GENERALISED OPTIMAL PERTURBATION APPROACH APPLIED TO DRAG REDUCTION BY WALL OSCILLATIONS IN TURBULENT FLOWS

Olivier Blesbois
Department of Aeronautics
Imperial College London
Prince Consort Rd., London SW7 2AZ
olivier.blesbois08@imperial.ac.uk

Sergei I. Chernyshenko
Department of Aeronautics
Imperial College London
Prince Consort Rd., London SW7 2AZ
s.chernyshenko@imperial.ac.uk

ABSTRACT

Drag reduction by wall oscillations in turbulent flows has recently been shown to be a promising technique. The reduction of the near wall streaks amplitude is known to play a significant role in the drag reduction mechanism. To gain a better understanding of the effect of wall oscillations on the streaks, the Generalised Optimal Perturbation (GOP) approach, based on the linearised Navier-Stokes equation, is used. Resemblance between drag and certain quantities arising in the GOP context is observed. It is found that for harmonic wall oscillations the streaks have an approximately constant angle to the main flow direction, with a jump in sign twice in the period. The mechanism of this phenomenon is clarified. The results are in a reasonable agreement with direct numerical simulations.

INTRODUCTION

Recent results have shown that the turbulent friction drag reduction by transverse wall oscillations is a promising technique. It has been shown that drag reduction of up to 40% and energy gain up to 18% can be achieved (Quadrio et al., 2009). However, the mechanisms leading to drag reduction are not yet well understood. To make a step towards improving the understanding of these mechanisms, this work focuses on the organised structures in turbulent flow past a transverse-oscillating wall and particularly on the near-wall streaks.

The near wall streaks in turbulent flow are well known to have an important role in sustaining turbulence. Streaks are structures elongated in the streamwise direction and composed of regions of low instantaneous velocity as compared to its mean value. In the present work the Generalised Optimal Perturbation (GOP) approach is used to study the near-wall streaks in turbulent flow with wall oscillations. The GOP approach was developed by Chernyshenko and Baig (2005) and used to explain the existence of streaks and to predict their spanwise spacing in turbulent channel flow without wall oscillations. The flow past an oscillating wall is considerably more complicated, and whether GOP can predict some of the features of the streaks in this case is an interesting question. Another important question is whether there is a correlation between the parameters of the solutions arising in GOP and the drag reduction, since, if such a correlation is confirmed, GOP can be used to improve the drag reduction techniques. The present work answers the first question and gives a preliminary indication of what the answer to the second question can be.

GOP APPROACH

For clarity, we will consider the case of a fully developed turbulent flow in a plane channel, as usually calculated in direct numerical simulations, that is in a finite domain with periodic boundary conditions. Generalisation of GOP to many other cases is straightforward.

The GOP theory is based on a particular idea of the reason why the linearised part of the Navier Stokes equation has the well-known ability to predict some of the turbulent structures, and particularly the near wall streaks in fully turbulent flows. If the turbulent velocity $\mathbf{u}$ is represented as the sum of the mean flow $\mathbf{U}$ and a fluctuation $\mathbf{u}'$, the Navier-Stokes equation can be written as

$$\frac{\partial \mathbf{u}'}{\partial t} + (\mathbf{U}.\nabla)\mathbf{u}' + (\mathbf{u}'.\nabla)\mathbf{U} + \nabla p - \frac{1}{Re}\Delta \mathbf{u}' = \mathbf{F} \qquad (1)$$

where the left-hand side is the linearised Navier-Stokes operator and $\mathbf{F}$ contains all the remaining terms including the terms nonlinear in $\mathbf{u}'$. The main idea of GOP is that the linear part of (1), on the left hand side, has strong enough selectivity properties to predict the streaks almost independently of the form of the right hand side $\mathbf{F}$. If the nonlinear term $\mathbf{F}$ is replaced by any reasonable representation, the streaks will still be present as a result of the simulation. Once this observation is made, only the selectivity properties of the linear operator need to be studied in order to predict the streaks. GOP then assumes that for the quantity of interest given (say, for the spanwise spacing of streaks at a certain distance from the wall) the selectivity properties will be reflected by the solution of the optimisation problem where the appropriate measure of the magnitude of the solution of (1) is maximized over all possible forms of $\mathbf{F}$ of a unit amplitude. Crucially, the measure of the solution should be selected in such a way that it measures the amplitude of that part of $\mathbf{u}'$ that determines the quantity of interest (thus, for streak spacing at a given distance to the wall this might be the energy $||\mathbf{u}'||^2$ averaged over a plane at this distance to the wall.)

The same simplification as in (Chernyshenko and Baig, 2005) is used in the present study, namely, instead of optimising over all possible $\mathbf{F}(t,\mathbf{x})$ the optimisation is performed only over right hand-sides of the form $\mathbf{F}=\mathbf{u}'_0(\mathbf{x})\delta(t-t_i)$. Then the optimal perturbation is the solution of an initial value problem for (1) with $\mathbf{F}=0$ such that it experiences the biggest possible transient growth (for such optimal perturbations to exist it is necessary that the base profile $\mathbf{U}$ is linearly stable). The perturbation starts at an initial time t_i, and its maximum magnitude will be attained at the "final time" t_f. To define the magnitude a quantity has to be used to measure the amplitude of the perturbation at any time $t>t_i$. The problem being linear, this quantity will be the ratio of a certain norm of the solution taken at time t to the (in general different) norm of the initial condition imposed at the initial time t_i. In the present study, as the structures to be predicted are the near wall streaks, and as their characteristics vary with the distance to the wall, a volume energy norm $\|\ \|_i$ is used for the initial condition, and a surface energy norm $\|\ \|_f$ is used for the solution. If x, y, and z are the streamwise, wall normal and spanwise coordinates respectively, with the corresponding velocity vector $\mathbf{u}'=(u,v,w)$, the initial and solution norms are:

$$\|\mathbf{u}'\|_i=\frac{1}{V}\int_V\left(u^2+v^2+w^2\right)dx\,dy\,dz \tag{2a}$$

$$\|\mathbf{u}'\|_f=\frac{1}{S}\int_{y=y_0}u^2\,dx\,dz \tag{2b}$$

where V is the volume of the flow domain and S is the area of the cross-section $y=y_0$ of the flow domain.

The choice of the surface energy norm as the solution norm means that more importance is given to the structures which have high energy in the plane $y=y_0$ and which, therefore, will dominate the flow in this plane. If, instead, both the solution norm and the initial value norm were chosen as the volume average of $\mathbf{u}'^2$, the resulting problem would be the widely known optimal perturbation problem, but then there would be no reason for the corresponding structure to dominate in the particular plane $y=y_0$. The GOP at a given distance to the wall is a solution of the maximisation problem

$$A(t_f)=\max_{\mathbf{u}'(t_i),t_i}\frac{\|\mathbf{u}'(t=t_f)\|_f}{\|\mathbf{u}'(t=t_i)\|_i} \tag{3}$$

The optimisation is performed here over all possible initial conditions and all possible initial times t_i. If the base profile is independent of time then the optimal perturbation is essentially independent of t_f. This was the case in (Chernyshenko and Baig, 2005), but the case of oscillating wall is more general. The structure predicted by the GOP is considered as the most probable streak at time t_f. Of course other structures will also be present in a real flow, but the flow structure in each of the planes $y=y_0$ will resemble the most prominent features of the corresponding GOP.

GOP in the spanwise-oscillating wall case

To apply the GOP approach to the flow with wall oscillations the mean velocity profile $\mathbf{U}$ was taken to be the sum of the mean profile in a turbulent flow in a plane channel along a non-oscillatory wall (Reynolds and Tiederman, 1967) and the time-dependent velocity profile corresponding to laminar Stokes layers due to the spanwise-oscillating walls of the channel oscillating in phase. Using Stokes layer as a representation of the phase-averaged spanwise velocity components is justified by the observations of the behaviour of the solution obtained using direct numerical simulations (DNS) for regimes when the drag reduction is substantial, see the companion paper (Touber and Leschziner, 2011). The transverse component of the wall velocity is given by the formula

$$w_{wall}=W_m\cos(2\pi t/T) \tag{4}$$

where T is the period of oscillations. The resulting phase-averaged mean velocity depends on time and the wall-normal coordinate, but is independent of x and z.

The optimisation for (3) is done in two steps. First, t_i and t_f are fixed, and the most amplified initial perturbation $\mathbf{u}'(t_i)$ for this set of parameters is calculated. Then the optimisation over t_i is performed. The first step of the optimisation is calculated using an adjoint optimisation algorithm. A description of the adjoint optimisation procedure is not given here, as an abundant literature on this subject exists (Hill, 1995; Farrell and Moore, 1991). The turbulent incompressible Navier-Stokes solver (Laizet and Lamballais, 2009) was modified to solve the linearised and adjoint equations and to perform the adjoint optimisation.

Since the mean flow is independent of the transverse coordinate, the solution of the linearised Navier-Stokes equation can be expanded into the Fourier series in z, and the numerical problem can be solved in 2D for each wavelength in z. The adjoint loop finds the optimal solution for given initial time t_i, final time t_f, and spanwise wavelength λ_z. Then, the second step the optimisation is performed over t_i and λ_z. The code was thoroughly validated by comparisons with Chernyshenko and Baig (2005) and by other means.

For the wall velocity given by (4) a similar decomposition is also possible in x. Hence, the optimal solution should have specific wavelengths both in transverse and streamwise directions. Therefore, the GOP will be an infinitely long structure in a direction that has a given angle with the mean flow axis.

RESULTS

The GOP was calculated for different oscillation periods and Reynolds numbers. The results are compared with DNS results obtained in the equivalent configuration by Touber and Leschziner (2011). Three cases are considered: the baseline case of no wall oscillation, the case with a period of oscillation $T^+ = 200$,[1] in which case prominent organised structures and moderate drag reduction are observed in DNS, and the case $T^+ = 100$, for which the drag reduction is close to the maximum but the structures can hardly be seen.

Comparison of the oscillating and non-oscillating wall cases

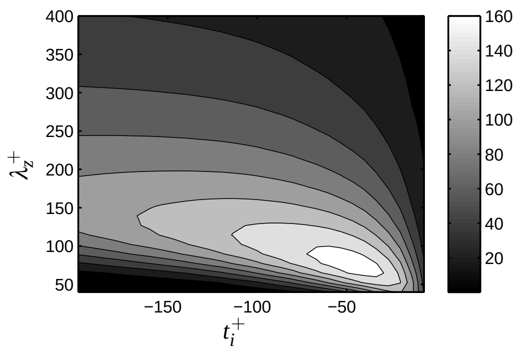


Figure 1. Maximum ratio $\|\mathbf{u}'\|_f/\|\mathbf{u}'_0\|_i$ after adjoint optimisation, $t_f = 0$, $Re_\tau = 180$, wall distance $y_0^+ = 11$, non-oscillating wall case.

In the case of non-oscillating wall the optimal perturbation is the same (up to a shift in time) for any t_f, but in the case of an oscillating wall optimal perturbations for different values of t_f differ. To illustrate an important point, we first arbitrarily fix $t_f = 0$, thus determining the phase of the wall oscillation at which the final time is. The features we are going to discuss are similar for all values of t_f. An adjoint optimisation is performed for a range of values of the initial time $t_i < t_f$ and the spanwise wavenumber λ_z. The obtained maximum ratio $\|\mathbf{u}'\|_f/\|\mathbf{u}'_0\|_i$ as a function of t_i and λ_z is plotted in figure 1 for non-oscillating wall and in figure 2 for oscillating wall. The GOP corresponds to the points on these plots at which the ratio $\|\mathbf{u}'(0)\|_f/\|\mathbf{u}'_0\|_i$ attains its maximum. For non-oscillating wall the maximum ratio is about 180 and it is attained at $(\lambda_z^+, t_i^+) = (80, -50)$. Note how different the gain plots are. In the non-oscillating wall case there is only one well defined maximum which corresponds to the GOP. In the oscillating wall case the figure shows two local maximums. The highest one corresponds to the GOP, but the maximum ratio for the second one is not much different from the first. This means that even if the structure corresponding to the GOP is more likely to be seen in the turbulent flow, the likelihood to see the structure corresponding to the second maximum is close to the likelihood of seeing the GOP. Hence, at t equal to this t_f it is likely that in the turbulent flow two structures will be seen at the same time; the one corresponding to the GOP and the one corresponding to the second maximum. Many simulations have been run in wall oscillation configurations with an oscillation period of $T^+ = 100$ and $T^+ = 200$, for different distance to the wall and for different t_f. This kind of behaviour with two maximums is found in many cases, and the presence of two dominant structures will be compared with DNS results in a later section.

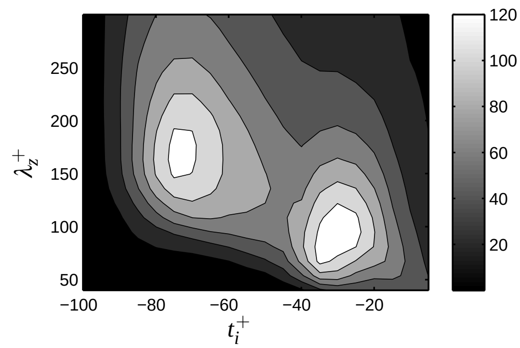


Figure 2. Maximum ratio $\|\mathbf{u}'\|_f/\|\mathbf{u}'_0\|_i$ after adjoint optimisation, $t_f = 0$, $Re_\tau = 180$, $y_0^+ = 11$, oscillations period $T^+ = 100$, and amplitude $W_m^+ = 12$.

Correlation with drag reduction

It is well known that when drag reduction is achieved by wall oscillations streaks also become weaker. However, while GOP was successful in predicting the existence and spanwise scale of the streaks (Chernyshenko and Baig, 2005), its ability to predict the response of the streak amplitude to wall oscillation and thus to predict drag reduction is far from obvious. Indeed, since GOP is a solution to a homogeneous problem for a linear equation, its amplitude is arbitrary. If one would nevertheless try to find a link between GOP and drag one might expect that there will be a correlation between the drag and the maximum amplification factor $\|\mathbf{u}'\|_f/\|\mathbf{u}'_0\|_i$. The results obtained in the present study show that this is definitely not the case. For example, $\|\mathbf{u}'\|_f/\|\mathbf{u}'_0\|_i = 180$ for the non-oscillating wall case in figure 1, whereas in the oscillating wall case, the maximum ratio can be sometimes as high as 400. Comparing figures 1 and 2 suggests an explanation: the maximums in figure 2 are narrower. If there could be a link between optimal perturbations and drag it should be a link between the drag and some integral characteristics of optimal perturbations rather than the height of a local maximum. Also, averaging in t_f of the amplification factor over a period of the wall oscillation should be made. This implies introducing a measure

$$A_{total} = \frac{1}{T}\int_{t_f=0}^{t_f=T}\int_{t_i=-\infty,\lambda_z^+=0}^{t_i=t_f,\lambda_z^+=\infty} \|\mathbf{u}'\|_f/\|\mathbf{u}'\|_i \, d\lambda_z^+ \, dt_i \, dt_f \quad (5)$$

the behaviour of which can be compared with the behaviour of the drag. It turns out that in the three cases calculated in the present study the behaviour is indeed similar: $A_{total} = 6.4 \cdot 10^4$

[1]Subscripts $^+$ denotes quantities expressed in wall units (non-dimensional units based on wall shear, density and viscosity)

and the drag reduction is 0% for non-oscillating case, $A_{total} = 5.1 \cdot 10^4$ and the drag reduction is 24.6% for $T^+ = 200$, and $A_{total} = 2.5 \cdot 10^4$ and the drag reduction is 31.7% for $T^+ = 100$, that is a decrease in A_{total} corresponds to a decrease in drag. This, however, is only the very first preliminary result, and the appropriate way of estimating drag reduction on the basis of GOP, if such a way exists, is yet to be found.

Note that the drag reduction values given above were obtained from DNS by Touber and Leschziner (2011) for $Re_\tau = 500$, while most of the GOP calculations described here are for $Re_\tau = 180$. However, for the value of y_0^+ considered here GOP, if expressed in wall units, appears to be almost independent of Re_τ.

Energy spectrum and GOP

The characteristic transverse scaling of the structures in a turbulent flow can be estimated by considering a pre-multiplied energy spectrum $\Phi(\lambda_z) = \lambda_z E(\lambda_z)$ at a given distance to the wall, where E is the standard energy spectrum depending on the transverse wavelength λ_z. In the case of non-oscillating wall and for small distances to the wall, $\Phi(\lambda_z)$ has usually a maximum around $\lambda_z^+ = 100$, which corresponds to the dominant structure at this scale, the near wall streaks. In the case of an oscillating wall it makes sense to consider the phase-averaged spectrum, which then is a periodic function of time. This provides an interesting opportunity for comparisons with GOP, as it also depends on the time t_f, so that even if the amplitude of the solution to the linearised equation is determined up to a constant factor, its behaviour as a function of time can be compared. The small width of the peaks, of course, represents the same difficulty as in the previous subsection, and it can be resolved by similar means, namely, by introducing

$$A_{GOP}(\lambda_z, t_f) = \int_{t_i < t_f} A_0(t_i, \lambda_z, t_f) dt_i \tag{6}$$

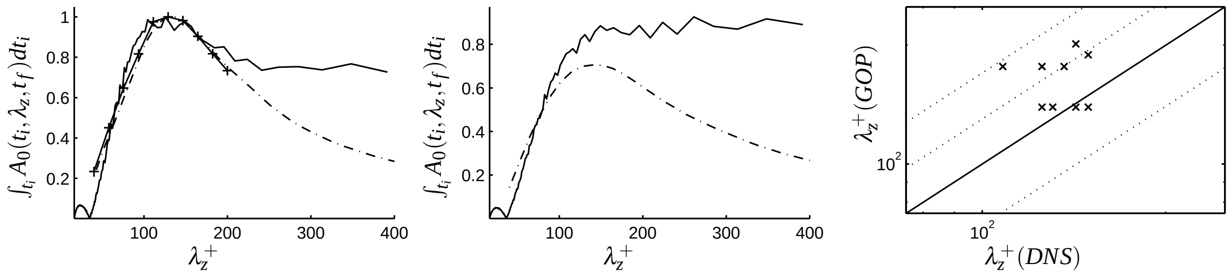
with $A_0(t_i, \lambda_z, t_f) = \max_{\mathbf{u}'_0}(\|\mathbf{u}'\|_f / \|\mathbf{u}'_0\|_i)$ being the result of an adjoint optimisation for the given t_i, t_f, and λ_z. This quantity is plotted as a function of the transverse wavelength λ_z together with $\Phi(\lambda_z)$ scaled for convenience of comparison. Numerous such plots were made for different values of t_f and y_0^+ for each of the cases $T^+ = 100$ and $T^+ = 200$. Two representative examples are given in figures 3(a) and 3(b). Figure 3(a) shows the behaviour typical for $T^+ = 200$ and for some part of the period for $T^+ = 100$. In these cases the streaks are well defined and a maximum in the pre-multiplied spectrum corresponding to these structures is always present around $\lambda_z^+ = 200$. For the oscillations at $T^+ = 100$, the streaks are more difficult to see in the DNS, and at some points in the period they seem to disappear completely. Figure 3(b) is representative for these cases. In figure 3(a) there is a clear local maximum of the pre-multiplied spectrum at relatively small scales, but in figure 3(b) the pre-multiplied spectrum increases with λ_z within the plot range, showing that a large part of the energy is concentrated in structures having much larger scale than the typical streak spacing. GOP, on the other hand, does not show this, may be because the mechanism responsible for generation of large-scale structures is different from the mechanism of formation of near-wall streaks, so that GOP does not describe it. From these two examples, we expect that the GOP and DNS results should be compared only in the case when there is a local maximum in the DNS pre-multiplied spectrum (as in figure 3(a)), but no comparison will be possible when there is no local maximum in the pre-multiplied spectrum (as in figure 3(b)).

Once again it is worth to point out that comparing quantities related to the amplification factor in GOP with quantities related to the magnitude of the turbulent fluctuations, as it is done in this section and the previous section, is new and quite different from comparing quantities linked only to the form but not the amplitude of the linearised solution, as for example the comparison between the spanwise period of GOP and the observed streak spacing. The comparisons of this and the previous section are only tentative, but the results appear to be promising, and further research in this direction is needed.

Figures 3(a) and 3(b) represent only a small fraction of such comparisons. In order to give a general idea of the degree of agreement or disagreement between the GOP and DNS curves in these figures we first discard all those cases when the DNS results do not exhibit the local minimum. Then we determine the values of λ_z at which the maximums are attained in the GOP and DNS curves, and plot all such values against each other. The result is shown in figure 3(c). If the positions of the maximums of the GOP and DNS curves were exactly the same, all the points would lie on the diagonal shown with the solid line. One can see, however, that the discrepancy can be as large as 70%. The positions of these maximums can be considered as a measure of streak spacing, and the agreement for streak spacing was noticeably better in (Chernyshenko and Baig, 2005). This might be due to several reasons. First, the mean profile of the longitudinal velocity used in the present work is the mean profile of the flow past a non-oscillating wall, while the prediction is attempted for the case of an oscillating wall. Second, Chernyshenko and Baig (2005) used the weighted initial condition norm requiring the knowledge of the normal Reynolds stresses in the flow. Third, the parameter compared in Figure 3(c) is based on the maximum of A_{GOP} as given by (6), rather than on the wavelength of GOP, as it were in (Chernyshenko and Baig, 2005). This last point is technical, but the first two are the result of the decision of using only the information on the non-oscillating wall flow for predictions about the oscillating-wall case.

Comparison for streak angle

After comparing the flow globally, it is interesting to focuss on the most probable streak itself, and particularly its evolution in time. As it has already been discussed, in the case of oscillating wall the GOP is an infinitely long structure inclined at a certain angle to the main flow direction in $x-z$ plane. DNS at $T^+ = 200$ shows streaks with a relatively well defined angles at most phases in the period (Touber and Leschziner, 2011). In the case $T^+ = 100$ the streaks are more difficult to see. To obtain the streak angle at a given distance to the wall, the distance between streaks in the streamwise and spanwise direction is obtained by extracting the local maximums corresponding to the streak in the streamwise and



(a) DNS pre-multiplied spectrum (-), GOP (-+). $T^+ = 100$, $y^+ = 11$, $t_f^+ = 40$.

(b) $T^+ = 100$, $y^+ = 6$, $t_f^+ = 10$

(c) Comparison of the characteristic transverse scales. The dashed lines correspond to 30% and 70% error margins.

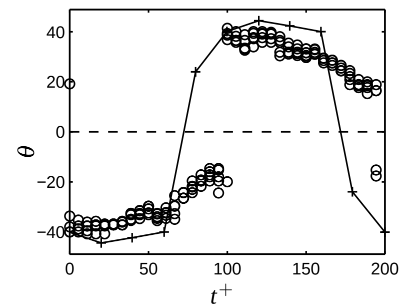
Figure 3. Comparison between the premultiplied energy spectrum $\Phi = E(\lambda_z)\lambda_z$ and the correspoding GOP parameter $A_{GOP} = \int_{t_i} A_0(t_i,\lambda_z,t_f)dt_i$. The maximum of each function represents the streaks spacing. In (c) the positions of the maximums for the DNS and the GOP are compared.

spanwise pre-multiplied energy spectra calculated in DNS[2]. Then a simple calculation gives the magnitude of the angle, and the sign of the angle has to be visually confirmed. As in the previous section, it is not always possible to extract the angles, particularly when there is no local maximum in the pre-multiplied spectrum, which explains the gaps in the DNS data in figures 4(a) and 4(b). These figures give two representative cases, with comparisons between GOP and DNS for other distances to the wall and periods of oscillations giving various degrees of agreement in between the levels of agreement in these two figures.

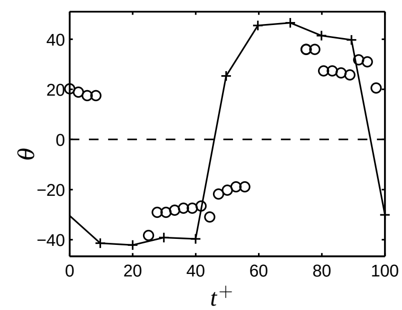
An obvious comment can already be made. In the case $T^+ = 100$, there are more phases in the period where the angle cannot be properly extracted. This is directly linked to the fact that in more cases in the $T^+ = 100$ simulations the pre-multiplied spectrum doesn't have a local maximum. Because of this issue, we will concentrate more on the observations in the case $T^+ = 200$, as the comparison are more meaningful.

To obtain the angles of the most probable streaks using the GOP approach, the GOP has to be calculated for different t_f over the period. As the mean flow is symmetric over half a period, there is also symmetry of the structures predicted by the GOP over half a period. Because of that, it is possible to calculate the result over only the first half of the period, and the value of the angle will be opposite during the next half period. The angles plotted in figures 4(a) and 4(b) correspond to the GOP for each t_f, that is to the higher of the peaks in figures similar to figure 1.

Figure 4 shows that in both the DNS simulations and GOP prediction, the streak angle remains relatively close to two values of the same magnitude but of opposite sign, and that at some phase in the period the streak angle jumps between these two values. There is a relatively good agreement between the angles predicted and calculated, but the more important fact is that the position of the jump is predicted with a relatively good accuracy by the GOP. As the jump exists in the DNS results and in the GOP predictions and is at a similar position, it shows that some of the important physical mechanisms leading to the streaks formation are captured by the GOP. The jump in the GOP result might be surprising, but it can relatively easily be explained by looking at figure 2. In this figure $t_f = 0$, which is just after the jump. It has already been pointed out that this figure shows two local maximums corresponding to two dominant structures, one being the GOP, and the other one being less likely than the GOP to be seen but with a large enough energy growth to be a very probable structure. In the figure for the slightly earlier t_f there also would be two local maximums, but the GOP would correspond to another maximum. This shows that each maximum presents a dominant structure, and depending on which maximum is higher the GOP will be one or the other of these dominant structures. The jump happens when the global maximum



(a) Simulation for $T^+ = 200$, observation at a distance to the wall $y^+ = 11$



(b) Simulation for $T^+ = 100$, observation at a distance to the wall $y^+ = 11$

Figure 4. Angles extracted from DNS simulations (o), and calculated using the GOP approach (+-)

[2]This was done in collaboration with Prof. M.A. Leschziner and Dr. E Touber (Imperial College London), whose work is also presented at this conference.

switches from one local maximum to the the other. From this result, the two dominant structures would be expected to be present in the DNS around the position of the jump. Two structures present at the same time can be expected to interfere and make the DNS picture difficult to analyse. This could explain why the angle of the structures cannot be calculated with a good accuracy in the DNS around the position of the jump.

There remain to be explained why the angle remains approximately constant between the jumps. To do this we will analyse the growth pattern of GOP corresponding to different values of t_f. In figure 5, the growth patterns for several

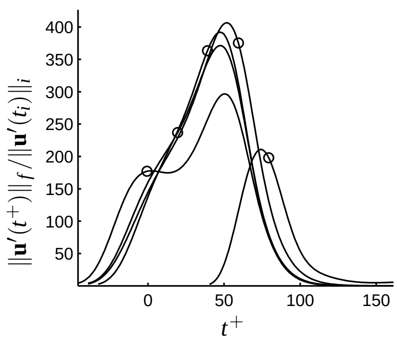


Figure 5. Growth pattern for the GOP for several values of t_f. Circles are place on each curve at the corresponding t_f. $T^+ = 200$ and $y_0^+ = 11$.

GOPs in the first half-period are given. It can be seen that for three of them the growth pattern is similar. Even though the values of t_f differ, the initial instants of these GOPs are within a rather narrow area, and the entire curves are close. This shows that the structures predicted for these different final times are essentially the same. As the same structure is present for almost half a period the angle remains almost constant and there is a discontinuity at the instant when the dominant structure switches from one to the other. More detailed analysis, which cannot be presented here because of the space limit showed that the period of intensive growth of this structure corresponds to the period when the phase-averaged mean flow has large wall-normal gradient of the velocity component parallel to the direction in which the structure is elongated while at the same time the wall-normal gradient of the velocity component parallel to the wall but transverse to the structure is small.

CONCLUSION

The Generalised Optimal Perturbation (GOP) approach of Chernyshenko and Baig (2005) was shown to give promising results for turbulent channel flow in the presence of transverse wall oscillations in the regime when these oscillations give drag reduction.

The calculated values of a certain measure of the streak energy obtained in the GOP context for three cases (the non-oscillatory wall case and two cases of wall oscillations) and the drag values calculated in DNS for these cases correlate.

The GOP approach predicts that in the flow past an oscillating wall the near wall streaks are elongated structures inclined to the streamwise direction at a certain angle. The angle remains approximately constant during certain parts of the period and suddenly switches sign twice in the period. The angle jump position and the value of the angle are in a reasonable agreement with direct numerical simulations.

The streak angle remains roughly constant between the jumps because there is a relatively short part of the period when a significant amplification of a particular initial perturbation occurs, so that this perturbation dominates over a large part of the remaining time within the period.

Aknowledgement This work was supported by EPSRC UK under the grant EP/G061556/1, together with Airbus operations Ltd and EADS UK Ltd. We are also grateful to Imperial College HPC service facilities for the access to the high performance computer cx1.

REFERENCES

Chernyshenko, S. I. and Baig, M. F. (2005). The mechanism of streak formation in near-wall turbulence. *Journal of Fluid Mechanics*, 544(-1):99–131.

Farrell, B. F. and Moore, A. M. (1991). An adjoint method to obtain most rapidly growing perturbations in oceanic flows. *Journal of Physical Oceanography*, 22:338–349.

Hill, D. C. (1995). Adjoint systems and their role in the receptivity problem for boundary layers. *Journal of Fluid Mechanics*, 292(-1):183–204.

Laizet, S. and Lamballais, E. (2009). High-order compact schemes for incompressible flows: A simple and efficient method with quasi-spectral accuracy. *Journal of Computational Physics*, 228(16):5989 – 6015.

Quadrio, M., Ricco, P., and Viotti, C. (2009). Streamwise-travelling waves of spanwise wall velocity for turbulent drag reduction. *Journal of Fluid Mechanics*, 627(-1):161–178.

Reynolds, W. C. and Tiederman, W. G. (1967). Stability of turbulent channel flow, with application to malkus's theory. *Journal of Fluid Mechanics*, 27(02):253–272.

Touber, E. and Leschziner, M. (2011). Near-wall streaks modifications by spanwise oscillatory wall motion. *TSFP7, this volume.*