INFLUENCE OF ACTIVE CONTROL ON STG-BASED GENERATION OF STREAMWISE VORTICES IN NEAR-WALL TURBULENCE

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ABSTRACT
The near-wall streamwise vortices are closely related with the generation of high skin friction at the wall in turbulent flows, and in the controlled, friction-reduced turbulent flows, streamwise vortices are greatly attenuated. In present study, the streak transient growth (STG) mechanism of generating near-wall streamwise vortices is employed, and the opposition control is imposed during the transient growth process of the perturbation to disclose how the active control affects the generation of quasi-streamwise vortices. It is found that in the transient growth stage, when the detection plane is located near the wall \(y^+_d = 15\), the control can suppress the generation of the streamwise vorticity through weakening the near-wall vertical velocity; when the detection plane moves away from the wall \(y^+_d = 28\), the control has opposite effects. In the vortex generation stage, the control can not change the dominance of the stretching effect. The \(y^+_d = 15\) control can achieve sustained overall attenuation of streamwise vortex generation via the suppression of the stretching term. The \(y^+_d = 28\) control, however, generates multiple extreme points in the stretching term distribution, which causes increasing the number of streamwise vortices by splitting the primary vortex into smaller and shorter fractions. The opposition control using the signal at \(y^+_d = 28\) with reduced strength has been proposed and tested in both minimal and full-scale channel flows. The effectiveness in turbulence suppression by the lessened \(y^+_d = 28\) control confirms present analysis.

INTRODUCTION
It is widely accepted that the near-wall quasi-streamwise vortices play a dominant role in both turbulence production and high skin-friction generation (Kim, Moin & Moser, 1987). By active manipulation of these near-wall coherent structures through blowing / suction at the wall in opposite to the velocity detected at a small distance from the wall, Choi et al. (1994) successfully suppressed turbulence intensity as well as reduced the friction drag. It is demonstrated by the vast amount of following works (recent reviews see Collis, Joslin, Seifert & Theofilis, 2004; Kim, 2003; Kim & Bewley, 2007; Kasagi, Suzuki & Fukagata, 2009) that in the actively controlled, friction-reduced turbulent flows, the quasi-streamwise vortices are greatly attenuated.

Although many interpretations have been proposed to the mechanism of turbulence suppression and skin friction reduction by active control from different viewpoints (see Choi, Moin & Kim, 1994; Hammond, Bewley & Moin, 1998; Farrell & Ioannou, 1996; Jimenez & Pinelli, 1999), how the control influence the generation of streamwise vortices still need further exploration. To do so, the mechanism of streamwise vortex generation in canonical wall-bounded turbulent flows need to be elucidated first.

The explanations for the generation of stream-wise vortices are categorized into two groups (Schoppa & Hussain, 2002): the so-called parent-offspring mechanism and the instability-based mechanism. Uniting the elements of parent-offspring and instability-based scenario, Schoppa & Hussain (2002) provided a more convincing mechanism - streak transient growth (STG), which is far more prevalent and energetic than normal-mode instability. In present work, the streak transient growth in a minimal channel subjected to the active opposition control is studied to disclose how the manipulations at the wall interfere with the formation and evolution of streamwise vortices, and why the difference in the detection position can have different even opposite effects on the flow. In the following, the numerical method and the STG problem formulation will be first described, and then the general response of disturbance energy and vortical structure to the control will be introduced. After that, the influence of the control on the STG-based generation process of streamwise vortices will be analyzed in details. Summary and discussion will be given at last.

PROBLEM FORMULATION
The flow of incompressible Newtonian fluid in a minimal channel subjected to active blowing/suction at the lower wall is studied by direct numerical simulation with Fourier-Chebyshev spectral method. The Reynolds number based on bulk mean velocity \(U_{in}\) and channel half width \(H\) is chosen to be \(Re = 2800\), corresponding to the friction Reynolds number around \(Re_{fr} = 180\). According to Jimenez & Moin (1991), the computational domain is selected to span \(\pi H \times 2H \times 0.2\pi H\) (approximately \(560 \times 360 \times 110\) wall units) in streamwise \(x\),
wall-normal ($y$) and spanwise ($z$) directions, respectively, to get sustained turbulence as well as to isolate only one low-speed streak in the domain. Accordingly, $32 \times 129 \times 32$ grids are used.

According to Schoppa & Hussain (2002), the flow field is initialized by the single-side turbulent profile with a low-speed streak as the base flow

$$U(y,z) = U_0(y) + \frac{1}{2} \Delta \cos(\beta z)g(y), \quad V = W = 0 \quad (1)$$

and the streamwise-dependent spanwise velocity $w'$ as the perturbation (referred to as STG perturbation in Schoppa & Hussain (2002) and hereinafter)

$$u' = 0, \quad v' = 0, \quad w' = A \sin(\alpha y)g(y). \quad (2)$$

Here the superscript $'$ denotes the perturbations to the two-dimensional base flow $U(y,z)$.

In Eq.(1), $U_0(y)$ is composed of a laminar profile near the upper wall and a turbulent profile near the lower wall as was adopted by Schoppa & Hussain (2002). The low-speed streak is represented by $\Delta \cos(\beta z)g(y)$, in which the spanwise wave number $\beta$ is chosen to make the streak span about $100$ wall units in spanwise direction, and $g(y)$ takes the form of $y \exp(-\eta y^2)$. We select $\eta$ to make $g(y)$ reach maximum value at $y' = 20$. Once $g(y)$ is fixed, $\Delta u$ is determined by the streak strength. According to Schoppa & Hussain (2002), the streak strength is quantified by the vortex line lift angle at $y' = 20$ as $\theta_0 = \tan^{-1}(|\Omega_x|_{y'=20}/|\Omega_z|_{y'=20})$, with $\Omega_x = \partial U/\partial z$ and $\Omega_z = -dU_0/dy$.

According to Schoppa & Hussain (2002), this STG-based vortex generation scenario may be one of the dominant mechanisms for near-wall streamwise vortex generation, since about 80% streaks extracted from fully developed near-wall turbulence are linearly stable. Hence in present study, the normal-mode stable streak with $\theta_0 = 45^\circ$ is chosen and the amplitude of the the initial STG perturbation takes the value of $w'_{max} = 0.5$. The opposition control is applied through the blowing / suction at the wall, i.e., the wall-normal velocity at the wall is specified as the opposite of the wall-normal velocity at a detection plane $v_{wall} = -v'$. In present study, two detection positions have been tested, $y_d' = 15$ and 28, respectively.

**EVOLUTION OF THE DISTURBANCE ENERGY AND VORTICAL STRUCTURE**

The time evolutions of the disturbance energy $E_{3D}$ and its components $u'^2$, $v'^2$ and $w'^2$ in no- and opposition-controlled minimal channel flow are shown in Fig. 1. The vortical structures are identified and visualized by the iso-surface of $\lambda_2 < 0$. Fig. 2 shows the iso-surface of $\lambda_2 = -3$ at $t^+ = 40, 65, 88, 125$ and 170 for the no-control and the two opposition control cases, respectively, to display the generation and evolution process of vortices.

According to the different characteristics in the evolution of disturbance energy and streamwise vortices, the whole process is divided into three stages: the transient growth dominated first stage ($0 < t^+ < 20$), the vortex generation dominated second stage ($20 < t^+ < 100$) and the turbulence dominated third stage ($t^+ > 100$). In the first stage, $0 < t^+ < 20$, the transient growth is the dominant mechanism, and the influence of the active control on the evolution of $E_{3D}$ is not obvious. Initially, $u' = v' = 0$ (Eq.2). After the flow begins, $u'$ is quickly generated, amplified and reaches a peak value at $t^+ = 20$, while the amplification of $v'$ by the transient growth mechanism is more mild, and it reaches a far less peak value later at $t^+ = 30$. Unlike $u'$ and $v'$, $w'$ is almost unchanged and keeps the initial amplitude until the end of the first stage. The second stage covers $20 < t^+ < 100$, during which the nonlinear effects comes into to play an important role and results in the generation of quasi-streamwise vortices, as shown in Fig. 2. The difference in the evolution of $E_{3D}$ for different cases begins to appear in this stage. For the two control cases, the dis-
turbances in $v'$ and $w'$ are both suppressed in comparison with the no-control case. But for $u'$, it is attenuated by the control with $y_d' = 15$, but evolves in a similar way to the no-control case for the control with $y_d' = 28$. In this stage, the influence of the control on the generation of streamwise vortices is obvious. For the no-control case, the quasi-streamwise vortices have been formed at $t^+ = 40$, and gradually elongated in the streamwise direction (Fig.2(a1)-(c1)). Compared with the no-control case, the stream-wise vortices are greatly attenuated by the $y_d' = 15$ control from the very beginning of the vortex generation process (Fig.2(a2-c2)). At $t^+ = 40$, the vortices are too weak to be displayed by $\lambda_2 = -3$, hence no structures are shown in Fig.2(a2). On the other hand, by the $y_d' = 28$ control for $t^+ < 125$, the streamwise vortices have a similar strength to the no-control case (Fig.2(a3-c3)), but with shorter streamwise length. The third stage refers to $t^+ > 100$, after which the flow becomes turbulent. At the beginning of this stage, $v'$ and $w'$ undergoes abrupt growth by the control with $y_d' = 28$, and $E_{3D}$ fluctuates afterwards with a magnitude similar to the no-control case. The growth in $v'$ and $w'$ as well as in $u'$ under $y_d' = 15$ control is very limited, and $E_{3D}$ keeps at a much lower level compared with the other two cases. In the no control case, at $t^+ = 125$ and $t^+ = 170$, as is shown by Fig.2(d1-e1), a spanwise arc is produced from the lifted downstream end of the vortices, forming a hair or hook vortex as was named by Robinson (1991). By the control with $y_d' = 15$, the vortical structures are attenuated and no hairpin vortices formed. However, at $t^+ \geq 125$, the vortices are increased in number but shortened in streamwise direction by the control with $y_d' = 28$, in comparison with the no-control case.

The response of the disturbance energy and streamwise vortices to the opposition control with different $y_d'$ in minimal channel flow (Choi, Moin & Kim, 1994). Additionally, it is worth to be addressed that the $y_d' = 15$ control influences the vortex generation process from the very primary stage and the vortices are attenuated all along; but the $y_d' = 28$ control seems only take effect after the vortices are generated and makes the number of streamwise vortices increased when the vortex tail moves away from the wall.

**INFLUENCE OF THE CONTROL ON THE GENERATION OF STREAM-WISE VORTICES**

How the control with different $y_d'$ takes affects on the flow in the three stages will be analyzed in detail in the following.

**The first stage**

The first stage can be treated as a linear process since the transient growth mechanism dominates the whole period. According to Schoppa & Hussain (2002), no streamwise vortices can be generated in this stage, but the streamwise vorticity $\omega_x$ can be transiently amplified, which are necessary to trigger the stretching effect in the generation of streamwise vortices in the following nonlinear stage. Especially, since the control is imposed through the normal velocity at the wall, its effect on $v'$ would be more straightforward and will be analyzed first. Then the influence of the control on the evolution and distribution of $\omega_x$ will be discussed.

We use the linearized Navier-Stokes equation with the continuity constraint to analyze the behavior of the flow. Recall that initially $u' = v' = 0$, and we only have $w'$ in the form of Eq.(2) upon the base flow $U(y,z)$. Since $v'$ and $\partial u' / \partial x$ is much smaller than $w'$ at the beginning, $\partial u' / \partial x \sim -\partial w' / \partial x \sim (\partial u' / \partial x)(\partial U / \partial z)$. And analysis shows that the variation in the spatial distribution of $w'$ is also very small. Hence $\partial w' / \partial z$ can be neglected compared with $\partial u' / \partial x$ in the continuity equation, and $\partial v' / \partial y = -\partial u' / \partial x \sim (\partial u' / \partial x)(\partial U / \partial z)$.

Considering the form of the base flow and the STG perturbation, Eq.(1) and Eq.(2), $v'$ should vary with $x$ and $z$ as $\cos(\alpha x)$ and $\sin(\beta z)$, respectively. The analysis has been confirmed by Fig.3(a1), in which the iso-surface of $|v'| = 0.01$ at $t^+ = 10$ is shown.

The variation of $v'$ with the wall normal coordinate $y$ is depicted by the contours of $v'$ in the $(y,z)$-plane across the peak position in $x$ direction, as is shown in Fig.3(b1-b3). One feature that should be noted is that $v'$ changes sign in $y$ direction around $y^+ = 20$, and reaches two extreme value around $y^+ = 9$ and $y^+ = 35$, respectively. Interestingly, if the detection plane is located at $y_d' = 15$, the blowing / suction velocity on the wall determined by the opposition control scheme is in opposite to $v'$ below $y^+ = 20$. Therefore, the control with $y_d' = 15$ weakens $v'$ below $y^+ = 20$ a little, as is shown in Fig.3(b2). However, when the detection plane is moved to $y_d' = 28$, the blowing / suction velocity on the wall is strong and its direction is the same as that of $v'$ blow $y^+ = 20$. That means the control with $y_d' = 28$ can enhance $v'$ below $y^+ = 20$. As can be seen in Fig.3(b3), $v'$ below $y^+ = 20$ is of the same magnitude as that above $y^+ = 20$. But compared with the other two cases, $v'$ above $y^+ = 20$ under the control is a little bit smaller.

According to Schoppa & Hussain (2002), the transient growth of the STG perturbations can lead to the formation of a sheet of streamwise vorticity $\omega'_x$. It is first generated in the streak trough region at the streamwise position corresponding to $w'$'s zero-crossing point, and convected in streamwise direction by mean flow advection. With time growing, $\omega'_x$ can also be generated at the streak flank and crest, and forms a $z$-continuous sheet. Fig.4 shows the distribution of $\omega'_x$ in $(y,z)$-plane through the zero-crossing position of $w'$ at $t^+ = 10$ and $20$ for no-control and the two control cases, respectively. Though the influence of the control on $\omega'_x$ very close to the wall is obvious, the $\omega'_x$ close to the wall is less important than the elliptical patch of $\omega'_x$ in the streak trough region in the nonlinear stage. The influence of the control on the elliptical patch of $\omega'_x$ in the streak trough region is not obvious at $t^+ = 10$, but can be distinguished at $t^+ = 20$. Compared with the no-control case, $\omega'_x$ is slightly attenuated by the $y_d' = 15$ control, but enhanced by the $y_d' = 28$ control. The underlying mechanism can be explained via vorticity perturbation equations in the streak-vortex-line coordinate system $(x,n,s)$ as is shown in Fig.5. According to Schoppa & Hussain (2002), the term $\Delta \partial \omega'_x / \partial x$ is dominant in the linear stage, in which $\omega'_x$ is the velocity component tangential to the vortex line. If $v'$ is negative near the left trough of the streak and positive near the right trough, just as the case shown in Fig.5, it will make favorable contribution to $\omega'_x$, vice versa. At $t^+ = 10$, the distribution of $v'$ displays a negative region near the left trough of the streak and a positive region near the right trough of the streak for all the three cases but with different magnitude, as
The second stage

After the transient growth in the first stage, the perturbations have been greatly amplified and the nonlinear effects are triggered and play an important role in the second stage, during which the streamwise vortices are generated by direct stretching the formerly produced, \( z \)-localized vorticity sheet (Schoppa & Hussain, 2002). As has been shown in Fig.2, the \( y''_2 = 15 \) control can greatly attenuate the streamwise vortices; while the \( y''_2 = 28 \) control can only affect the location and the length of streamwise vortices. How the control influence the vortex formation in the nonlinear stage will be analyzed through the inviscid evolution equation for streamwise vorticity \( \omega_z \):

\[
\frac{\partial \omega_z}{\partial t} = -u \frac{\partial \omega_z}{\partial x} - v \frac{\partial \omega_z}{\partial y} - w \frac{\partial \omega_z}{\partial z} + \frac{\partial v}{\partial x} \frac{\partial u}{\partial y} + \frac{\partial w}{\partial x} \frac{\partial u}{\partial z} + \frac{\partial w}{\partial y} \frac{\partial u}{\partial z} + \omega_z \frac{\partial u}{\partial z} + \omega_u \frac{\partial u}{\partial x}
\]

On the right-hand side of the above equation, the advection terms due to \( u, v \) and \( w \) are denoted by \( \text{ADX, ADY and ADZ} \), respectively. \( \text{T11 and T12} \) represent the contributions from tilting, and \( \text{ST} \) represent the stretching contribution.

According to Schoppa & Hussain (2002), stretching effect (ST) is the dominant factor in the generation of streamwise vortices. In present study, the vortex with \( \omega_z > 0 \) at \( t^+ = 40 \) is studied. The vortex core, which is identified by the position of minimum \( \omega_z \), is located at \( (200,17,69) \) for no-control, \( (230,20,57) \) for \( y''_2 = 15 \) control and \( (230,22,65) \) for \( y''_2 = 28 \) control cases, respectively, see Fig.2(a1-c1) for reference. In Fig.6, the vortex core is denoted by a black dot. The bold line identifies the vortex-related \( \omega_z > 0 \) region, and the fine lines stand for the terms concerned (solid for positive value and dashed for negative value). If ST has the same sign as \( \omega_z \) at the same place, the term will make favorable
Figure 6. Contours of ST on (a1-a3) (y,z)-plane and (b1-b3) (x,y)-plane across the vortex core at $t^+ = 40$ for (a1-b1) no-control, (a2-b2) $y_d^+ = 15$ control and (a3-b3) $y_d^+ = 28$ control. Bold solid line is the iso-contour of $\omega_x = 2$, which identifies the vortex-related $\omega_x > 0$ region. The black dot shows the position of vortex core. Thin solid lines show positive ST and thin dashed lines show negative ST with contour level increments of 0.5.

Figure 7. Contours of $\partial u/\partial x$ on (a1-a3) (y,z)-plane and (b1-b3) (x,y)-plane across the vortex core at $t^+ = 40$ for (a1-b1) no-control, (a2-b2) $y_d^+ = 15$ control and (a3-b3) $y_d^+ = 28$ control. Thin solid lines show positive $\partial u/\partial x$ with contour level increments of 0.1.

Figure 8. Contours of $\partial w/\partial x$ on (a1-a3) (y,z)-plane and (b1-b3) (x,y)-plane across the vortex core at $t^+ = 40$ for (a1-b1) no-control, (a2-b2) $y_d^+ = 15$ control and (a3-b3) $y_d^+ = 28$ control. Thin dashed lines show $\partial w/\partial x \leq -0.25$ with contour level increments of 0.1.

that in the no-control case, as is evidenced by Fig.2(a2-e2). Further analysis shows that $\partial u/\partial x$, which is directly related to ST production, is mainly generated by $-(\partial w/\partial x)(\partial u/\partial z)$ term. For $y_d^+ = 15$ control, both $-\partial w/\partial x$ and $\partial u/\partial z$ are attenuated. For $y_d^+ = 28$ control, no suppression can be observed in both $-\partial w/\partial x$ and $\partial u/\partial z$, but it has been clearly elucidated that the more extreme points in ST is originated from $-\partial w/\partial x$.

It is also interesting to note that when the vortex is generated around $y^+ = 20$, the vertical velocity sensed at $y_d^+ = 15$ and $y_d^+ = 28$ is in the same phase, but with different magnitude. Their different influence on ST, and finally on streamwise vortices suggests that the strength of wall blowing / suction is crucial to the control, not only the detection position. This will be further discussed in the final section.

The third stage
In the third stage, the flow in no control and $y_d^+ = 28$ control cases becomes turbulent, while it tends to be laminar by $y_d^+ = 15$ control. As has been shown in Fig.2, the streamwise vortices under $y_d^+ = 28$ control are similar to no control case before $t^+ = 125$, but are broken into smaller and shorter fractions afterwards. We first resorted the explanation to the stretching term ST. As is discussed in the previous section, multiple extreme points in ST can be found within the primary elongated vortices. This means that the stretching strength is not uniform in the primary vortex, which can be broken into many fractions under scattered strong stretching effect. It should be addressed that the multiple extreme points in ST originating from $\partial w/\partial x$, as is elucidated in the previous section, is the result of the interaction between the flow injected on the wall and the primary vortices.

SUMMARY AND DISCUSSION
The effect of the opposition control on the STG-based generation of the near-wall streamwise vortices is studied by the direct numerical simulation of the minimal channel flow at $Re_x = 180$. The normal mode stable streak with $\theta_{00} = 45^\circ$ is considered. The initially imposed STG perturbation is set to
reach maximum value at $y^+ = 20$. It is found that in the linear transient growth period, $\nu'$ changes sign around $y^+ = 20$, and hence $y_d^+ = 15$ control can weaken $\nu'$ below $y^+ = 20$, while $y_d^+ = 28$ control can strengthen $\nu'$ there, resulting in the suppression and enhancement of the streamwise vorticity generated in the $z$-localized elliptical patch of streamwise vorticity sheet, respectively. In the nonlinear vortex generation stage, it is found that the stretching effect is still dominant over advection and tilting effects, no matter whether the control is imposed or not. By $y_d^+ = 15$ control, the alleviated streamwise vorticity production in the linear stage takes effect directly to the suppression of the stretching term, and iteratively causing the sustained overall attenuation of streamwise vortex generation. By $y_d^+ = 28$ control, however, the stretching term keeps similar strength as that in no-control case, but with more extreme points originating from the interaction between the strong wall blowing / suction and the streak meandering. The streamwise vortices generated under $y_d^+ = 28$ control are similar to those in no-control case in early time, but are split into smaller and shorter fractions in later time due to the multiple extreme regions in stretching term distribution within the primary elongated streamwise vortices.

Although the vertical velocity imposed at the wall by $y_d^+ = 28$ control is opposite in sign to that by $y_d^+ = 15$ control in the linear stage, after the vortices are generated, the wall blowing/suction determined by the signals sensed at $y_d^+ = 15$ and 28 are in the same phase but with different strength. It is reasonable to suppose that the opposition control using the signal at $y_d^+ = 28$ but with a reduced strength can also suppress turbulence through the attenuation of the already-existing streamwise vortices. To test this conjecture, we performed the simulations to the minimal channel flow with $v_{wall} = -0.5v_{y_d}^{+28}$. It is found that the lessened opposition control with $y_d^+ = 28$ cannot effectively suppress the generation of streamwise vortices in the first and second stage just as $y_d^+ = 15$ control does, as can be expected, but exhibit its effects in turbulence stage. We use the time history of the plane-averaged wall skin friction to signify the turbulence levels in the channel, see Fig.9(a). For the lessened $y_d^+ = 28$ control, it follows the curves of no-control and $y_d^+ = 15$ control when $t^+ < 60$, and then grows slightly higher than the no-control case until $t^+ = 260$. After that it differs from the no-control case, and evolves towards the $y_d^+ = 15$ control case, and hence our conjecture is confirmed. That is because the the lessened opposition control with $y_d^+ = 28$ can only take effect to the already-existing streamwise vortices, while the $y_d^+ = 15$ control can interfere in the generation process of the vortices. We also tested the lessened $y_d^+ = 28$ control in the full-scale turbulent channel flow at $Re = 180$, and compared with the $y_d^+ = 15$ control, see Fig.9(b) for the time evolution of wall skin friction. It can be seen that in the full-scale turbulent channel flow, the lessened $y_d^+ = 28$ control is still effective, as is supposed to, and the skin friction is reduced by about 10%, in comparison with the 22% reduction in skin friction achieved by $y_d^+ = 15$ control.

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**REFERENCES**


**Figure 9.** Time history of the plane-averaged wall shear stress. (a) minimal channel; (b) full-scale channel.