavior is complicated and has implications for dispersion at the very least.

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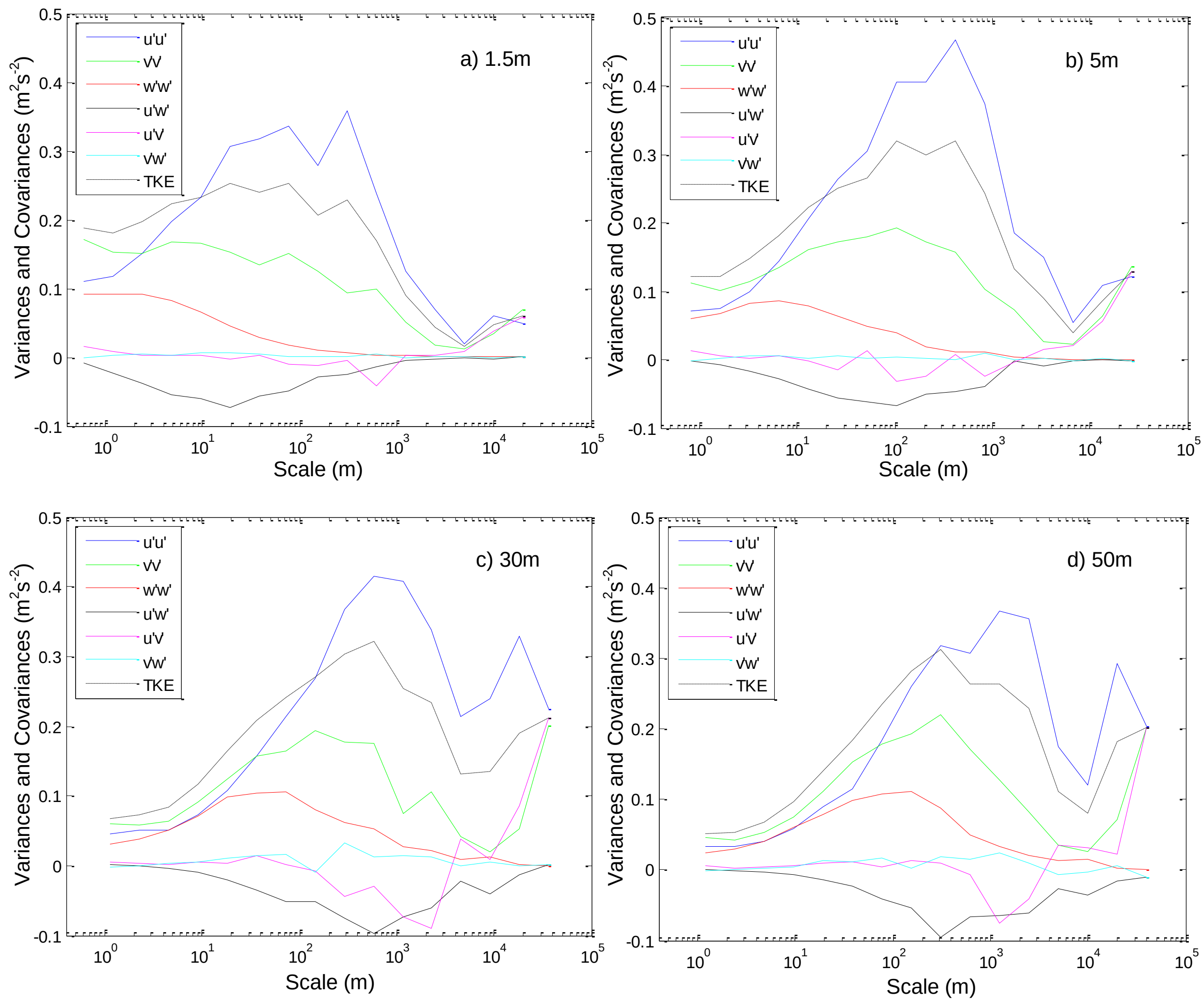

Figure 2: Multiresolution decompositions of the variances and covariances of the Reynolds stress tensor and TKE. a) 1.5m above ground level, b) 5m agl, c) 30m agl, d) 50m agl. The spectral gap at 1.5m and 5m is quite well defined, while at 30m and 50m, the gap scale still has a significant amount of energy compared to the other scales. Also note that the peak TKE scale is typically slightly smaller than the peak scale for the streamwise variance, u'u'. The peak scale for u'w' is about the same as the peak TKE scale. The covariance between the streamwise direction and lateral cross-stream direction (u'v') is not always negligible, especially at larger scales and farther from the surface.

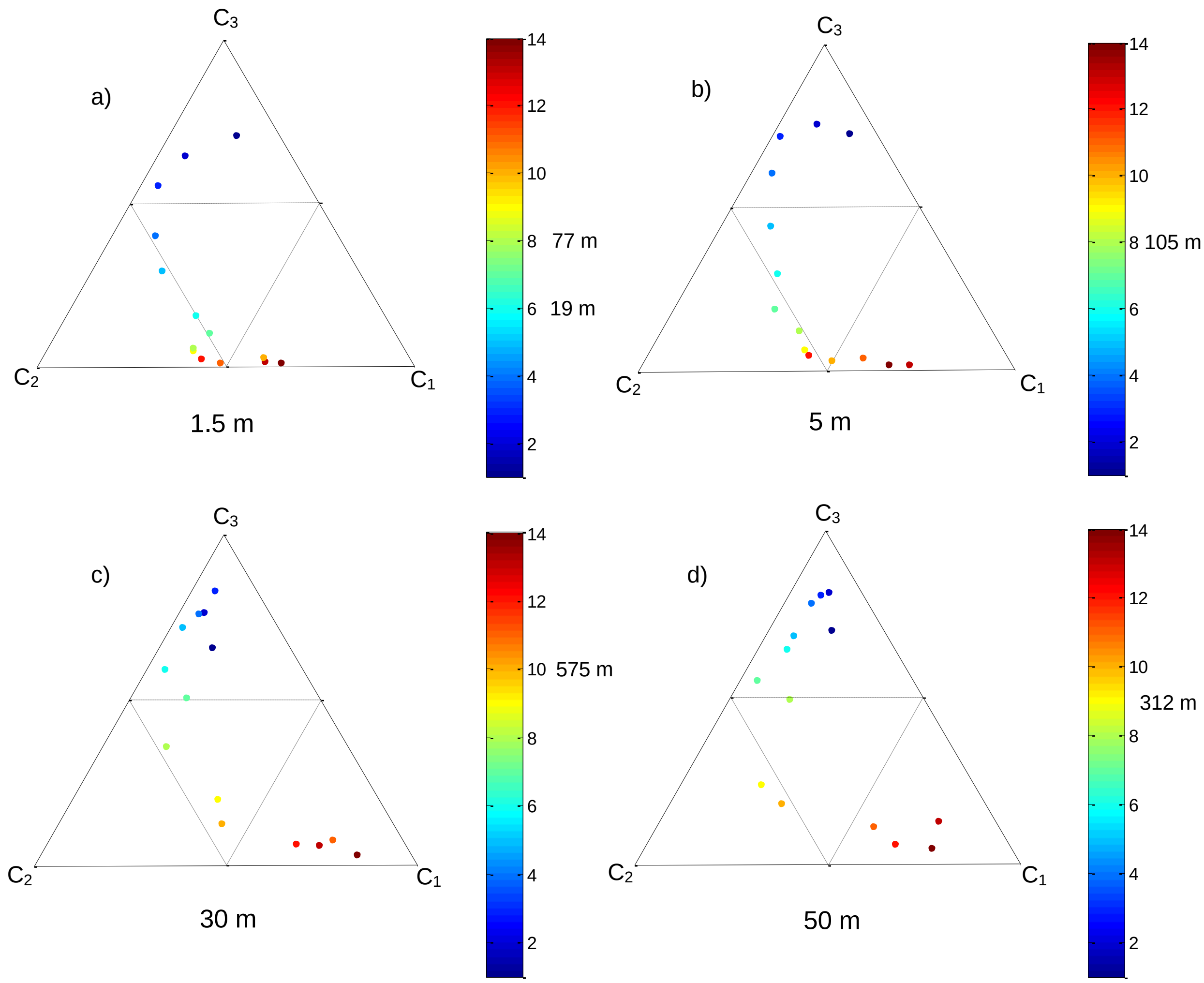

Figure 3: Barycentric turbulence plots of the multiresolution decompositions of the Reynolds stress tensor. The numbers on the color bars refer to n in the scale 2^n data points. The equivalent spatial scale is listed at each level for the scale of peak TKE (Fig. 2). a) 1.5m above ground level, b) 5m agl, c) 30m agl, d) 50m agl. The plots are consistent with the concept that the surface restricts motion perpendicular to the surface. The smallest scales at the lower elevations are less isotropic than the same scales farther from the surface. There is also a broader range of near isotropic scales (C_3>0.5) at higher elevation. In addition, the largest scales are much closer to two dimensional near the surface, another indication that vertical motion is more restricted closer to the surface. Also of note, refering to the plots in Figure 2, the maximum TKE scales tend to plot at about the same location near the C_2=0.5 line. This is not universal. Turbulence anisotropy behaves differently in the vicinity of two perpendicular surfaces such as in an urban street canyon.

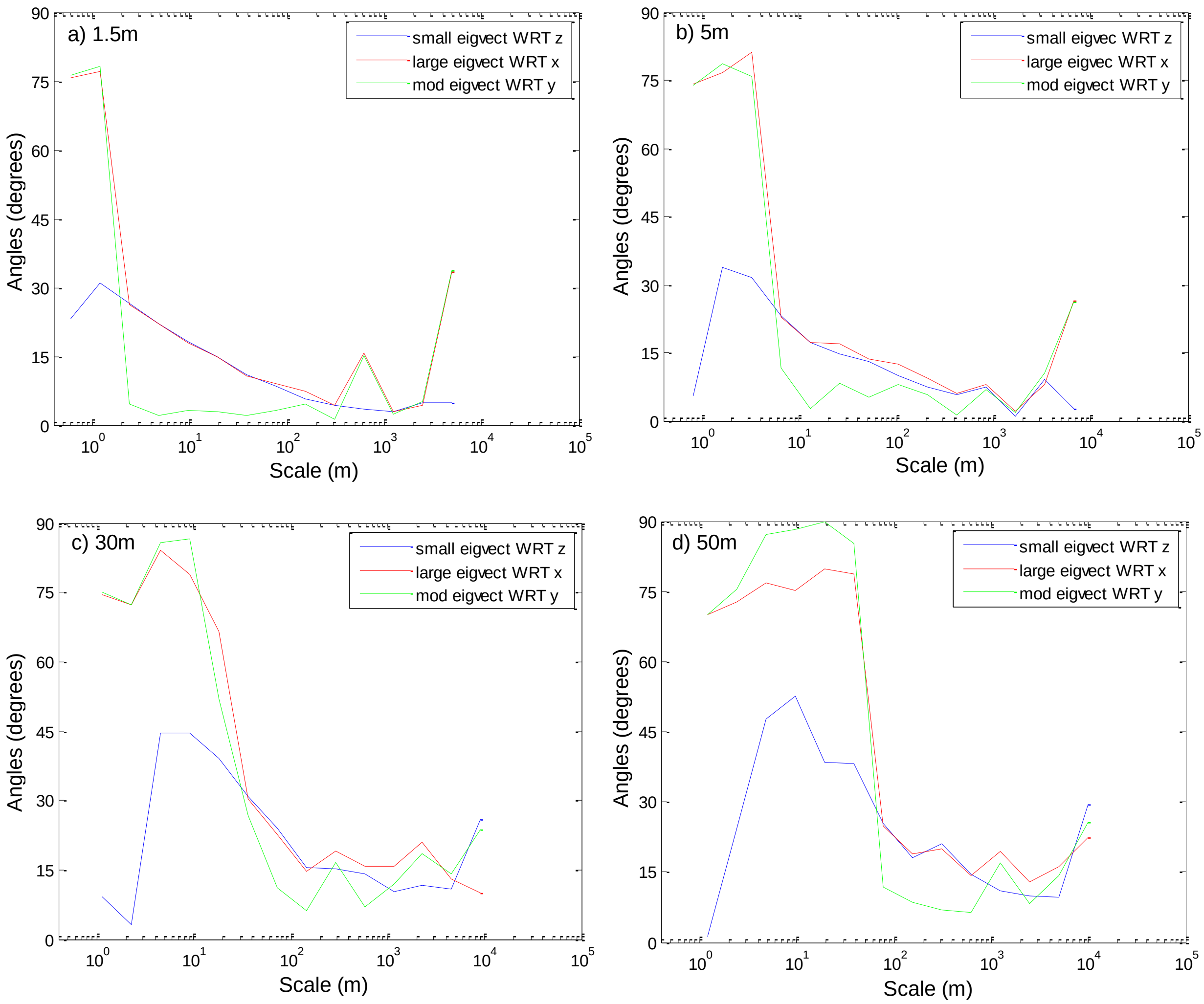

Figure 4: Angles between the eigenvector directions and the streamwise coordinate system axes. Blue line is the angle between vertical (wall normal) and the eigenvector associated with the smallest eigenvalue. Red line is the angle between the streamwise direction and the eigenvector associated with the largest eigenvalue. Green line is the angle between the lateral cross-steam direction and the middle-valued eigenvalue. Laboratory reported values for the full Reynolds stress tensor for the red and blue lines are 17°, and 0° for the green line. Multiresolution Reynolds stress tensor values are similar to this for the scales near peak TKE values. a) 1.5m above ground level, b) 5m agl, c) 30m agl, d) 50m agl.