INLET CONDITIONS AND WAVE-PACKETS IN SUBSONIC JET NOISE

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ABSTRACT

The acoustic field of Mach 0.4, 0.5 and 0.6 jets, measured using an azimuthal ring array and then decomposed into azimuthal Fourier modes, is found to comprise a superdirective axisymmetric component (exponential decay with radiation angle) for low Strouhal numbers. This is shown to be consistent with an axially non-compact, wave-like source, the marked directivity being a result of axial interference over a source extent that spans several jet diameters. The source is then modelled using parabolised stability equations (PSE) for the axisymmetric mode, the experimental mean velocity field (obtained from measurements with a traversing Pitot) being used as a base flow. The PSE results closely match the velocity data on the jet centerline. Calculation of the axisymmetric mode of the acoustic field using a source term constructed from the PSE modes leads to agreement to within 3dB of the experimental values at low axial angles for Strouhal numbers between 0.3 and 0.9, and for all three Mach numbers, suggesting that linear instability waves constitute the flow mechanism responsible for the said radiation, and that PSE is thus a pertinent reduced-order model that connects fluctuations at the nozzle inlet, via a wave-packet sound-source mechanism, to low-angle sound emission.

INTRODUCTION

The observation of a certain degree of order in turbulent flows, previously thought to be comprised of purely stochastic eddies, has motivated numerous studies aimed at extracting the features of coherent structures in jets (see for instance Crow & Champagne, 1971 or Hussain & Zaman, 1981).

The existence of such coherent structures has significant implications for aeroacoustics. Instead of convected eddies with random phase relative to one another, source models for coherent structures should explicitly account for the convected-wave form of the source (Michalke, 1970). Such a source model—a convected wave modulated by a Gaussian—is presented, for instance, by Crow (1972) (see also Crighton, 1975). The Gaussian envelope accounts for amplification of the wave near the nozzle exit, this being understood as due to the instability of the velocity profile, and the subsequent saturation and decay of the wave amplitude as the mixing layer thickens. Similar models have been proposed which explicitly account for the jitter of the coherent structures (Ffowcs Williams & Kempton, 1978; Cavalieri *et al.*, 2011*b*). A notable feature of such wave-like sources is their directivity, which peaks at low axial angles and presents an exponential polar decay. This has been labelled *superdirectivity* (Crighton & Huerre, 1990).

One would like to ascertain what is an appropriate dynamic model for such behaviour. One possibility is linear stability theory, the time-averaged mean flow being used as a base-flow (a scale separation is assumed between the large-scale, coherent structures and the more random, background tubulence). The dynamics of instability waves in jets were first studied with the assumption of parallel flow. Batchelor & Gill (1962) studied the temporal stability problem for a cylindrical vortex sheet, the analogue spatial stability problem was studied by Crow & Champagne (1971), and Michalke (1971) studied the stability of a jet with finite mixing layer thickness. However, in order to obtain the complete spatial envelope of the instability waves, non-parallel flows must be considered. This has been done by Crighton & Gaster (1976), who studied slowly-diverging, incompressible jets. Similar ideas were followed by Tam & Morris (1980) and Tam & Burton (1984), respectively, for compressible mixing layers and axisymmetric jets. Whereas most comparisons between stability theory and experiments have been performed for forced flows, more recently, Suzuki & Colonius (2006) have provided evidence that instability waves are present in unforced jets. In this sense, stability theory may be useful as a framework for understanding the formation and dynamics of coherent structures in jets on one hand, and for sound prediction (where low-angle emission is concerned) at lower computational cost than DNS and LES.

Parabolised Stability Equations (hereafter PSE) (Herbert, 1997) and Global mode calculations (see Chomaz, 2005 for a review) both allow non-parallel effects to be accounted for. However, the PSE approach involves considerably lower computational cost. Examples of the applications of PSE to compressible jets can be found, for instance, in the works of Sandham & Salgado (2008) and Gudmundsson & Colonius (2009).

In the present work we assess the suitability of PSE as a dynamic *Ansatz* that can link nozzle inlet conditions to the acoustic farfield. We first investigate the degree to which the acoustic field of unforced subsonic jets is compatible with a wave-packet sound source: measurements of the acoustic field using an azimuthal ring of microphones allow a decomposition into azimuthal Fourier modes; the directivity of the axisymmetric mode is then related to a wave-packet source *Ansatz*, and this allows us to estimate the axial extent of the source. A PSE calculation is then performed, using the mean velocity field as a base-flow: the instability waves so obtained agree well with the velocity fluctuations on the jet centerline. By then solving a wave equation, driven by a source term constructed from the axisymmetric component of the PSE solution, we compute a model sound field. Good agreement with measurements suggest that instability waves are an important contributor to the sound field at low axial angles, and that PSE is a pertinent reduced-order model for describing the evolution of inlet fluctuations into coherent structures, in the form of linear instabilities; these then driving low-angle sound emission via a wave-packet sound-production mechanism.

EXPERIMENTAL SETUP

The experiments were performed in the Bruit et Vent anechoic facility at the Centre d'Etudes Aérodynamiques et Thermiques (CEAT), at Institut Pprime, Poitiers, France. A photo of the setup is shown in figure 1(*a*). Measurements were made of the acoustic field of three unheated jets, with acoustic Mach numbers $M = U/c$ of 0.4, 0.5 and 0.6, where U is the jet exit velocity and c is the ambient sound speed. The nozzle diameter, D, is 0.05m. The Reynolds number $\rho UD/\mu$ varies from 4.2×10^5 to 5.7×10^5, where ρ and μ are, respectively, the density and viscosity at the nozzle exit. A boundary layer trip was used to force transition upstream of the nozzle exit. Velocity measurements were performed using a Pitot tube for the mean velocity and a hot wire for the fluctuations; we have used the *in situ* calibration for the hot wire described by Tutkun *et al.* (2009). Measurements of the nozzle inlet conditions were performed 2.5mm (D/20) downstream of the nozzle exit. The resulting profiles, representative of the nozzle boundary layer, are shown in figure 2. The results are consistent with a turbulent boundary layer at the nozzle exit.

Six microphones were deployed on an azimuthal ring in the acoustic field at constant polar angle θ to the downstream jet axis. The setup is shown in figure 1(*a*). The ring has diameter of 35D. For different values of θ, the ring was displaced along the jet axis, leading to differences in the distance r between the nozzle exit and the microphones. We have therefore used the $1/r$ scaling for the acoustic pressure to correct all results to a fixed distance of $r = 35D$.

(*a*)

(*b*)

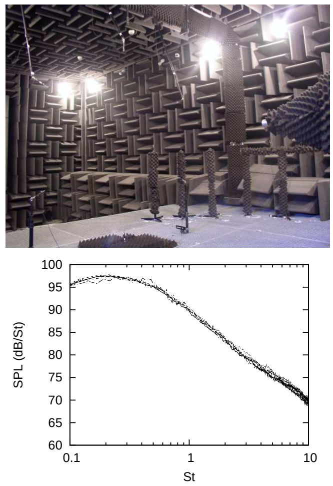

Figure 1. (*a*) Experimental setup for the acoustic measurements; (*b*) spectra of the six azimuthally distributed microphones at polar angle $\theta = 30^\circ$ and $M = 0.6$

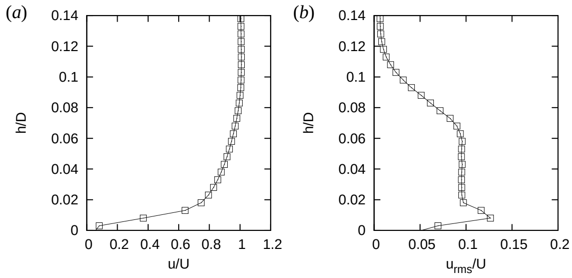

Figure 2. Boundary layer profiles at the nozzle exit for the Mach 0.6 jet: (a) mean velocity and (b) rms value

It is assumed that the nozzle and upstream flow conditions are sufficiently axisymmetric such that the jet and its sound field present circumferential homogeneity (Michalke & Fuchs, 1975). A verification of this hypothesis in the acoustic field was performed by comparing the microphone spectra (figure 1(*b*)). The good agreement indicates that there is no preferred azimuthal direction for sound radiation.

ACOUSTIC RESULTS

Figure 3 shows the directivity of the Mach 0.6 jet for the measured angles, as well as the contributions of the different azimuthal modes. We note that the axisymmetric mode presents a marked directivity towards the low axial angles: there is a 7.8 dB increase in sound intensity between 45° and 20°. The other azimuthal modes increase more gradually over $45^\circ \leq \theta \leq 90^\circ$, with a slope close to that of mode 0 in the same angular sector. For lower angles, modes 1 and 2 decay. Similar directivities for the azimuthal modes 0, 1 and 2 have been observed in a large eddy simulation of a Mach 0.9 jet (Cavalieri *et al.*, 2011*a*).

Spectra for angles 20°, 30° and 40° are shown in figure 4. The increase of mode 0 is mostly concentrated in the lower frequencies. For Strouhal numbers greater than 1, there is still a dominance in the total spectra of modes 1 and 2.

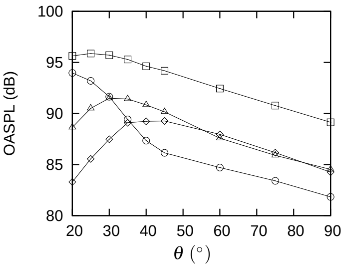

Figure 3. Directivity for $M = 0.6$: squares, total; circles, mode 0; triangles, mode 1; and diamonds, mode 2.

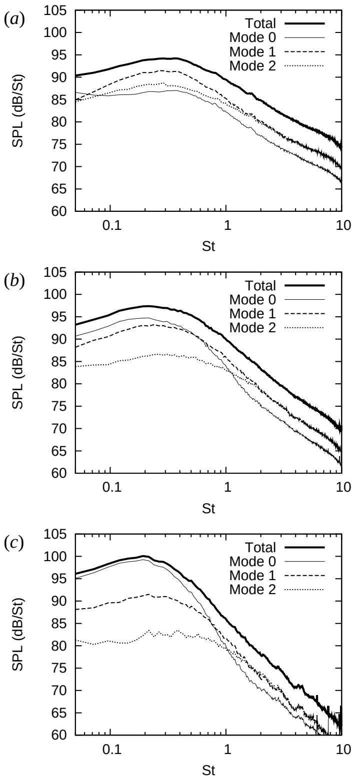

Figure 4. Spectra at M=0.6 of individual modes for (*a*) $\theta = 40^\circ$, (*b*) $\theta = 30^\circ$ and (*c*) $\theta = 20^\circ$

To evaluate the directivity of the spectral peak, the SPL for St=0.2 is shown in figure 5. We see that for this frequency there is an even higher directivity of mode 0, with an increase of 15.4 dB from 45° to 20°, i.e. a factor of 34 in the acoustic intensity.

The superdirective behaviour of wave-packet models (Crow, 1972; Ffowcs Williams & Kempton, 1978; Cavalieri *et al.*, 2011*b*) leads to an exponential decay of the acoustic intensity as a function of $(1 - M_c \cos\theta)^2$. To verify if the measured directivity has the same trend, we have plotted the SPL at St=0.2 as a function of $(1 - M_c \cos\theta)^2$ in figure 5(b). The fit with a straight line shows that the axisymmetric mode is indeed superdirective, in agreement with the cited models. The same trend is also present for $M = 0.4$ and $M = 0.5$ jets, in figure 6.

The axisymmetric superdirectivity is observed over a frequency range centered on the peak frequency. This can be

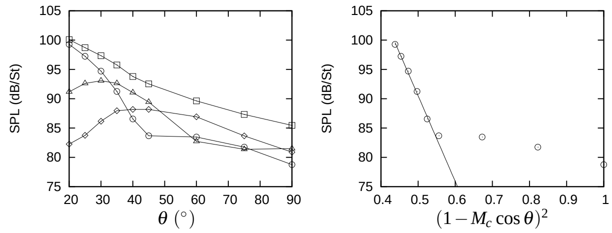

Figure 5. SPL for St=0.2 for the Mach 0.6 jet as a function of (a) θ and (b) $(1 - M_c \cos\theta)^2$. Same conventions of figure 3.

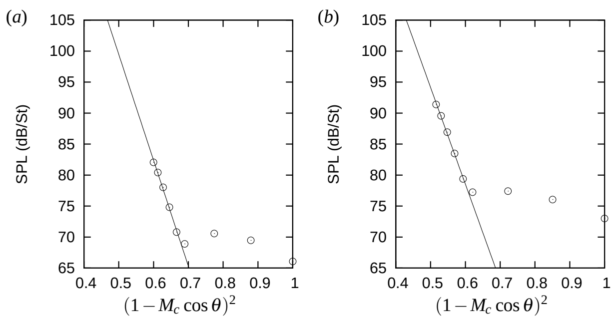

Figure 6. SPL for St=0.2 and m=0: (*a*) M=0.4 and(*b*) M=0.5 jets.

seen in figure 7. We note that for $0.1 \leq \mathrm{St} \leq 0.3$ the narrow-band directivities have similar shapes, and in figure 7(b) the linear fit made for $\mathrm{St} = 0.2$ closely matches the directivity for $\mathrm{St} = 0.1$ and $\mathrm{St} = 0.3$. The exponential decay is thus observed for a frequency range around the peak. For higher frequencies at low angles the SPL values are lower than the peak, but as the angle is increased the SPLs tend to merge with the exponential decay of the peak frequency. As the frequency is increased this decay is progressively less significant: whereas a decay of 15.4dB from 20° to 45° was observed for $\mathrm{St} = 0.2$, for $\mathrm{St} = 0.4$ we have a decay of 10.7dB, and for $\mathrm{St} = 0.6$ we have 7.7dB (now between 25° and 45°, for the maximum level is obtained for $\theta = 25^\circ$). This trend is possibly associated with an increase in the axial compactness of the source as the frequency is increased.

As the directivity of the peak frequencies is exponential between $\theta = 20^\circ$ and $\theta = 45^\circ$ for the three Mach numbers considered, we can estimate the wave-packet envelope size (and thus the number of spatial oscillations implicated in the source interference) by evaluating the ratio L/D using the wave-packet *Ansatz* of Crow (1972) (see also Crighton 1975), which comprises a wavelike, axially-coherent line distribution of axially-aligned longitudinal quadrupoles (of a form consistent with the T_{11} components of Lighthill's stress tensor); the source is a convected wave, of frequency ω and wavenumber k, modulated by a gaussian envelope with characteristic length L:

$$T_{11}(\mathbf{y}, \tau) = 2\rho_0 U \tilde{u} \frac{\pi D^2}{4} \delta(r) \mathrm{e}^{\mathrm{i}(\omega\tau - k y_1)} \mathrm{e}^{-\frac{y_1^2}{L^2}} \qquad (1)$$

where ρ_0 is the ambien fluid density and $\tilde{u}$ the streamwise

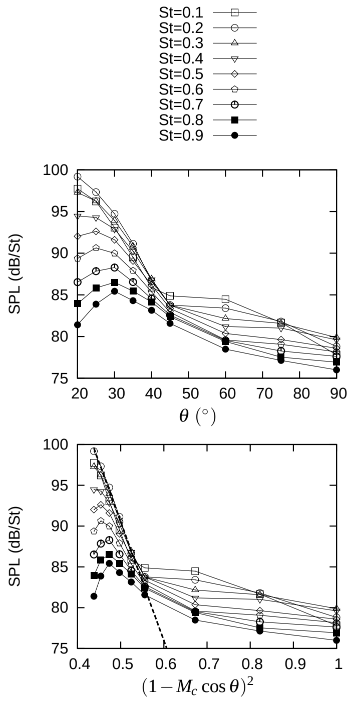

Figure 7. Directivity for the axisymmetric mode for $M = 0.6$ as a function of Strouhal number and of (a) θ and (b) $(1-M_c\cos\theta)^2$.

velocity fluctuation amplitude.

Evaluation of the far field pressure leads to

$$p(\mathbf{x},t) = -\frac{\rho_0 U \tilde{u} M_c^2 (kD)^2 L\sqrt{\pi}\cos^2\theta}{8|\mathbf{x}|}\mathrm{e}^{-\frac{L^2k^2(1-M_c\cos\theta)^2}{4}}\mathrm{e}^{\mathrm{i}\omega\left(t-\frac{|\mathbf{x}|}{c}\right)}, \quad (2)$$

where M_c is the Mach number based on the phase velocity, U_c, of the convected wave.

Since $Lk = 2\pi \mathrm{St}(L/D)(U/U_c)$, when the directivity follows $\cos^2\theta\exp(-L^2k^2(1-M_c\cos\theta)^2/4)$, as in eq. (2), for St=0.2 and $U_c = 0.6U$, we obtain the results shown in table 1.

Table 1. Source extension of axisymmetric mode at $\mathrm{St} = 0.2$

M	$\mathrm{SPL}(\theta = 20°) - \mathrm{SPL}(\theta = 45°)$	kL	L/D
0.4	13.2dB	6.50	3.10
0.5	14.1dB	6.34	3.03
0.6	15.4dB	6.40	3.06

Use of Crow's wave-packet model with the three Mach numbers results in a consistent estimation of L/D for all cases, with values of 3–3.1. The values of L/D are related to the gaussian envelope in eq. (1), and indicate that this wave-packet spans an axial region of 6-8D. This modulation is such that three oscillations are present in the source: there is thus significant axial interference in the source, leading to the observed superdirectivity in the radiated sound field.

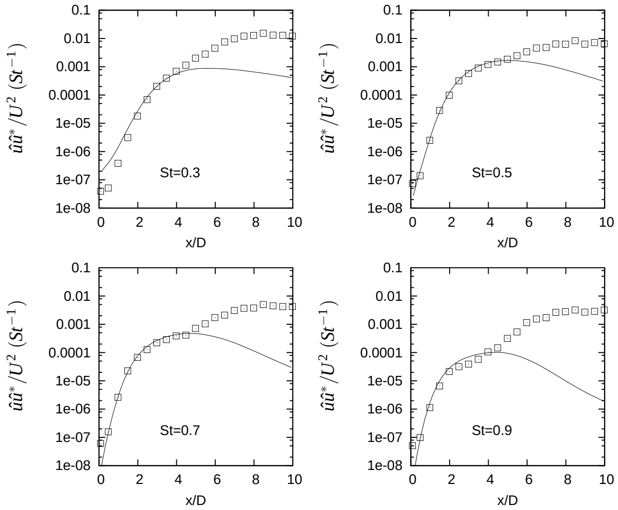

Figure 8. Comparison between PSE (lines) and experiment (points) for $M = 0.4$ and (a) St=0.3, (b) St=0.5, (c) St=0.7 and (c) St=0.9

The present estimation is in agreement with Hussain & Zaman (1981), who by means of phase-averaging, estimate that the organised component of jet excited at $\mathrm{St} = 0.3$ comprises a train of three coherent structures spanning an axial region of 7D. This also agrees with the experimental observation of Tinney & Jordan (2008), in the near pressure field of unforced coaxial jets, of a subsonic convected wave extending up to 8 secondary jet diameters downstream of the nozzle exit: the first two POD modes of the near field pressure take the form of a sine and a cosine, modulated by an envelope function, and comprising three axial oscillations.

PSE RESULTS

Linear PSE was performed using the experimental mean velocity field. The approach followed is described by Gudmundsson & Colonius (2009). The linear modes have a free constant, and this has been adjusted using the velocity spectra on the jet centerline. On the centerline, the kinematic boundary conditions are zero transverse velocity and finite axial velocity for azimuthal mode 0, zero axial velocity and finite transverse velocity for mode 1, and zero velocities for all higher modes (Batchelor & Gill, 1962). As the velocity measurements were performed with a single hot wire, we expect that in the potential core the measurements will be of the axial velocity, allowing thus the comparison between the mode 0 from linear PSE and the hot wire spectra.

Figure 8 shows comparison, between PSE and experiment, of the amplitude for the streamwise velocity component for the Mach 0.4 jet. Similar agreement is found for the other jets over the Strouhal number range of 0.3–0.9. Close agreement is obtained in the region extending to the end of the potential core ($x \approx 5.5D$). Downstream of this point, the differences are due to the full turbulence spectrum being measured by the hot wire.

The acoustic field of a PSE mode, for a frequency ω, is computed by means of the line source, obtained following a

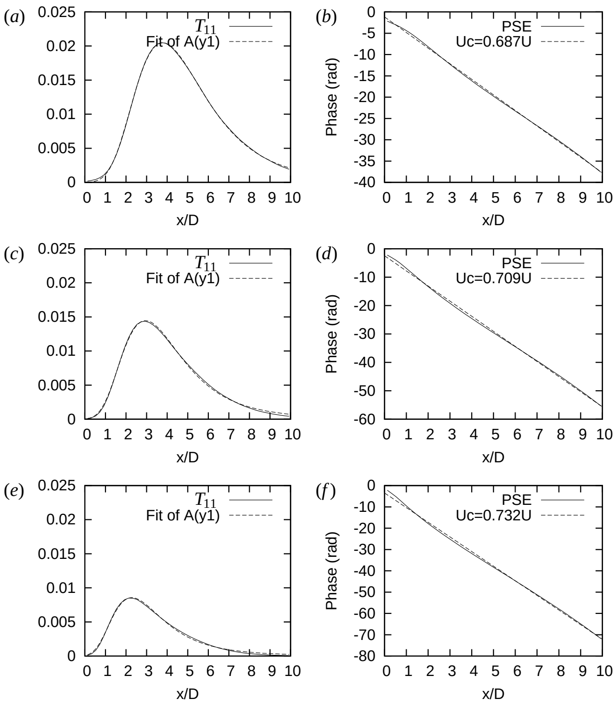

Figure 9. Fits of amplitudes and phases for the PSE modes of the $M = 0.4$ jet and (a) and (b), $St = 0.4$; (c) and (d), $St = 0.6$; and (e) and (f), $St = 0.8$

radial integration:

$$T_{11}(y_1,\omega) = 4\pi\rho_0\delta(r)\int_0^\infty U(y_1,r)u(y_1,r,\omega)r\mathrm{d}r, \quad (3)$$

where $u(y_1,r,\omega)$ is the axial velocity fluctuation taken from the PSE modes calibrated with the centerline velocity data. This expression in (3) gives a source consistent with eq. (1). The radial integration is justified, for the present Mach and Strouhal numbers, by an assumption radial source compactness (Cavalieri *et al.*, 2011*b*). With this source, the acoustic field is obtained by

$$p(\mathbf{x},t) = \frac{1}{4\pi}\iiint\left[\frac{1}{|\mathbf{x}-\mathbf{y}|}\frac{\partial^2 T_{11}}{\partial y_1^2}(y_1,\omega)\mathrm{e}^{\mathrm{i}\omega\tau}\right]_{\tau=t-\frac{|\mathbf{x}-\mathbf{y}|}{c}}\mathrm{d}\mathbf{y} \quad (4)$$

To compute the unbounded integral in equation (4), the radially integrated source is fit to a simpler analytical function whose amplitude is given by $A(y_1) = C\mathrm{e}^{-(y_1-y_c)^2/(L+ay_1)^2}$, and whose phase speed is taken to be constant. The sample fits shown in figure 9 demonstrate the efficacy of this approach for the current PSE data. We note that the Gaussian function is similar to that used by Reba *et al.* (2010) or Kœnig *et al.* (2010), but in the present case we are not using it as a model, but simply as a device to accurately compute the integral given the PSE source term.

Comparison of the sound radiation using eqs.(3) and (4) with the measured sound field for the axisymmetric mode is shown in figures 10 and 11, respectively, for the $M = 0.4$ and $M = 0.6$ jets. There is good agreement, and the present curves are typical of the agreement found for Strouhal numbers of 0.3–0.9. This suppoorts the contention that instability waves are responsible for the sound radiation at low axial angles, even for these low Mach number jets. The results for the $M = 0.5$ jet (not shown) present similar agreement.

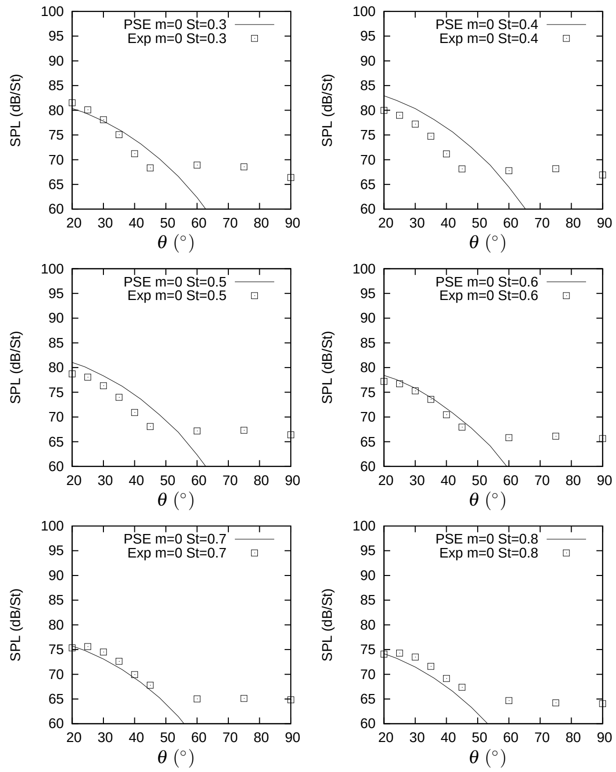

Figure 10. Comparison between the acoustic field calculated with PSE (lines) and the experiment (points) for $M = 0.4$ and (a) St=0.3, (b) St=0.4, (c) St=0.5, (d) St=0.6, (e) St=0.7 and (f) St=0.8

CONCLUSION

A study of the azimuthal modes of the sound radiated by subsonic jets ($0.4 \leq M \leq 0.6$) is presented; results show the axisymmetric mode to be superdirective at low frequencies. This is shown to be consistent with a source taking the form of a wavelike, axially non-compact, line-distribution of axially-aligned longitudinal quadrupoles, this convected wave form undergoing spatial amplification, saturation and decay.

The source is modelled as an ensemble of linear instability waves, calculated using parabolised stability equations for the axisymmetric mode. Close agreement is found between the amplitudes of the stability modes and the experimental velocity data on the centerline. The acoustic field is obtained by solving the free-space wave equation, driven by this source *Ansatz*, and comparison with the axisymmetric component of the experimental acoustic field shows good agreement over $0.3 \leq St \leq 0.9$ and for the Mach numbers studied.

The results suggest that even for low Mach number jets sound radiation at low angles is due to an axially extended pat-

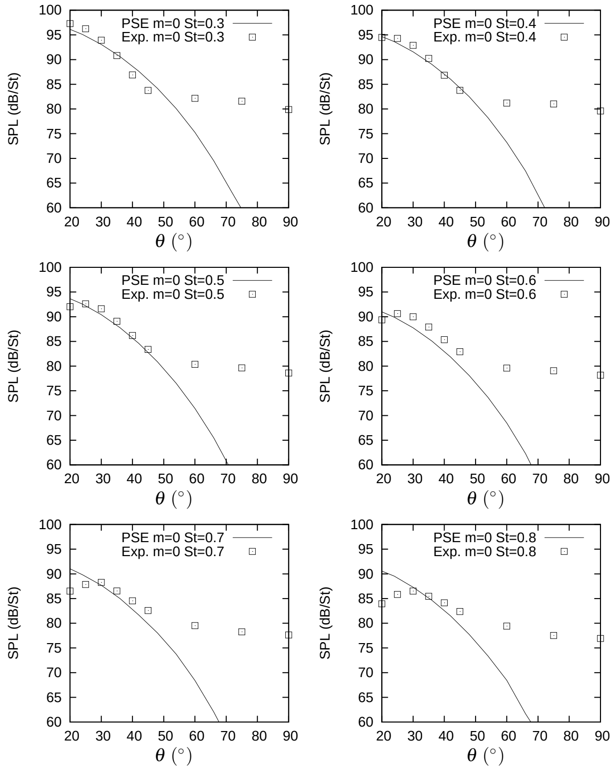

Figure 11. Comparison between the acoustic field calculated with PSE (lines) and the experiment (points) for $M = 0.6$ and (a) St=0.3, (b) St=0.4, (c) St=0.5, (d) St=0.6, (e) St=0.7 and (f) St=0.8

tern of convecting axisymmetric structures, that radiate sound by means of a wave-packet mechanism. The underlying flow mechanism can be modelled as linear instability waves using the mean turbulent velocity field as the base flow.

ACKNOWLEDGMENTS

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