INVESTIGATION OF TRANSITIONAL AND TURBULENT HEAT AND MOMENTUM TRANSPORT IN A ROTATING CAVITY

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ABSTRACT

The paper gives the results of the DNS/LES which was performed to investigate the 3D turbulent and transitional non-isothermal flows within a rotor/stator cavity. Computations were performed for the cavity of the aspect ratio L=3.0- 35.0 and curvature parameter Rm=1.8, for different rotations and Prandtl numbers. Two flow cases were considered: i) flow fully dominated by centrifugal and Coriolis forces (with negligibly small Earth accelerations term); ii) the Rayleigh – Bénard convection with superimposed moderate rotation. The main purpose of the investigations was to analyze the influence of rotation and aspect ratio on the flow structure and heat transfer. In the paper we analyzed distributions of the Reynolds stress tensor components, Nusselt number and others structural parameters, which can be useful for modeling purposes.

INTRODUCTION

The instability structures of the flow in the rotor/stator and rotor/rotor cavity have been investigated since the sixties of the last century, mostly with reference to the applications in turbomachinery. The flow between the rotor and stator is also an interesting fundamental problem, which allows us to investigate the influence of mean flow parameters on the strongly 3D boundary layers. The flow in rotor/stator cavity was investigated experimentally and numerically among others by Schouveiler et al. (2001), Serre and Pulicani (2001), Lygren and Anderson (2004). The non-isothermal flow conditions were also taken into consideration in some investigations (Randriamampianina et al. 1987, Tuliszka-Sznitko et al. 2009a, b). This showed that the rotation-induced buoyancy influences the stability characteristics and the critical conditions. Tuliszka-Sznitko et al. (2009a) performed the LES of the non-isothermal flow in the rotor/stator cavity, delivering distributions of the local Nusselt numbers along the stator and rotor for different configurations and Reynolds numbers. Pellé and Harmand (2007) performed measurements over the rotor (in the rotor/stator configuration), using a technique based on infrared thermography. A very detailed experimental investigation of the turbulent flow around a single heated rotating disk was performed by Elkins and Eaton (2000).

The flow in the cavity between two disks heated from below (the Rayleigh - Bénard convection) with superimposed moderate rotation is mostly used as a model problem for predicting geophysical phenomena (solar and giant planetary convection, deep oceanic convection). The flow with moderate rotation undergoes a series of consecutive bifurcations starting with unstable convection rolls at moderate Rayleigh number. The transition culminates at the state dominated by coherent plume structures. The Rayleigh - Bénard convection with superimposed rotations been studied, among others, by Kunnen et al. (2009).

In the present paper, we investigate the flow with heat transfer in the rotor/stator annular cavity of the aspect ratio from the range L=3.0 – 35.0 and curvature parameters Rm=1.8. Computations are performed for two main flow cases: In the first group we consider flows with heat transfer fully dominated by centrifugal and Coriolis forces; we analyze the influence of the rotational Reynolds number and aspect ratio on the flow structure and heat transfer (distributions of the local Nusselt number). In the second flow case we investigate the influence of superposition of moderate rotation on Rayleigh – Bénard convection. We analyze the influence of the Rossby, Rayleigh and Prandtl number on the distributions of the local and averaged Nusselt numbers and on the statistics.

MATHEMATICAL AND GEOMETRICAL MODEL

We investigate the non-isothermal flows in the cavity between stationary and rotating disks of the inner and outer radius R_0 and R_1, respectively. The interdisks spacing is denoted by 2h (Fig. 1). The rotor rotates at uniform angular velocity $\Omega = \Omega e_z$, e_z being the unit vector on the axis. The flow is described by the Navier-Stokes, continuity and energy equations, written in a cylindrical coordinate system (R, φ, Z) with respect to the rotating frame of reference:

$$\nabla \cdot V = 0 \qquad (1a)$$

$$\rho\frac{\partial V}{\partial t}+\rho(V\cdot\nabla)V+\rho\Omega\times(\Omega\times R)+2\rho\Omega\times V= \quad (1b)$$
$$-\nabla P+\mu\Delta V-q\rho Z$$
$$\frac{\partial T}{\partial t}+(V\cdot\nabla)T=a\Delta T \quad (1c)$$

where t is dimensional time, R is radius, P is pressure, ρ is density, q is the gravitational acceleration pointing downward, V is the velocity vector, a is the thermal diffusivity and μ is the dynamic viscosity. The flow is governed by the following dimensionless geometrical parameters: aspect ratio $L=(R_1-R_0)/2h$ and curvature parameter $Rm=(R_1+R_0)/(R_1-R_0)$. The dimensionless axial and radial coordinates are: $z=Z/h$, $z\in[-1,1]$, $r=(2R-(R_1+R_0))/(R_1-R_0)$, $r\in[-1,1]$. To take into account the buoyancy effects induced by the involved body forces, the Boussinesq approximation is used, i.e. the density associated with the terms of centrifugal and Coriolis forces due to the disk rotation, the curvilinear motion of the fluid and the gravitational acceleration is considered to be a variable.

In the flow cases dominated by the centrifugal and Coriolis forces, in which the Earth acceleration is negligibly small, the velocity components and time are normalized as follows: ΩR_1, $(\Omega)^{-1}$. The governing parameters are: the rotational Reynolds number $Re=\Omega R_1^2/\nu$, the thermal Rossby number $B=\beta(T_2-T_1)$, where $\beta=-1/\rho_r(\partial\rho/\partial T)_p$, T_1 and T_2 are two chosen reference temperatures. The dimensionless temperature is defined in the following manner: $\Theta=(T-T_1)/(T_2-T_1)$. The dimensionless components of the velocity vector in radial, azimuthal and axial directions are denoted by: u, v, w and dimensionless pressure is denoted by p. The no-slip boundary conditions are used with respect to all rigid walls, u=w=0. For the azimuthal velocity component, the boundary conditions are as follows: $v=0$ on the rotating disk and v=-(Rm+r)/(Rm+1) on the stator. In the paper we consider the following configuration: the rotating bottom disk is attached to the inner cylinder and the stator is attached to the outer cylinder. T_1 is the temperature of the upper disk and the inner cylinder, and T_2 is the temperature of the bottom heated rotating disk and the outer cylinder. The thermal boundary conditions are as follows:

$\Theta=1$ for z = -1.0, $-1.0\le r\le 1.0$ and for outer cylinder

$\Theta=0$ for z =1.0, $-1.0\le r\le 1.0$ and for inner cylinder

The thermal Rossby number equals B=0.1 in all considered flow cases.

When the flow between two disks heated from below with superimposed moderate rotation is considered (Rayleigh - Bénard convection) the velocity components are normalized with the free-fall velocity $\sqrt{q\beta\Delta T(2h)}$, time is normalized by a convection time scale $(2h)/\sqrt{q\beta\Delta T(2h)}$ and temperature by $\Delta T=(T_2-T_1)$.

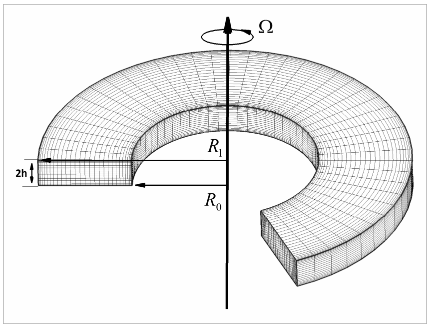


Figure 1. Schematic picture of computational domain, meridian section

NUMERICAL APPROACH

The numerical solution is based on a pseudo-spectral Chebyshev-Fourier-Galerkin collocation approximation. In the time approximation we use a second-order semi-implicit scheme, which combines an implicit treatment of the diffusive terms and an explicit Adams-Bashforth extrapolation for the non-linear convective terms. In the non-homogeneous radial and axial directions we use Chebyshev polynomials with the Gauss-Lobatto distributions to ensure high accuracy of the solution inside the very narrow boundary layers at the disks.

In the LES we use a version of the dynamic Smagorinsky eddy viscosity model proposed by Meneveau et al. (1996), in which the required averaging is performed over the pathlines of the fluid particles, instead of averaging over the direction of statistical homogeneity. The Smagorinsky coefficient is determined by minimizing the modeling error over the pathlines of the fluid particles. The numerical algorithm used for the LES of the non-isothermal flow in the annular cavity, proposed in the papers Tuliszka-Sznitko et al., (2009a, b), is an extended version of the DNS algorithm developed by Serre and Pulicani (2001).

In the present paper for the flow cases of the high aspect ratio and the large Reynolds number we use about $6\cdot10^6$ collocation points.

RESULTS OBTAINED FOR LARGE ROTATION

The problem addressed in this section is the turbulent flow between a rotating and a stationary disk with heat transfer. Computations are performed for wide range of aspect ratio L=3.0–35.0 and for the curvature parameter Rm=1.8. According to Schouveiler et al. (2001), who performed experimental investigations for cavity of Rm=1 and different L and Re, in this range of aspect ratio, the flow is of Batchelor type. That means that the flow consists of two boundary layers on each disk separated by an inviscid rotating core in which the velocity gradient is weak. However, when the aspect ratio L is increasing the inviscid core between two boundary layers is shrinking. The exemplary axial profiles of the averaged radial velocity component obtained in the present paper for different L in the middle section of cavities are showed in Fig. 2.

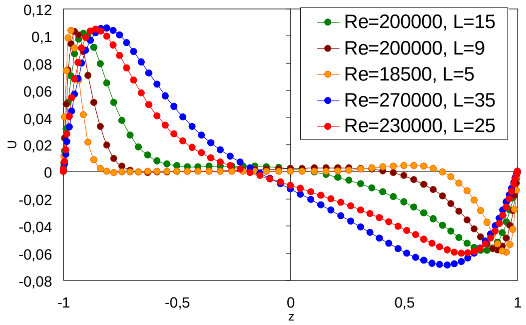
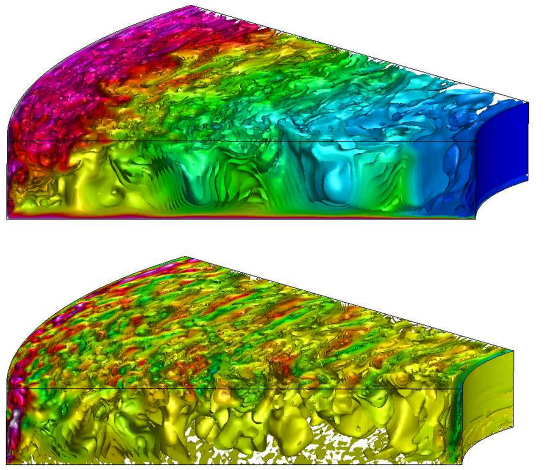


Figure 2. Axial profiles of radial velocity component obtained in the middle sections of cavities, B=0.1.

Fig. 3 shows the iso-surfaces of the temperature and axial velocity component obtained for: a) L=5, Rm=1.8, Re=195000, Pr=0.71, b) L=9, Rm=1.8, Re=230000, Pr=2.71, c) L=35, Rm=1.8, Pr=0.71, Re=280000. For all considered in Fig. 3 flow cases the fluid is pumped radial outward along the heated rotor $(\Theta(z=-1)=1.0)$ towards the heated outer stationary cylinder $(\Theta(r=1)=1.0)$. The fluid recirculates along the cooled stator $(\Theta(z=1)=0.0)$ towards the inner rotating cylinder. We can see from Fig. 3 that the heated fluid of higher temperature is concentrated near the outer cylinder, so that it causes large temperature gradients in this area. The Reynolds stress tensor components also reach their maximum values in the vicinity of the outer cylinder. The distributions of three main Reynolds stress tensor components $\sqrt{\overline{u'u'}}$, $\sqrt{\overline{v'v'}}$, $\sqrt{\overline{w'w'}}$ show strong anisotropy in both boundary layers (with the larges azimuthal component). Fig. 4a, b and c show the axial distributions of $\sqrt{\overline{v'v'}}$ obtained in three sections of cavity: in the section near the inner cylinder, in the middle section and in the section near the outer cylinder (results are obtained for L=5, 25, 35 and different Reynolds numbers). We can see that the maximum of the azimuthal component of the Reynolds stress tensor for all analyzed flow cases occurs near the outer cylinder in the rotor and stator boundary layer. In the middle section $\sqrt{\overline{v'v'}}$ is larger in the stator boundary layer.

We analyze the influence of the aspect ratio L and rotational Reynolds number Re on distributions of the local Nusselt numbers. The exemplary distributions of the local Nusselt numbers along dimensionless radius r obtained for Re =230000 and for L=25.0 and Re=195000 and L=5 are showed in Fig. 5. We observe the increase of the heat transfer near the outer cylinder where the turbulence is the largest.



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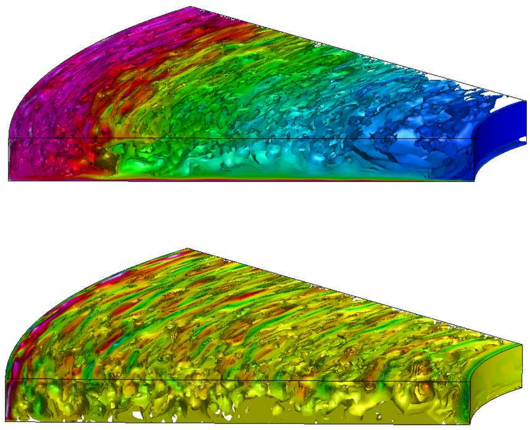

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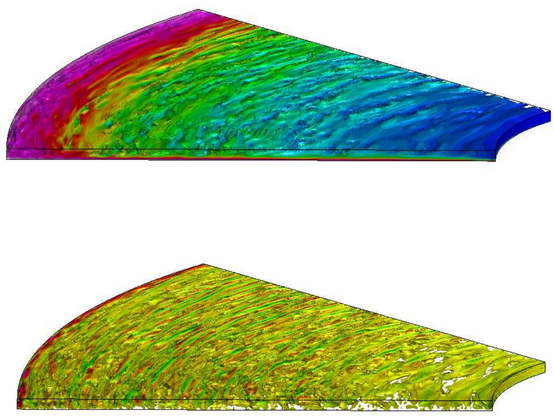
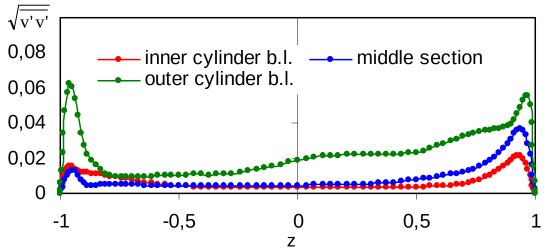


c)

Figure 3. The iso-surfaces of temperature and axial velocity component obtained for the following parameters: a) L=5, Rm=1.8, Re=195000, Pr=0.71, b) L=9, Rm=1.8, Re=230000, Pr=2.71, c) L=35, Rm=1.8, Pr=0.71, Re=280000.



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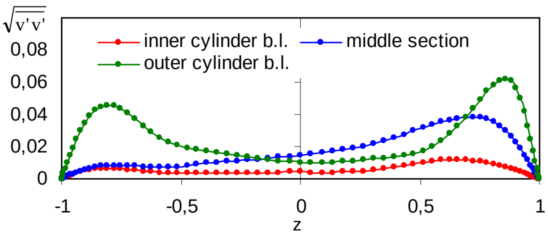

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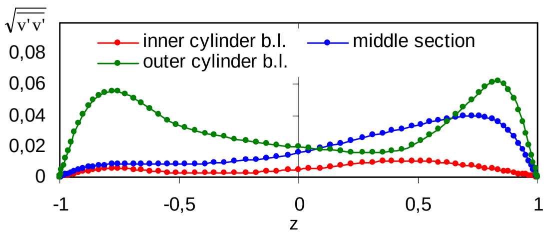
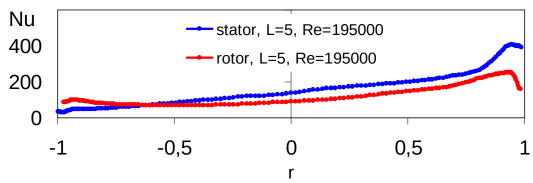


c)

Figure 4. The axial profiles of the azimutal component of the Reynolds stress tensor. a) L=5, Rm=1.8, Re=195000; b) L=25, Rm=1.8, Re=230000; c) L=35, Rm=1.8, Re=280000.



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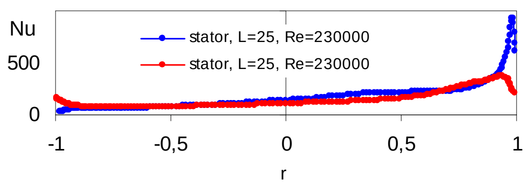


b)

Figure 5. Exemplary Nusselt distributions obtained for: a) L=5, Rm=1.8, Re=195000, b) L=25, Rm=1.8, Re=230000.

Fig.6 shows the distribution of dimensionless temperature fluctuations, normalized by the friction temperature, obtained in the middle sections of cavities. The results obtained in the present paper are compared to the results of Elkins and Eaton (2000) obtained experimentally for heated single rotating disk and also with 2DBL results of Wroblewski and Eibeck (1990) and Blair and Bennett (1987). We observe good agreement with the Elkins and Eaton (2000) in the outer part of the boundary layer. The differences in the area close to the disk result from different thermal boundary conditions used in the present paper (isothermal) and in Elkins and Eaton (2000) experiment (constant heat flux).

The turbulent Prandtl number is defined as the ratio of the eddy diffusivity for momentum to the eddy diffusivity for heat. The above definition is not a strict definition for strongly 3DTBLs, however, we use it to compare our results with the data published by Elkins and Eaton (2000) for the flow along a single heated rotating disk. In many 2DTBLs the turbulent Prandtl number equals 1 in the area near the wall and decreases to 0.8 with increasing z. Elkins and Eaton (2000) showed that for a single rotating disk Pr_t equals 1 near the disk and then decreases to the value of about 0.6 at the half width of the boundary layer δ . In Fig.7 we present the comparison of axial distributions of Pr_t obtained in the present investigations in the middle sections of the cavities (the stator boundary layer) with the results obtained by Elkins and Eaton (2000) and Littell and Eaton (1991), and with the distributions obtained in some 2DTBLs investigations (Wroblewski and Eibeck, 1990, Subramanian and Antonia, 1981).

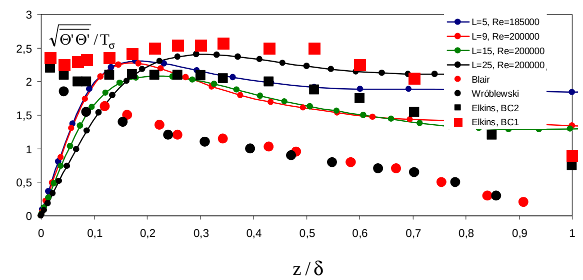


Figure 6. Distributions of temperature fluctuations normalized by friction temperature obtained in the stator boundary layer in the middle section of cavities. Comparison with the data from literature.

In the paper we also analyze another structural parameters proposed by Elkins and Eaton (2000), for example, $R_{\varphi\Theta} = (\overline{v'\Theta'}^2 + \overline{u'\Theta'}^2)^{0.5} / (\overline{\Theta'^2}(\overline{v'^2} + \overline{u'^2}))^{0.5}$. In most structural parameters we observe the agreement in axial distributions, however, the largest differences have been observed while comparing parameter $(\overline{u'^2} + \overline{v'^2}) / \overline{w'^2}$. The discrepancies in distribution of $(\overline{u'^2} + \overline{v'^2}) / \overline{w'^2}$ are particularly significant for cavities of large aspect ratio L. In our computations

$\overline{(u'^2} + \overline{v'^2}) / \overline{w'^2}$ parameter reaches a peak near the disks and then decreases rapidly to the value of about 2 near the edge of the boundary layers, showing that the vertical motion is very weak close to the disks

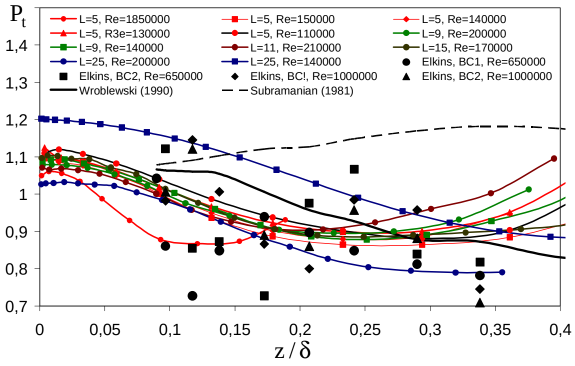


Figure 7. Axial distributions of the turbulent Prandtl number .

RAYLEIGH – BĖNARD CONVECTION WITH SUPERIMPOSED ROTATION

The Rayleigh - Bénard convection is defined as an unbounded horizontal layer of fluid heated from bellow. The problem is governed by Rayleigh number $Ra = q\beta\Delta T(2h)^3 / \nu a$ and Prandtl number $Pr = \nu / a$. Where $\Delta T = (T_2 - T_1)$. The Raylaigh number is the measure of the importance of the buoyancy coming from the Earth acceleration relative to diffusion in the fluid. To describe the effect of superimposed rotation on the Rayleigh - Bénard convection the Taylor number is used $Ta = (2\Omega(2h)^2 / \nu)^2$. In the analysis of the results the Rossby number is often introduced, which is defined in the following manner $Ro = \sqrt{Ra / Pr \cdot Ta}$. The Rossby number is the measure of importance of buoyancy resulting from the gravitational acceleration relative to the importance of superimposed rotation. For $Ta \to 0$ $(Ro \to \infty)$ we have classic Rayleigh – Bénard convection. In the present paper calculations are obtained with no-slip conditions on both disks and cylinders. Two thermal conditions are applied on cylinders: adiabatic and isothermal. Computations are performed for Ro=0.1, 0.25, 0.5, 0.75 and 1.0 and for Rayleigh number up to $3 \cdot 10^6$. For Ro=0.1 convection is suppressed by rotation (for geometrical parameters considered in this section: L=5, Rm=1.5). As Ra increases the flow undergoes through a sequence of bifurcation, starting with unstable convection rolls at moderate Rayleigh and culminating in the state dominated by coherent plume structures (iso-surfaces showing plumes structures are presented at Fig. 8).

The differences between structures obtained for different Ro and for the same Ra=250000 are visible in the Fig. 9a, b and c where iso-surfaces of temperature are analyzed.

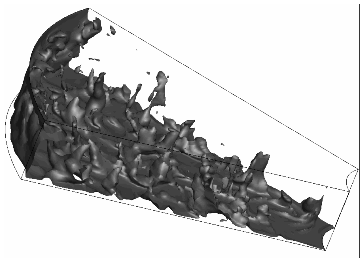


Figure 8. Iso-surfaces of temperature showing structure of plumes.

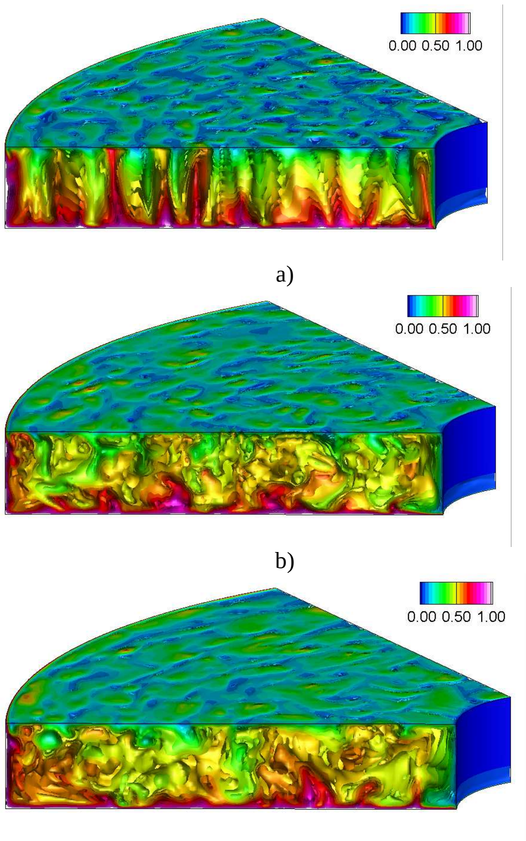
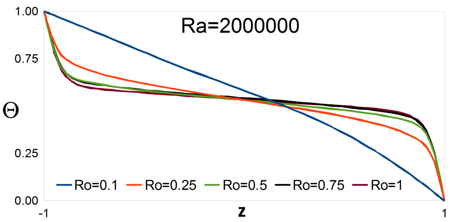


Figure 9. Iso-surfaces of temperature obtained for L=5, Rm=1.5 , Ra=2500000 and different Rossby number: a) Ro=0.25, b) Ro=.0.75, c) Ro=1.

The superimposed rotation on the Rayleigh-Bénard convection stabilizes the fluid layer. Simultaneously the modest rotation increases the Nusselt number compared to the non-rotating case flow. It is explained by so called Ekman pumping. Fig. 10a presents profiles of temperature averaged in time and horizontal direction. In classic Rayleigh - Bénard convection averaged temperature profile consists of two boundary layers and the bulk of zero temperature gradient. As rotation is superimposed, a negative temperature gradient in the bulk appears; its value depends on Rossby number (Fig. 10a). In Fig. 10b we compare the axial profiles obtained in the present computations for adiabatic and isothermal boundary conditions on the cylinders with the results of Kunnen et al. (2009) and Julien et al. (1996), who used periodicity conditions in horizontal direction. The results were obtained for Ro=0.75, Pr=1 and $Ra = 2.0 \cdot 10^6$. In Fig.10b we observe that the difference between the results obtained for adiabatic and isothermal boundary conditions on the cylinder is negligibly small. Profiles obtained by Kunnen et al. (2009) and Julien et al. (1996) are almost identical in the bulk; the differences are observed only in the boundary layers. In the bulk area our results show more negative temperature than Kunnen et al. (2009) and Julien et al. (1996), which can be attributed to confinement of the domain in horizontal direction.

In Rayleigh - Bénard investigations we analyzed: the distributions of the Reynolds stress tensor components, the distribution of turbulent heat fluxes and the distributions of local and averaged Nusselt numbers.



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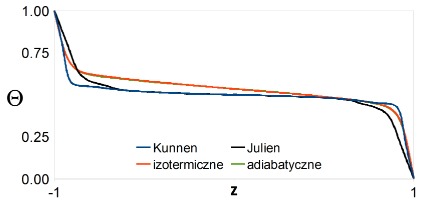

b)

Figure 10. a) Distributions of averaged temperature Ra=2000000, L=5, Rm=1.5. b) Comparison with Kunnen et al. (2009) and Julien et al. (1996) results.

CONCLUSIONS

In the paper we have analyzed the influence of rotation and aspect ratio on flow structure and heat transfer in the rotor/stator cavity. Computations have been performed for large rotations (large Reynolds numbers) and for Rayleigh-Bénard convection with superimposed moderate rotations. The results (the Reynolds stress tensor components, the components of the turbulence energy tensor, the turbulent Prandtl numbers and other structural parameters) were compared with the results obtained by Elkins and Eaton (2000). We have found significant influence of the aspect ratio on the flow structure and the distribution of Nusselt numbers.

The results obtained for Rayleigh-Bénard convection showed large influence of Rossby number on statistics and the flow structure. The results have been compared with the data published by Kunnen et al. (2009).

ACKNOWLEDGEMENT

We are grateful to The Poznan Supercomputing and Networking Center, where the computations have been performed.

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