LES ONE-WAY COUPLING OF NESTED GRIDS USING SCALE SIMILARITY MODEL

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ABSTRACT

The method for coupling between nested grids is proposed for LES turbulent flows. In this method fluctuating velocity simulated in a coarse grid is imposed to a fine grid and the grid-scale velocity fluctuation of the fine grid is generated based on the scale similarity model. The a-priori test of a turbulent boundary layer flow over a rough surface is conducted to validate this method. In order to fulfill spatially developing turbulent boundary layer flows with no pressure gradient we apply the quasi-periodic boundary condition to the streamwise direction. In the test coarsely resolved velocity data which is generated filtering finely resolved LES data are applied for reproducing subgrid-scale components of the coarsely resolved LES. In this a-priori test the technique for the immersed boundary method is applied to force the averaged fluctuation velocity component to zero. The reproduced fluctuation velocity agrees well with the true value which can be derive by subtracting the generated coarsely resolved velocity data from the finely resolved LES data and the kinetic energy spectra of the reproduced streamwise fluctuation velocities fit to the -5/3 power law for the inertial subrange.

The one-way coupling method is also applied for a free convective boundary layer flow with no heat flux from the ground nesting coarse grid WRF-LES and fine grid LES. In this method velocity data simulated in WRF-LES is imposed to a fine grid LES and the grid-scale velocity fluctuation of the fine grid LES is generated based on the scale similarity model. Although the total kinetic energy spectra of the fine grid LES streamwise fluctuation velocity slightly underestimates the -5/3 power law for the inertial subrange in the high wavenumber region, the connection between the coarse grid WRF-LES and the fine grid LES works well without large interpolation errors.

INTRODUCTION

The grid nesting technique are used in many LES simulations, such as a flow around buildings in an atmospheric boundary layer flow, where the grid is refined at the region adjacent to the buildings, while the atmospheric boundary layer flow is simulated using coarsely resolved grids.

In the LES when the grid is suddenly refined at the interface of nested grids which is normal to the mean advection the resolved shear stresses decrease due to the interpolation errors and the delay of the generation of smaller scale turbulence that can be resolved on the finer mesh (Piomelli et al, 2006). The adjustment region is required to regenerate the high wavenumber part in the energy spectrum of the fluctuating velocity. In the grid nesting approach for LES the commutation errors are thought to be very significant near the nest inflow interfaces. The method of applying the one-way coupling method using the scale similarity model to get rid of these numerical errors at the interface between coarse grid and fine grid is proposed.

The fine grid velocity could be decomposed into mean and fluctuating components. In this method the mean velocity component of the fine grid is equal to the velocity of the coarse grid and the averaged fluctuating velocity component of the fine grid is zero. Meanwhile the high wavenumber fluctuating velocity component of the fine grid which is implicitly estimated as subgrid-scale turbulence in the coarse grid is reproduced explicitly in the fine grid near the nest inflow interface. The fluctuating components of the fine grid velocity could be estimated solving the momentum equations which could be derived by subtracting coarse grid filtered Navier-Stokes equations from fine grid filtered Navier-Stokes equations. We applied the scale similarity model to the subgrid scale Reynolds stresses and the Smagorinsky model is applied to the fine grid subgrid scale Reynolds stresses. The filter width of the coarse grid has to be in the inertial subrange of energy spectrum to predict the production terms in the momentum equations properly.

One-way Coupling Method

The Navier-Stokes equations which are filtered with the fine grid filter width ($\bar{\Delta}$) are written as below:

$$\frac{\partial \bar{U}_i}{\partial t} + \frac{\partial \bar{U}_i \bar{U}_j}{\partial x_j} = -\frac{\partial \bar{P}}{\partial x_i} - \frac{\partial \bar{\tau}_{ij}}{\partial x_j} \qquad (1)$$

where $\bar{U}_i$ is a velocity in ith-direction filtered with fine grid filter; $\bar{P}$ is a filtered pressure; and $\bar{\tau}_{ij}$ is subgrid scale Reynolds stresses. In this filtered Navier-Stokes equations, the diffusion terms is neglected because of high Reynolds number. The filtered Navier-Stokes equations for coarser grid system which are filtered with filter width ($\widetilde{\Delta}$) are written as below:

$$\frac{\partial \widetilde{U}_i}{\partial t} + \frac{\partial \widetilde{U}_i \widetilde{U}_j}{\partial x_j} = -\frac{\partial \widetilde{P}}{\partial x_i} - \frac{\partial \tilde{\tau}_{ij}}{\partial x_j} \qquad (2)$$

where $\widetilde{U}_i$ is a filtered velocity in ith-direction; $\widetilde{P}$ is a filtered pressure; and $\tilde{\tau}_{ij}$ is subgrid scale Reynolds stresses. We assume the filter width $\tilde{\Delta}$ is much larger than that of the fine grid filter ($\bar{\Delta}$). The filtered velocity $\bar{U}_i$ can be decomposed into $\overline{\widetilde{U}}_i$ and u_i as below:

$$\bar{U}_i = \overline{\widetilde{U}}_i + \bar{u}_i \qquad (3)$$

The velocity $\overline{\widetilde{U}}$ is fluctuating very slowly due to the filter width $\bar{\tilde{\Delta}}\left(= \sqrt{\tilde{\Delta}^2 + \bar{\Delta}^2}\right)$. The velocity $\bar{u}_i$ is a fluctuation component which consists with subgrid scale turbulence of the coarser grid system. If the difference between the two filter widths is large enough we can obtain the equation:

$$\bar{u}_i \approx \bar{U}_i - \tilde{U}_i \qquad (4)$$

By using this relation, we can extract the equations of motion for fluctuation velocity components $\bar{u}_i$ by subtracting equation (2) from equation (1).

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial}{\partial x_j}\{\bar{U}_i\bar{U}_j - \widetilde{U}_i\widetilde{U}_j\} = -\frac{\partial(\bar{P} - \widetilde{P})}{\partial x_i} - \frac{\partial(\bar{\tau}_{ij} - \tilde{\tau}_{ij})}{\partial x_j} \qquad (5)$$

These equations could be solved in the same manner to solve the Navier-Stokes equations by adding an equation of continuity for the fluctuation velocity. Additional restriction that time-space average of fluctuation velocity components $\bar{u}_i$ is zero will make averaged $\bar{U}_i$ follow the mean value of $\widetilde{U}_i$. The technique for the immersed boundary method proposed by Goldstein et al (1993) is applied to force the averaged fluctuation velocity component $\bar{u}_i$ to zero. Goldstein et al proposed the method to determine the velocity at particular point to the desired value in an unsteady viscous flow. The body force f_i which is controlled by the feedback system is imposed to the equation (5):

$$f_i(t, x_i) = \alpha \int_0^t err(\tau, x_i) d\tau + \beta err(t, x_i) \qquad (6)$$

.

The quantities α and β are negative constants and $err(t, x_i)$ is a residual error between the simulated velocity and the determined velocity. The error being feed back is the velocity integral and the velocity itself. In this study we applied this feedback system to control u_i to zero. The quantities are $\alpha = -2000$ and $\beta = -30$ in this study.

The subgrid scale Reynolds stresses implemented in coarser grid Navier-Stokes equation (2) is the production term in equation (5). In this method the scale similarity model (Bardina et al, 1980) is applied to the subgrid scale stresses: $\tilde{\tau}_{ij}$ and the Smagorinsky model is applied to the subgrid scale stresses: $\bar{\tau}_{ij}$ in equation (1). The filter width of the coarse grid has to be in the inertial subrange of energy spectrum to predict the production terms in equation (5) properly.

A-PRIORI TEST

In order to validate this approach an a-priori test is carried out. A turbulent boundary layer flow over a rough surface is simulated using LES (Figure 1). The simulated velocity data is filtered to generate coarser grid data in the a-priori test. The filter width $\tilde{\Delta}$ of the low-pass filter to generate the coarser grid data is two times as large as the filter width $\bar{\Delta}$ of the LES simulation. Then the low-pass filtered coarser grid data $\widetilde{U}_i$ is introduced into the equations (5) to derive the finer grid components of velocity fluctuations $\bar{u}_i$ (Figure 2). Then the simulated velocity fluctuations $\bar{u}_i$ could be compared with the estimated values from equation (4). The LES grid filtered velocities $\bar{U}_i$ in the equation (5) can be derived explicitly summing up a low-pass filtered velocity $\widetilde{U}_i$ and a fluctuation velocity component $\bar{u}_i$ at the last time step.

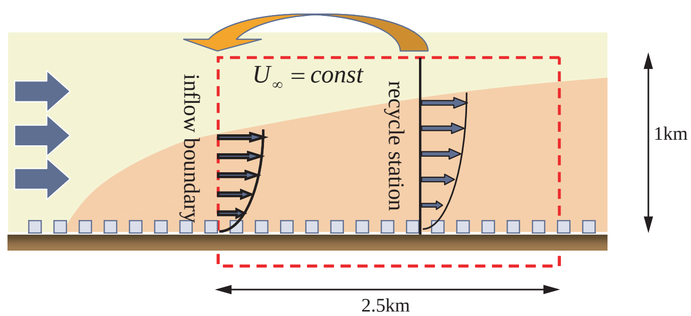


Figure 1. Computational region and concept of quasi-periodic boundary condition for a turbulent boundary layer flow.

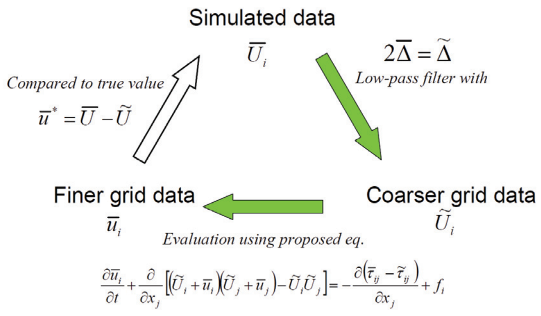


Figure 2. Schematic of the a-priori test.

Numerical Method

The computational region is 2.5km length, 1km height and 0.5km width. Roughness blocks of the rough surface are simulated using the immersed boundary method (Goldstein et al, 1993). In order to fulfill spatially developing turbulent boundary layer flows with no pressure gradient we apply the quasi-periodic boundary condition to the streamwise direction (Nozawa and Tamura, 2001). In this method the flow data at the recycle station are rescaled and re-introduced to the inlet of the computational region. The outflow boundary layer is set downstream of the recycle station. The free stream velocity above the boundary is U_0 and the periodic boundary condition is set to the spanwise boundary. The boundary conditions to solve equation (5) are that except the spanwise boundary conditions the fluctuation velocities are zero at the boundary and no quasi-periodic boundary condition is applied to the streamwise direction.

In this study the Adams-Bashforth method is applied to the convection term and the pressure term of the equation (5) could be derived using the projection method. The fourth-order central differencing scheme is applied to the convection and other remaining terms.

Results

Figure 3 shows the time history of low-pass filtered non-dimensional streamwise velocity $\widetilde{U}$ of the turbulent boundary flow over rough surface at height 100m. By imposing this filtered velocity to the equation (5) and by solving the equation the fluctuation component velocity $\overline{u}$ could be derived. The simulated fluctuation component velocity $\overline{u}$ has high frequency oscillation compared to those of the low-pass filtered velocity $\widetilde{U}$ (Figure 4). Time averaged velocity of fluctuation component $\overline{u}$ is almost zero and low frequnecy component in $\overline{u}$ is damped compared to the low-pass filtered streamwise velocity $\widetilde{U}$. These results indicate that the feedback system of the equation (6) works well to reduce the low frequency oscillation. The simulated fluctuation component velocities $\overline{u}$ agree well with the true value which can be derived by subtracting the generated coarsely resolved velocity data from the finely resolved LES data $\overline{U} - \widetilde{U}$. Not only the amplitude of the oscillation but also the phase of the simulated $\overline{u}$ coincides with true value $\overline{U} - \widetilde{U}$.

Figure 5 shows the streamwise velocity contours of $\widetilde{U}$ and $\widetilde{U} + \overline{u}$. The large scale turbulence structures could be found in both figures of $\widetilde{U}$ and $\widetilde{U} + \overline{u}$. While the small scale

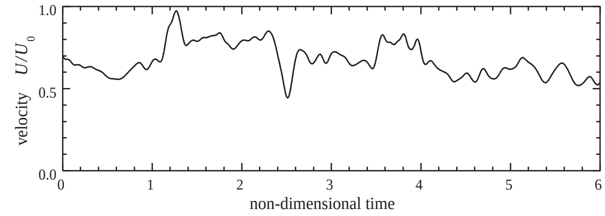


Figure 3. Time history of filtered streamwise velocities $\widetilde{U}$ for the a-priori test.

velocity fluctuations which could be found in the contour figure of $\widetilde{U} + \overline{u}$ are not seen in the contour figure of $\widetilde{U}$.

Figure 6 shows the kinetic energy spectra for streamwise velocity fluctuations at a height of 100m. The spectrum on coarser grid velocity fluctuations $\widetilde{U}$ has inertial subrange up to wavenumber 10km^{-1} and it decreases due to the explicit low-pass filter. The kinetic energy spectrum of the simulated fluctuation velocity component $\overline{u}$ enlarges in the range where the kinetic energy of the fluctuation coarsely resolved velocity $\widetilde{U}$ decreases and fits well with the simulated data $\overline{U}$ in the high wavenumber region. The total turbulence energy which could be obtained adding up the coarser grid turbulence energy $\widetilde{U}$ and the finer grid turbulence energy $\overline{u}$ extends its inertial subrange region and fits very well with the simulated one with the fine grid. From the a-priori test the one-way coupling method could connect coarser grid and finer grid reproducing the grid-scale turbulence in the finer grid.

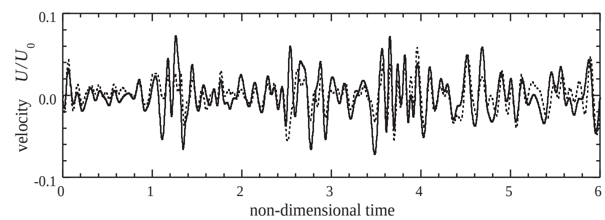
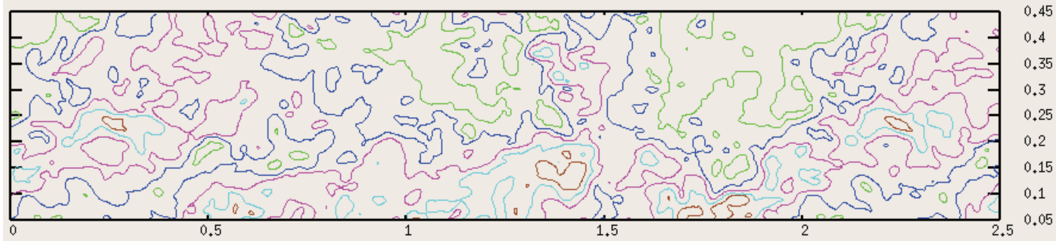


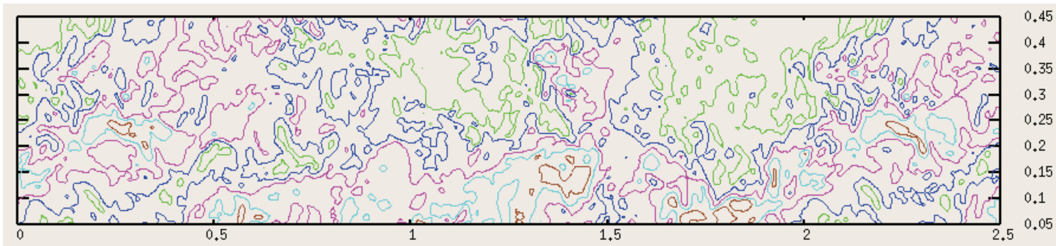
Figure 4. Time history of fluctuating streamwise velocity for the a-priori test, solid line: $\overline{u}$, dashed line: $\overline{U} - \widetilde{U}$.

A-POSTERIORI TEST

The results of the a-priori test indicated that the proposed one-way coupling method based on the scale similarity model could generate the small scale turbulence at the interface of the nested grids and smoothly connect the fine grid and the coarse grid. As the next step the one-way coupling method is applied for a turbulent flow nesting WRF-LES and LES in an a-posteriori test. The Advanced Research WRF is widely used mesoscale meteorological model and in this study we carry out an idealized free CBL (convective boundary layer) flow using WRF-LES mode. The velocity simulated in a coarse



(a) coarse grid streamwise velocity component, $\widetilde{U}$



(b) finely resolved streamwise velocity component, $\widetilde{U}+\overline{u}$

Figure 5. Streamwise velocity contours for the a-priori test at the horizontal section $z = 100$m.

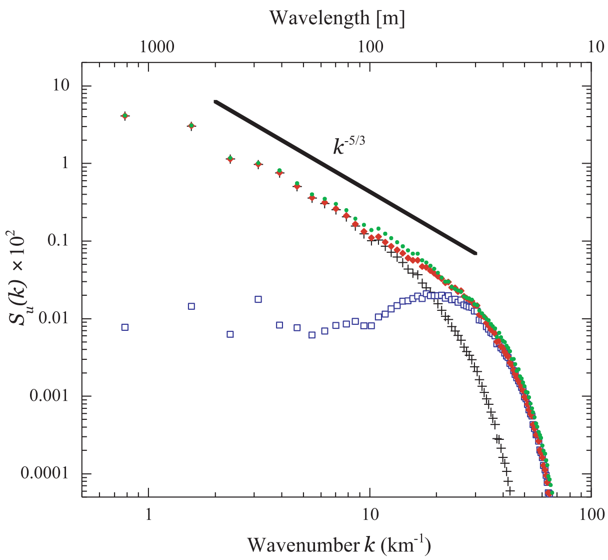


Figure 6. Kinetic energy spectra for a-priori test $+$:$\widetilde{U}$, $\square$:$\overline{u}$, red $\bullet$:$\overline{u}$, green $\bullet$:$\widetilde{U}+\overline{u}$.

grid by WRF-LES is imposed to a fine grid LES and the grid-scale velocity fluctuation of the fine grid is simulated using the proposed one-way coupling method.

Numerical Method

The WRF-LES is one of the two modes of the Advanced Research WRF (version 3.1). In the WRF-LES the 3-dimensional eddy viscosity predicted by the TKE equation for the 1.5-order turbulence closure is used to represent subgrid-scale diffusion. The PBL scheme is not used in the WRF-LES. In this study the WRF-LES is carryed out in a single domain without nesting with the conventional WRF-weather model.

In the idealized free CBL simulation the periodic boundary condition is set in horizontal plane and the heat flux from the ground is zero. The subgrid scale model in this simulation is TKE scheme proposed by the Deardroff. The geostropic wind speed is 10m/s in the east-west direction and the Coriolis parameter f are set to 10^{-4}s^{-1}. The computation domain is 13km in east-west direction and 6.5km in north-south direction. The mesh resolution is 50m in horizontal directions. The upper and lower limits of the domain is 2090m and 130m heights respectively and the domain is divided into 44 layer in vertical direction. The simulation was run for 10 hours as a pre-run and after that the simulated data of the WRF-LES is imposed to the finer mesh LES.

The computational domain of the fine grid LES is 6.5km in east-west direction, 2.7km in north-south direction and 1km in vertical direction. The computational grid resolution is 25m in horizontal directions. The inertial subrange of the streamwise velocity fluctuation in the WRF-LES is limited up to wavenumber 2km^{-1} and it is 10 times as large as the horizontal mesh resolution of the WRF-LES. In this study the filter width of the scale similarity model ($\widetilde{\Delta}$) of equation (5) is set 8.5Δ.

This proposed one-way coupling method is formulated for incompressible fluids. Although the idealized free CBL WRF-LES is a compressible fluid simulation, in this a-posteriori test we assume that the influence of the compressiblity of the fluid is small enough to ignore and the formation of the proposed method is available. In this one-way coupling method the WRF-LES velocity data $\widetilde{U}$ must be imposed to

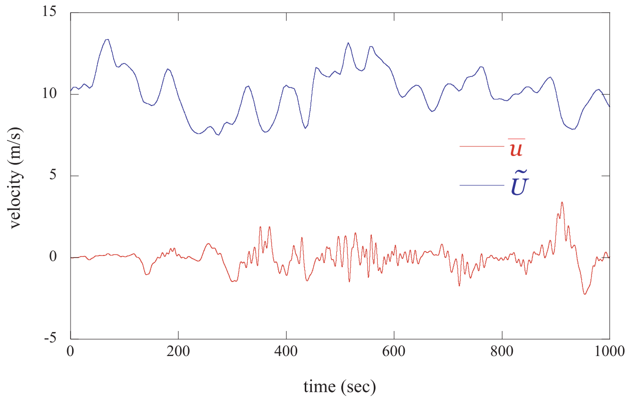


Figure 7. Time history of streamwise velocity for the a-posteriori test at height 740m.

the point where the velocity location of the finer mesh LES is defined. We used B-spline method to interpolate WRF-LES veolcity data to the finer LES mesh. The time step of the LES is 0.1sec and while that of the WRF-LES is 5sec in this study and a linear interpolation is used to interpolate the velocity data from the WRF-LES. The parameter α and β in equation (6) is -16.0, -3.0 respectively.

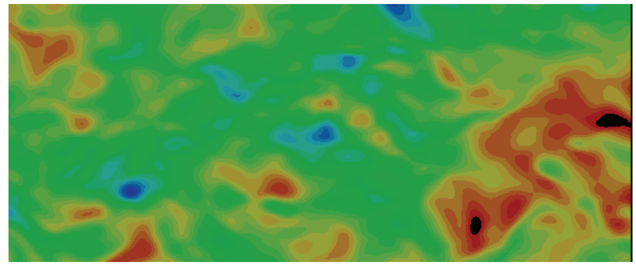
Results

Figure 7 shows the time history of streamwise velocities at height 740m. The mean velocity of the WRF-LES simulation $<\widetilde{U}>$ is 10m/s and its standard deviation is 1.3m/s. While the time averaged of the fine grid LES streamwise velocity $\overline{u}$ is 0.005m/s and its standard deviation is 0.71m/s. The high frequnecy oscillation could be found in $\overline{u}$ while the $\widetilde{U}$ has only lower frequency oscillation compared to that in $\overline{u}$.

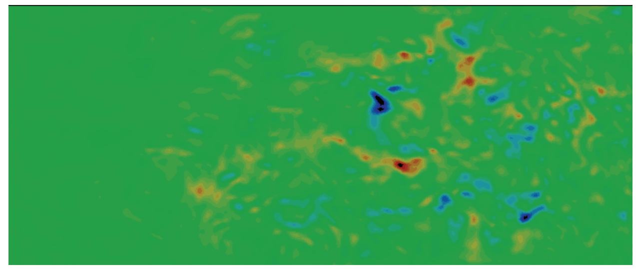
Figure 8 shows the streamwise velocity contours at the horizontal section at height 800m. The contour of $\widetilde{U}$ have large scale turbulence structures while only the small scale structures could be seen in the contour of $\overline{u}$. While the small scale turbulence structures don't distribute widely in the whole domain they only locate near the region where large scale turbulence structures in $\widetilde{U}$ contour locate. This indicates that the high frequnecy turbulence structures generated by the one-way coupling method are strongly influenced by the large scale turbulence structures simulated in the coarse grid WRF-LES.

Figure 9 shows the kinetic energy spectra for streamwise velocity fluctuations at height 740m. The inertial subrange of the kinetic energy spectrum is limited up to wavenumber 2km^{-1} in the coarse grid WRF-LES and the kinetic energy has decrease at the higher wavenumber due to the artificial damping of the model.

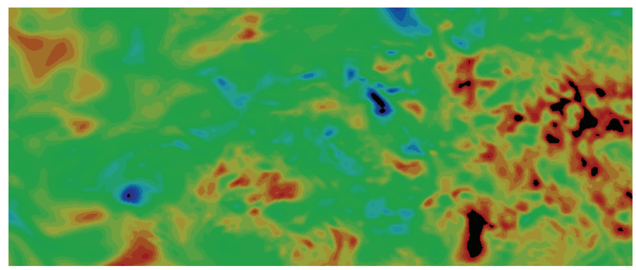
The kinetic energy spectrum for streamwise velocity fluctuations of $\overline{u}$ has large kinetic energy in a high wavenumber region where the kinetic energy of WRF-LES velocity fluctuation $\widetilde{U}$ decreases while the low wavennumber kinetic energy of $\overline{u}$ is small. The total kinetic energy which could be obtained adding up the both $\widetilde{U}$ and $\overline{u}$ slightly decreases the energy comparing with the inertial subrange -5/3 slope in the high wavenumber region. The reason of this underestimation



(a) WRF-LES grid-scale component, $\widetilde{U}$



(b) LES grid-scale component, $\overline{u}$



(c) all component, $\widetilde{U}+\overline{u}$

Figure 8. Streamwise velocity contours at the horizontal section for the a-posteriori test at height 800m.

of kinetic energy in high wavenumber region is not clear but it may be caused by the uneven distribution of the small scale turbulence structures which is mentioned above. The spectrum of the coarse grid WRF-LES slightly overestimates the -5/3 slope of the inertial subrange at wavenumber 1km^{-1} and then slightly underestimates it in higher wavenumber region. This might cause the underestimation of the total kinetic energy in high wavenumber region. Although these results it may be said that the coarse grid WRF-LES and fine grid LES could be nested without large interpolation errors by applying the proposed one-way coupling method using scale similarity model.

CONCLUSIONS

The method for coupling between a coarse grid and a fine grid was proposed. In the method the subgrid-scale turbulence of the coarse grid is generated in the fine grid whose average velocity corresponds to that of the coarse grid. The a-priori test was conducted to validate this method. The fluctuating

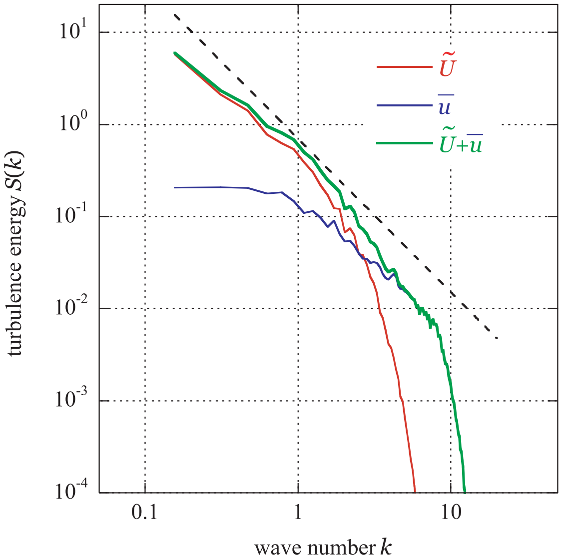


Figure 9. Kinetic energy spectra of the fluctuating velocities for the a-posteriori test.

velocity component agreed well with the true value which can be derived by subtracting the generated coarsely resolved velocity data from the finely resolved LES velocity data. The kinetic energy spectrum of total velocity by summing up the fluctuating velocity component and coarsely resolved velocity fited well to the -5/3 power law for the inertial subrange.

As the a-posteriori test the free convective boundary layer flow velocity data which were conducted simulating the WRF-LES were imposed to the finer mesh LES. The finer mesh LES generates the high wavenumber turbulence which were damped in the WRF-LES. Those small scale turbulence structures were located near the region where the large scale turbulence structures in WRF-LES distribute. The kinetic energy spectra for streamwise velocity fluctuations of the fine mesh LES had power in the high wavenumber region where the energy by WRF-LES turbulence decreased due to the artificial damping.

The results indicate that the coarse mesh and the fine mesh could be smoothly connected by using the proposed one-way coupling method based on the scale similarity model.

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