A stochastic extension of the Approximate Deconvolution Model

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ABSTRACT

The approximate deconvolution model (ADM) for large-eddy simulation exploits a range of represented but non-resolved scales as buffer region for emulating the subgrid-scale energy transfer. ADM can be related to Langevin models for turbulence when filter operators are interpreted as stochastic kernel estimators. The objective of this paper is to introduce the concept of the Eulerian formulation of the Langevin model in a consistent form, allowing for stable numerical integration, and to show how this model can be used for a modified way of subfilter-scale estimation. An initial verification of the concept has been performed for the tree-dimensional Taylor-Green vortex.

THE STOCHASTIC APPROXIMATE DECONVOLUTION MODEL (SADM)

The theoretical derivation of SADM is given in Adams (2011). Here, a summary of the model is given. For modelling the effect of non-represented scales the buffer-scale range of represented non-resolved scales is employed, which is also the underlying concept of ADM (Stolz & Adams, 1999). Instead of applying (linear) deconvolution to that scale range the small-scale field in that range is reconstructed from a generalized Langevin model GLM (Pope, 1983). The GLM can be transformed to Eulerian referene frame (Adams, 2011), leading to transport equations for a stochastic-momentum field that resemble the conservation equations of weakly compressible flow. The transport equation for a number-density field n is

$$\frac{\partial n}{\partial t}+\frac{\partial m^{\alpha}}{\partial x^{\alpha}}=0 \ , \tag{1}$$

where m^{α} is the α -component of the stochastic-momentum field. The stochastic-momentum field is transported by

$$\begin{aligned}&\frac{\partial m^{\alpha}}{\partial t}+\frac{\partial}{\partial x^{\beta}}\frac{m^{\alpha}m^{\beta}}{n}=-\overline{F}^{\alpha}n-\frac{\partial p_S}{\partial x^{\alpha}}-\\&-\chi^{\alpha\beta}\left(m^{\beta}-\frac{\overline{m^{\beta}}}{n}n\right)+\sqrt{\gamma}n\varsigma^{\alpha}\end{aligned} \ . \tag{2}$$

For consistency of the transformation a repulsive force is introduced by $p_S=p_0+c^2 n$, where c^2 is a pseudo speed of sound, for the computations in this paper it is $c=10$. ε is an estimated filtered-scale dissipation rate, and $\chi^{\alpha\beta}=C_1/T_L$ in the context of the current paper is an isotropic inverse relaxation time scale. $-\overline{F}^{\alpha}$ is the force exerted by the filtered field on the stochastic field, i.e. the filtered-field pressure gradient and the filtered-field stress (the latter is neglected here, as its effect is expected to be small).

The filtered field is transported by

$$\begin{aligned}&\frac{\partial\overline{\rho}}{\partial t}+\frac{\partial\overline{\rho u^{\alpha}}}{\partial x^{\alpha}}=0\\&\frac{\partial\overline{\rho u^{\alpha}}}{\partial t}+\frac{\partial}{\partial x^{\beta}}\frac{\overline{\left(\rho u^{\alpha}\right)^{*}\left(\rho u^{\beta}\right)^{*}}}{\overline{\rho}}=\\&=-\frac{\partial\overline{p^{*}}}{\partial x^{\alpha}}-\frac{\partial\overline{\sigma^{*\alpha\beta}}}{\partial x^{\beta}}\\&\frac{\partial\overline{E}}{\partial t}+\frac{\partial}{\partial x^{\alpha}}\frac{\overline{\left(\rho u^{\alpha}\right)^{*}\left(E^{*}+p^{*}\right)}}{\overline{\rho}}=\\&=\frac{\partial\overline{q^{*\beta}}}{\partial x^{\beta}}-\frac{\partial}{\partial x^{\beta}}\overline{\sigma^{*\alpha\beta}\frac{\left(\rho u^{\alpha}\right)^{*}}{\overline{\rho}}}\end{aligned} \ . \tag{3}$$

The starred momenta denote quantities where the non-resolved represented scales are reconstructed as given below. Starred pressure and total energy are computed from their physical definitions but using the reconstructed momenta.

From the computed momentum field $\overline{\rho u^{\alpha}}$ residual scales with the non-resolved represented scales are removed by a high-order filter G_2 (in this case 12th order) that can be constructed from applying a 5th-order deconvolution to the primary filter G . The primary filter is the same as in Stolz & Adams (1999). The small-scale field within the non-resolved represented range is reconstructed from

$$\left(\rho u^{\alpha}\right)^{*} = G_2 * \overline{\rho u^{\alpha}} + \bar{\rho}\left(I - G_2\right) * \left(\frac{m^{\alpha}}{n}\right). \quad (4)$$

APPLICATION OF SADM TO THE THREE-DIMENSIONAL TAYLOR-GREEN VORTEX

We consider as test computation the three-dimensional Taylor-Green vortex, with initial conditions as defined by Drikakis et al. (2006)

$$\rho(x,y,z,t=0)=1$$

$$\underline{u}(x,y,z,t=0)=$$

$$=\begin{bmatrix} \sqrt{\gamma}Ma\sin(x)\cos(y)\cos(z) \\ -\sqrt{\gamma}Ma\sin(x)\cos(y)\cos(z) \\ 0 \end{bmatrix} \quad (5)$$

$$p(x,y,z,t=0)=1+\frac{\gamma Ma^2}{16}\cdot$$

$$\cdot\left(\left(\cos(2x)+\cos(2y)\right)\left(2+\cos(2z)\right)-2\right)$$

An ideal-gas equation of state is used, reference Mach number is $Ma = 0.25$. The computational domain has size 2π in each direction. Resolutions are 32^3, 48^3, and 64^3. Computations are performed until 10 reference time units $t^* = 0.2985$.

In figure 1 we compare the evolution of the total dissipation rate for this case with direct numerical simulations of Brachet et al. (1983).

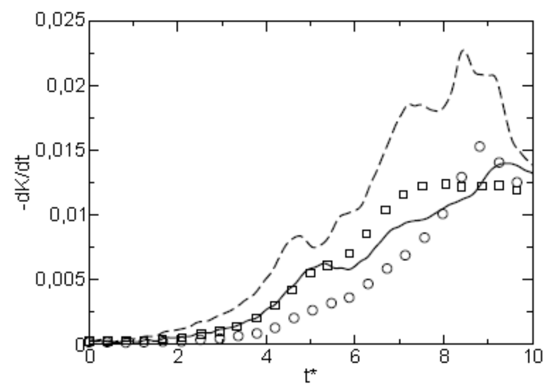

Figure 1: Dissipation-rate evolution over time for 3D TGV. Solid line: SADM with 48x48x48 resolution. Dashed line: ADM (same resolution). Square symbols: dynamic Smagorinsky model with 64x64x64 resolution. Circle: DNS.

It is evident that SADM shows a significant improvement as compared to ADM, and recovers approximately the prediction quality of the dynamic Smagorinsky model at higher 64^3 resolution. The main reason for the significant difference between ADM and SADM is that with ADM the small-scale range is suppressed too strongly, preventing small-scale structures to develop properly during the transitional period of the flow.

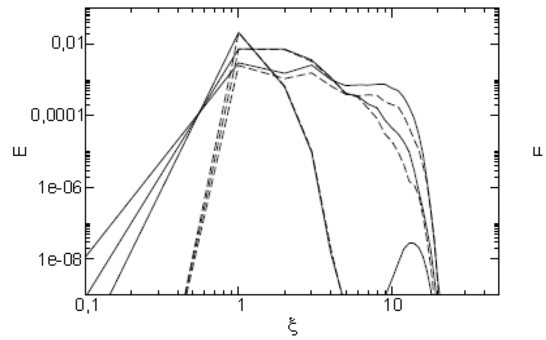

Figure 2: Evolution of the 3D kinetic-energy spectra at times t=t∗ = 0.7; 5; 10 with resolution 48x48x48. Solid line: SADM, dashed line ADM.

This observation is confirmed from the evolution of the three-dimensional energy spectra given in figure 2.

CONCLUSIONS

The stochastic extension of ADM provides a physical rationale for the relaxation term in the later versions of ADM (Stolz et al. 2001a. 2001b). It is easy to show that ADM plus relaxation term model closely resembles a deterministic version of the simplified Langevin model of Pope (1983), transformed to Eulerian reference frame. Consequentially, one may ask how the generalized Langevin model can be incorporated into the ADM framework. The transformation to Eulerian reference frame leads to the introduction of a stochastic number-density field, as proposed by Soulard & Sabelnikov (2006). Unlike their derivation, a derivation based on the delta-function calculus (Adams, 2011), motivates the introduction of a repulsive force for the stochastic-momentum evolution equations, which allows for stable numerical integration. The generated stochastic-momentum field gives an estimate for turbulent fluctuations on the represented, non-resolved range of scales. Instead of employing regularized deconvolution of the LES filter, as with ADM, this range of scales can be extracted and inserted as estimated small-scale field when generating the approximately deconvolved momentum field. Numerical experiments show that such a stochastic extension of ADM for compressible isotropic turbulence recovers the excellent prediction capabilities of ADM, and for the transition of the three-dimensional Taylor-Green vortex delivers significantly better results. Further work needs to focus on the extension to non-isotropic turbulence and on exploiting the formal connection to Reynolds-averaged modelling via the generalized Langevin model.

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