RAPIDLY SHEARED HOMOGENEOUS MHD TURBULENCE IN A ROTATING FRAME

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ABSTRACT

Rapid distortion theory is applied to magneto-hydrodynamic turbulence that is sheared in a rotating frame. We describe analytically the modification of the three-dimensional velocity spectra due to the presence of an external magnetic field, using the quasi-static approximation. Using this analytical solution, we investigate the effect of the frame rotation in the evolution of one-point statistics, under the linear theory. For initial fields that are two-dimensional (but three-componential), with the axis of independence aligned with the flow direction, we derive analytically one-point statistics, such as the Reynolds stresses and the structure dimensionality tensor in physical space. The analytical results are compared with the linear three-dimensional exact numerical solution for initially isotropic homogeneous turbulence, and they show remarkable agreement. They describe accurately the tendencies in the morphology of the turbulent structures that develop as a result of the two competitive mechanisms of the frame rotation and the Joule dissipation. These results are in qualitative agreement with existing non-linear DNS results.

INTRODUCTION

The impact of a strong magnetic field on the turbulent flow of an electrically conductive fluid that is sheared in a rotating frame is encountered in diverse applications. Examples include liquid-metal cooling systems for fusion reactors, electromechanical brakes in continuous steel casting, solar wind turbulence and coronal heating, and the optimization process of semiconductor crystal growth. At a fundamental level it is well known that, when acting alone, mean shear and frame rotation or external magnetic fields modify the turbulence structure and induce strong anisotropy. However, the coupled effects of mean shear and rotation in the presence of magnetic fields have not been well studied so far. In the limit of low magnetic Reynolds numbers $R_m = uL/n << 1$, where u is the r.m.s. of the fluctuating velocity, L is the integral length scale of the flow and n is the magnetic diffusivity, the Lorentz force can be treated in a quasi-static (QS) approximation[1,2] and expressed as a linear function of the velocity fluctuations. This simplified picture of the interaction between the magnetic field and homogeneous turbulence highlights the key role played by the turbulence structure in magneto-hydrodynamic (MHD) flows. When acting alone, the magnetic field modifies the angular distribution of turbulent kinetic energy in spectral space, and hence, the anisotropy of the componentality and dimensionality of the turbulence (for a clarification of dimensionality and componentality see [3,4,5]). The Lorentz force preferentially counteracts velocity fluctuations perpendicular to the direction of the magnetic field, in the process causing a net dissipation of turbulent kinetic energy, called the Joule dissipation. The Joule dissipation is highly anisotropic, and affects more strongly those modes that have their wave numbers aligned with the magnetic field [6,7,8]. Overall the magnetic field tends to eliminate gradients in the direction of the magnetic lines and in the process lengthens turbulent eddies in that direction. Thus, it tends to produce two-dimensional (2D), but three component (3C) turbulence, where the velocity fluctuations depend only on the coordinates in the plane perpendicular to the magnetic field. This phenomenological explanation is also supported by numerical studies such as the Direct Numerical Simulations (DNS) of Zikanov and Thess[9], Kassinos et al.[5] and Rouson et al.[10].

On the other hand, in the purely hydrodynamic case in a non-rotating frame, it is well documented that homogeneous mean shear tends to elongate and align the turbulent structures in the direction of the mean flow [11,12]. DNS results of Bardina et al.[13], Salhi and Cambon[14], and Brethouwer[15] had clearly shown that rotation of the frame can act to either stabilize or destabilize turbulent shear flow, depending on the ratio of the frame rotation rate to the shear rate. Also, Salhi[16] and Akylas et al. [4], studied in detail the linear response of sheared turbulence to frame rotation using Rapid Distortion Theory (RDT), in the limit of strong shear and/or rotation rates, and helped to clarify global features of homogeneous shear flow in a rotating frame. The combined effects of mean shear, system rotation, and an externally imposed magnetic field on the structural morphology of homogeneous MHD shear flow have been examined in two recent DNS studies by Kassinos et al.[2,17]. Their basic results showed that, in general, one of the key parameters determining eddy alignment is the ratio of the time scale of the mean shear $\tau_S=S^{-1}$, to the Joule time, $\tau_M=nB^{-2}$, where B is the intensity of the constant magnetic field in Alfven units, n is the magnetic diffusivity and S is the mean shear. When $\tau_S<<\tau_M$, they found that the turbulence structures tend to align preferentially with the stream-wise direction irrespective of the magnetic Reynolds number, R_m. On the contrary, at the other limit when $\tau_S>>\tau_M$, and at low R_m, they reported that the turbulent eddies became elongated and aligned with the magnetic field. For $\tau_S\approx\tau_M$, the picture was not clear and competing mechanisms tended to produce different structural anisotropies. However, they reported that strong span-wise rotation, in combination with a span-wise magnetic field, tends to promote a stream-wise alignment of the turbulent structures, at least when $\tau_S\approx\tau_M$.

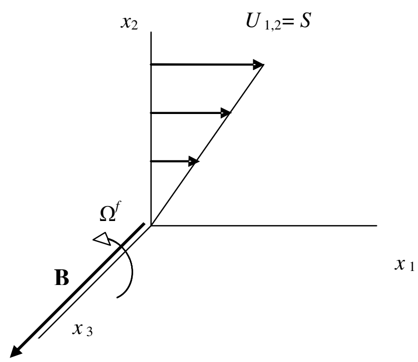

Figure 1. Illustration of the general case investigated here.

In this work, we present a simplified approach to MHD turbulence sheared in a rotating frame (Fig. 1), using the QS approximation coupled with RDT. This approach allows the analytical study of the coupled effects of the frame rotation and the magnetic field in the evolution of homogeneous shear turbulence. We restrict the study to inviscid RDT. Since nonlinear effects are absent from the RDT solutions (no energy cascade is present), any viscous effects are of secondary importance – even though simple to introduce as an integrating factor. The present analytical outcomes enhance our understanding on these competitive coupling effects and offer a simplified representation that can be used in incorporating the proper physics in structure-based models[18].

LINEAR EQUATIONS

We consider here homogeneous turbulence that is sheared in a rotating frame in the presence of a magnetic field parallel to the axis of rotation (see Fig. 1). Using the QS approximation, and neglecting non-linear terms in the full Navier-Stokes equations (equations 4.1 and 4.2 in reference[2]) the inviscid RDT transport equations become [2,4,5]

$$\begin{gathered} \partial_t u_i + Sx_2\partial u_i/\partial x_1 = \varepsilon_{ij3}2\Omega^f u_j - \delta_{i1}Su_2 - \partial p/\partial x_i\rho + B\partial b_i/\partial x_3 \\ nb_{i,kk} = -B\partial u_i/\partial x_3 \end{gathered} \qquad (1)$$

In the above expressions $S=\mathrm{d}U_1/\mathrm{d}x_2$ is the constant mean shear rate, Ω^f is the frame rotation rate around the x_3 axis, and B is the intensity of the constant magnetic field in applied in the x_3 direction (Fig. 1). Note that the magnetic field has been normalized into Alfven units, and appears with dimensions of velocity. The pressure term p, is modified compared to the hydrodynamic case, in order to include magnetic interactions. The parameter $n=1/(\sigma\mu)$ is the magnetic diffusivity, where σ is the electric conductivity of the fluid and μ is the fluid magnetic permeability. The continuity for the velocity and the magnetic fields in system (1) imposes that $u_{i,i}=0$, $b_{i,i}=0$. Following the Rogallo[19] method in order to transform into a frame that deforms with the mean shear, we set the deforming coordinates $\xi_1=x_1-x_2St$, $\xi_2=x_2$, $\xi_3=x_3$ and after applying the continuity equation for the Fourier transformed variables (denoted with ^) in the Rogallo coordinates, $k_iu_i=0$, we can solve for the pressure term

$$ik^2\hat{p}/\rho = -k_1 2\Omega^f\hat{u}_2 + \left(k_2^0 - Stk_1\right)2\Omega^f\hat{u}_1 + 2k_1S\hat{u}_2 \qquad (2)$$

At this point we may note that the modified pressure has exactly the same form as the one derived for the hydrodynamic case in[4]. However, the effect of the magnetic field is implicitly introduced through the modification of the velocity components due to the Joule effect in (1). Introducing equation (3) into the system (4), we obtain the linear evolution of the Fourier transformed velocity components

$$\frac{d\hat{u}_i}{d\beta} = \frac{\left(k_2\eta\hat{u}_1 + 2k_1\hat{u}_2 - k_1\eta\hat{u}_2\right)k_i - mk_3^2\hat{u}_i}{k^2} - \eta\delta_{i2}\hat{u}_1 + (\eta-1)\delta_{i1}\hat{u}_2 \quad (3)$$

In the above equation, $\beta=St$ is the total shear applied, and $k^2=(k_1^2+k_2^2+k_3^2)^{1/2}$ is the wave number vector magnitude. One may note that in the transformed Rogallo coordinates, the wave number components evolve as $k_i=k_i^0-\delta_{i2}Stk_1$ (the superscript 0 denotes initial values), as a result of the deformation of the frame due to the mean shear applied. The two parameters that control the evolution of the turbulence in equation (3) are the dimensionless rotation rate (the reciprocal of Rossby number), $\eta = 2\Omega^f/S$, and the dimensionless magnetic interaction parameter, or Stuart number, $m=\tau_S/\tau_M$ $= B^2/nS$. The first dimensionless parameter gives the relative strength of the rotation to the shear applied, and the second is the ratio between the time scales imposed by the shear $\tau_S=S^{-1}$, and the magnetic field $\tau_M=nB^{-2}$, respectively. As it has been underlined by the numerical studies of Kassinos et al.[2,17], these two ratios play a key, rather competitive, role on the evolution of the morphology of turbulence, at least for low Reynolds magnetic numbers, when the QS approximation is valid.

LINEAR SOLUTION FOR THE JOULE DISSIPATION

The terms $-u_imk_3^2/k^2$ on the RHS, of the system of equations (3) introduce the effect of the Joule dissipation, within the QS approximation. The presence of this term is what distinguishes the MHD system of equations (4) from the respective *hydrodynamic* RDT system (without an external magnetic field), studied in detail by [4,16]. After some algebra, it can be shown that the general solution of (4) $u_i(\beta,\eta,m,\boldsymbol{k})$ can be written in the form

$$\hat{u}_i\left(\beta,\eta,m,\boldsymbol{k}\right) = \hat{u}_i\left(\beta,\eta,m=0,\boldsymbol{k}\right)M\left(\beta,m,\boldsymbol{k}\right) \qquad (4)$$

where $u_i(\beta,\eta,m=0,\boldsymbol{k})$ is the general linear solution for sheared *hydrodynamic* turbulence in a rotating frame (m = 0). The multiplying function $M(\beta,m,\boldsymbol{k})$ modifies the general linear hydrodynamic case due to the presence of the external magnetic field. Under the QS approximation $M(\beta,m,\boldsymbol{k})$, satisfies

$$dM\left(\beta,m,\boldsymbol{k}\right)/d\beta=-mM\left(\beta,m,\boldsymbol{k}\right)k_3^2/k^2 \qquad (5)$$

and the solution of equation (5) yields

$$M=\exp\left\{\frac{-mk_3^2/k_1}{\sqrt{k_1^2+k_3^2}}\left[\tan^{-1}\left(\frac{k_1\beta-k_2^0}{\sqrt{k_1^2+k_3^2}}\right)+\tan^{-1}\left(\frac{k_2^0}{\sqrt{k_1^2+k_3^2}}\right)\right]\right\} \qquad (6)$$

Equation (6) describes analytically how the presence of an external magnetic field (under the QS approximation) modifies the general RDT spectral solutions for sheared hydrodynamic turbulence in a rotating frame [4,16]. More specifically, in the linear limit of the MHD equations and under the QS approximation, the magnetic effects are introduced solely through the multiplicative function $M(\beta,m,k)$, which corrects the hydrodynamic solutions for the effect of the magnetic field. From equation (6) it is clear that the influence of the magnetic field in the linear limit is not uniformly distributed over all the wave number components, but shows a profound preference on the direction of the magnetic field (large values of k_3 component). This is an immediate consequence of the underlying physics of the Joule dissipation, which is strongly anisotropic and tends to elongate the structures towards the direction of the magnetic field. Furthermore, using spherical coordinates ($k_1=k_0\cos\alpha$, $k_2=k_0\sin\alpha\sin\varphi$, $k_3=k_0\sin\alpha\cos\varphi$), we see that $M(\beta,m,k)$ is independent on the wave number magnitude, k_0, and it only depends on the direction of the wave number components; that is, on the angles of the spherical coordinates, α and φ. This is in agreement with the role of the Lorenz force, which demands that despite its angular anisotropy, Joule dissipation acts equally at all scales[2], and hence modifies the standard Kolmogorov phenomenology of the turbulent spectra.

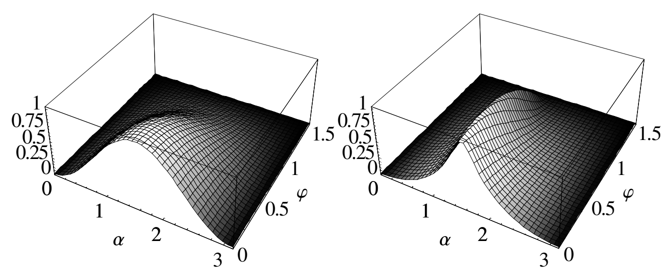

Figure 2. The dependence of the ratio A=-ln[$M(\beta,m,k)$]/$m\beta$ on the direction of the wave number components for values of total shear β=St equal to 0.1 (left) and 5.0 (right).

In figure 2 we present the dependence of the modifying function $M(\beta,m,\boldsymbol{k})$ on the two angles of the spherical coordinates ($0\le\alpha\le\pi$, $0\le\varphi\le\pi/2$) for two different values of the total shear, β = 0.1 and 5. More specifically we give the distribution of the ratio A=-ln[$M(\beta,m,\boldsymbol{k})$]/$m\beta$, along the plane determined by α and φ. The dependence of $M(\beta,m,\boldsymbol{k})$ on the total shear applied, and consequently on the time, is introduced though the wave number component $k_2=k_2^0-k_1\beta$, which appears in the *arctan* term at the RHS of equation (9). As the value of the total shear increases, for a non-zero k_1 component, this term tends to the constant value of 2π, and thus the direct dependence of $M(\beta,m,k_1\neq0)$ on the shear and on the time (through β) becomes gradually of secondary importance (Fig. 2). Consequently, for the modes with $k_1\neq0$, the primary dependence is on the wave number orientation and the magnetic interaction parameter, m. Furthermore, the function $M(\beta,m,k)$ has a profound peak in a range which narrows as ~ β^{-1} around $\alpha=\pi/2$ (k_1 = 0). At k_1 = 0, $M(\beta,m,k)$ is maximized (Fig. 2), and evolves exponentially with time as $M\left(\beta,m,k_1=0\right)=\exp\left(-\beta m\sin^2\varphi\right)$. In the other two-dimensional (2D) limit where k_2^0=0, for φ =0, the dependence on the total shear is less profound and $M\left(\beta,m,k_2^0=0\right)=\exp\left[-mk_3^2/\left(k_1\sqrt{k_1^2+k_3^2}\right)\tan^{-1}\left(k_1\beta/\sqrt{k_1^2+k_3^2}\right)\right]$. As already noted, for the modes with $k_1\neq0$ the dependence of $M(\beta,m,\boldsymbol{k})$ on β vanishes gradually as its value increases, and the correction of the linear hydrodynamic solution for the effect of the magnetic field approaches the form $\lim_{\beta\to\infty}M\left(m,k_2^0=0\right)=\exp\left(-m\pi k_3^2/2k_1\sqrt{k_1^2+k_3^2}\right)$. For all cases, as shown from equation (6), when the φ coordinate is $\pi/2$ (k_3=0) the function $M(\beta,m)$ is unity, and the turbulence becomes independent of the magnetic field. Note that in this 2D limit the dependence on the rotation rate also vanishes[4], in agreement with the principle of *material indifference*[20] for turbulent motion independent of the direction of the rotation, and thus, only the shear drives the turbulence evolution.

TWO-DIMENSIONAL SOLUTION FOR THE TUBULENCE EVOLUTION

In the following, we present and investigate solutions for initially three-component (3C), but two-dimensional (2D) turbulence, independent of the flow direction x_1, $u_i=u_i(x_2,x_3)$, $b_i=b_i(x_2,x_3)$ for $i=1,2,3$. Such solutions of the RDT system (3) in the absence of a magnetic field (m = 0) have been proved accurate for capturing the basic characteristics of the initially isotropic hydrodynamic 3D-3C case[4], for which the shear imposes rapidly a state where the turbulence is aligned with the direction of the mean flow. Taking the 2D limit of (6) for k_1= 0, and multiplying by the corresponding 2D RDT solution (m = 0) solved in detail in [4], yields the general solution of the system (3). Calculating the velocity spectra $E_{ij}\sim\overline{\hat{u}_i\hat{u}_j^*}$, we can integrate over all the wave numbers to obtain (in physical space) the development of the stress components $R_{ij}=\overline{u_iu_j}=\iiint E_{ij}d^3\mathbf{k}$ and the structure dimensionality tensor components[3,4,5] $D_{ij}=\iiint E_{nn}(\mathbf{k})k_ik_j/k^2d^3\mathbf{k}$. As it has been pointed out, the combined use of these two tensors gives a better description of the morphology of turbulent fields[3,4,5]. However, the form of the spectral solutions is such that it does not allow for fully analytical integrations, for the calculation of one-point statistics in real space. Still, we may draw significant information for the dependence of the form of the evolution of the turbulent kinetic energy (TKE) on the controlling parameters η and $m = B^2/Sn$, and for the limiting states of the structure tensors[2,3], which describe the trends in the morphology of the turbulent field.

In the absence of a magnetic field ($m = 0$), the above solutions simplify to the form of 2D sheared turbulence in a rotating frame studied in [4]. As it is known for the purely hydrodynamic case, the form of the energy growth is determined by the sign of the Bradshaw parameter [4,14,19], $Br = \eta(1-\eta)$. Positive values of this parameter (meaning that the frame is counter-rotating at a rate that is smaller than the rotation rate associated with the mean shear itself) correspond to destabilization of the TKE, while negative values (co-rotation or counter-rotation at higher rates) drive TKE to a diminishing behavior. If the magnetic field is present, the parameter m is always positive and thus, as will be shown, the stability of the TKE is still governed by the sign of the Br. However, the limiting states of the turbulence, from the linear MHD solutions, are drastically influenced by the value of dimensionless parameter m.

In the case of negative values of Br (which means that $\eta > 1$ or $\eta < 0$) the TKE is mainly driven by the magnetic field to approach asymptotically $q^2/q_0^2 \approx (2\pi m\beta)^{-1}$. We also calculate the limiting values of the normalized Reynolds stresses, $r_{ij} = R_{ij}/R_{kk}$, and the normalized structure dimensionality tensor, $d_{ij} = D_{ij}/R_{kk}$, in order to describe the limiting states of the morphology of the turbulence[3,4,5]. These limits for negative values of the Br parameter are

$$\begin{gathered} r_{11} = \frac{\eta-1}{2\eta-1},\ r_{22} = 0,\ r_{33} = \frac{\eta}{(2\eta-1)},\ r_{12} = 0 \\ d_{11} = d_{33} = d_{12} = 0,\ d_{22} = 1 \end{gathered} \tag{7}$$

and they correspond to an 1D-2C state, where all the dependence is confined along the cross-flow direction x_2 (see Fig. 1). At the same time, the distribution of the energy in the plane normal to that axis depends on the actual value of η. Such a state corresponds to horizontal sheets extending perpendicular to the cross flow direction x_2, when the turbulent motion is aligned with the other two directions. This limiting state is different than the 2D-3C obtained in the respective purely hydrodynamic case[4], and shows that the presence of the magnetic field parallel to x_3-axis, forces drastically turbulence to become uniform in this direction.

The picture is modified when the rotation rate is in the range $0 < \eta < 1$, which corresponds to positive values of Br parameter (counter-rotation at a rate that is smaller than the rotation rate associated with the mean shear itself). For such cases, the energy spectrum $E_{\alpha\alpha} = E_{11}+E_{22}+E_{33}$, for large times, and for values of $\sqrt{Br} = \sqrt{\eta(1-\eta)} < 2m$, peaks at the value of the critical angle

$$\varphi^* = a\sin\left(\sqrt{Br}/2m\right) \tag{8}$$

At this value of φ, the spectra evolve exponentially, $\sim \exp\left[2\beta\left(\sqrt{Br}\sin\varphi^* - m\sin^2\varphi^*\right)\right]$ with the time. In fact, the critical angle φ^* is a dimensionless measure of the relative strengths of the destabilizing rotation effect to the stabilizing external magnetic field effect, for negative Br values. Using the method of the steepest descent[20] (also known as Laplace's method) we expand the spectral solutions around the value of φ^* and we approximate the asymptotic behavior of the stresses, for large values total shear. By doing so, we see that for $Br > 0$ the TKE finally tends to evolve exponentially $q^2/q_0^2 \approx \beta^{-1/2}\exp\left(2\beta\sqrt{Br}\sin\varphi^* - 2\beta m\sin^2\varphi^*\right)$, no matter how strong is the magnetic field applied (note that this cannot be derived correctly applying the more simplistic 1D pressureless approach [4,21]). However, the time needed for the energy in order to finally reach an increasing behavior is much more when the magnetic field is strong. Nevertheless, the magnetic field has a profound influence on the limiting states of the turbulence, in terms of the dimensionality and componentality[3,4,5] which are

$$\begin{gathered} r_{11} = 1-\eta,\ r_{22} = \eta\sin^2\varphi^*,\ r_{33} = \eta\cos^2\varphi^* \\ d_{11} = 0,\ d_{22} = \cos^2\varphi^*,\ d_{33} = \sin^2\varphi^* \end{gathered} \tag{9}$$

From the above equation we see, that in the case of relatively weak magnetic fields, when $\varphi^* = \pi/2$, the frame rotation forces the turbulence to reach the same 1D-2C same asymptotic states as in the case without any magnetic field, $m = 0$, i.e. with $r_{11} \to 1-\eta$, $r_{22} \to \eta$, $d_{33} \to 1$. This result is in qualitative agreement with the DNS evidences of Kassinos et al.[3], for strong spanwise rotation, in combination with a spanwise magnetic field, tends to promote a streamwise alignment of the eddies, at least when $m \approx 1$, leading eventually to a state characterized by vertical slabs (figures 5c and 6c in reference[15]). In contrary, when the magnetic field is relatively strong and $\varphi^* < \pi/2$, the asymptotic states are modified showing a 2D-3C picture, with a gradual increase of the uniformity in the x_3 direction.

DISCUSSION

In this section we compare the present analytical results based on the 2D-3C initialization with the exact linear RDT numerical solution of the 3D-3C initially isotropic case calculated using the Particle Representation Model (PRM) [3,24,25,26,27], with large enough numbers of particles in order to ensure the accuracy of the solution. The Particle Representation Model (PRM)) (introduced in [24] and discussed in [27]) is a set of equations for the evolution of the properties of a hypothetical "particle". Each particle can be visualized as representing a 1D-1C flow, similar to a vortex sheet. The equations for the particle properties have an one-to-one correspondence with the respective linear RDT equations in Fourier space. With the PRM we follow the evolution of an ensemble of particles, determine its statistics and use these to calculate the one point statistics of an evolving field. It is important to note that, as shown by [24], the linear version of the

PRM does not incorporate any modelling, that is, it solves exactly the RDT system.

In figures 3, we show the respective TKE evolutions corresponding to the present 2D-3C solution, and the initially isotropic case for several combinations of the driving parameters η and m. From the comparisons it turns out that the present 2D-3C approach, although overestimating the TKE, explains accurately the type of the TKE growth, and determines correctly the linear criterion for the destabilization of the turbulent flow. The profound overestimation of the energy in the 2D-3C analytical solution, as compared to the 3D-3C initially isotropic case, can be attributed mainly to the combined effect of rotation and shear [4].

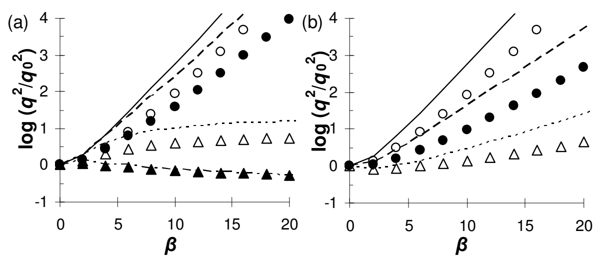

Figure 3. Evolution of the TKE. On the left side (a) the magnetic field is fixed to m=0.09 and the frame rotation is: η=0.5 (continuous, open circles), η=0.25 (long dashed, solid circles), η=0 (short dashed, open triangles), η=-0.5 (dotted dashed, solid triangles). On the right side (b) the frame rotation is fixed to η = 0.5 and magnetic field is: m=0.09 (continuous, open circles), m=0.25 (long dashed, solid circles), m=0.49 (short dashed, open triangles). The results are calculated from the 3D-3C initially isotropic exact PRM numerical solution (symbols) and the present 2D-3C analytical solution with k_1=0 (lines).

In terms of the stress evolution (not shown here) it maybe shown that for all the cases with $0 < \eta < 1$ (positive Bradshaw parameter), despite initial differences (due to the different initializations) the limiting states reached by the analytical 2D solution are in excellent agreement with the corresponding limiting states obtained numerically for the initially isotropic turbulence, where the d_{11} component tends relatively quickly to zero. The parameter that determines the limiting states is the value of the critical angle φ^*. In the case of relatively weak magnetic fields, when $\varphi^* = \pi/2$, the frame rotation forces the turbulence to reach the same 1D-2C asymptotic states as in the case without any magnetic field, m =0, i.e. with $r_{11}\rightarrow 1-\eta$, $r_{22}\rightarrow\eta$, $d_{33}\rightarrow 1$. This result is in qualitative agreement with the DNS results of Kassinos et al.[2,17]. They found that strong spanwise rotation in combination with a spanwise magnetic field tends to promote a streamwise alignment of the eddies, at least when $m \approx 1$, leading eventually to a state characterized by vertical slabs (figures 5c and 6c in reference[17]).

When the magnetic field is relatively strong (when $\varphi^* < \pi/2$), the asymptotic states of the linear solution are modified showing a 2D-3C picture, with a gradual increase of the uniformity in the x_3 direction. As the time scale ratio m increase and φ^* tends to zero, the turbulence tends to form 1D-2C structures elongated in the directions of the mean flow and the magnetic field.

When $Br < 0$, for $\eta < 0$ or $\eta > 1$, the linearly coupled action of the magnetic field and the shear, in the RDT solution, results in an 1D state, with horizontal sheets extending perpendicular to the cross flow direction x_2, when the turbulent motion is aligned with the other two directions. However, for $Br < 0$ the initial 3D character of the turbulence becomes more important. In this case, the limiting states reached by the present analytical 2D-3C solution diverge from the linear 3D-3C initially isotropic results (although both 1D-2C), in terms of the anisotropy of the componentality. Despite of this disagreement concerning the r_{ij} evolution, both initializations result in decay of the TKE (Fig. 3).

In conclusion, the combination of shear with the presence of a magnetic field in a rotating frame introduces two complex competing mechanisms in terms of their tendencies in producing 2D turbulence. The linear theory cannot take into account the non-linear interactions in the turbulence evolution. An immediate consequence of the linearity of the solution is that irrespectively of the value of the dimensionless ratio m, or the rotation rate η, the shear guides turbulent structures to elongate and finally align in the direction of the mean flow (x_1-axis) at large values of total shear. That is, the dimensionality component d_{11} finally vanishes. This is in contrast to the DNS results[2,17] which show that only for very small values of the ratio m, the turbulence structures finally align with the direction of the mean flow. This DNS trend is reflected by the initial responses of the linear solutions. For very small values of the magnetic interaction parameter, m ~ 0.1, the dimensionality of the turbulence initially tends to align with the axis of the mean flow, due to the shear. As m increases the picture is reversed and the Lorentz force is initially more effective in forcing the turbulence to align with the spanwise direction

CONCLUSIONS

Summarizing, in this work we presented a simplified approach for the study of MHD sheared turbulence in a rotating frame (Fig. 1), using the QS approximation coupled with RDT, for studying analytically the coupled effects of the frame rotation and the magnetic field in the evolution of homogeneous turbulence. We have derived the exact modification of the corresponding purely hydrodynamic 3D-3C spectra (equation 6) due to the presence of the external magnetic field using the quasi-static approximation. This modification is strongly anisotropic, affecting mainly the modes that are in the direction of the magnetic field and thus have larger values of the k_3 wave number component. On the other hand, the modification is independent on the wave number magnitude, correctly capturing the proper physics of the Joule dissipation. Furthermore, for initial fields that are 2D-3C, with the axis of independence aligned with the flow direction, we derived the one-point statistics in physical space. The 2D-3C analytical results of this study are in good agreement with the numerical solution for initially isotropic 3D-3C homogeneous turbulence. In fact the analytical 2D solution describes accurately the destabilizing effects in terms of the TKE evolution due to the frame rotation, and captures perfectly the RDT asymptotic states of the morphology of the turbulence. Also, in agreement with recent DNS evidences, the linear theory predicts that when

$\tau_S \approx \tau_M$, strong spanwise rotation tends to promote a stream-wise alignment of the turbulent structures. The present analytical outcomes enhance our understanding on these competitive coupling effects and offer a simplified representation that can be used in incorporating the proper physics in structure-based models.

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