COHERENT STRUCTURES IN THE TRANSITION OF COMPRESSIBLE PLANAR WAKES

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ABSTRACT

Temporally evolving direct numerical simulations of transitioning plane wakes are conducted to study the influence of compressibility on the developed structures in the flow. Four cases are investigated with free stream Mach numbers of 0.3, 0.8 1.2 and 2.0. The growth rate of the inviscid linear stability approximation collapses the turbulence statistics in the linear region of the flow for the low compressible cases. At higher Mach, the scaling does not provide as good of an agreement. The investigation of the preferential wavelengths reveals an increased three-dimensionality with Mach number, a result that is supported by experimental observations but contradicts the linear stability results. We attribute the increased three-dimensionality to the receptivity of the wake to the symmetric (varicose) mode. The varicose mode is two-dimensional at low Mach number but becomes oblique in the supersonic regime. In opposition to the mixing layer, the high-speed wake undergoes a spreading rate increase with increased Mach number during transition, despite a reduced linear growth rate. As the principal instability wavelength increases with compressibility, the developed rollers are larger with a higher circulation. The pairing of these structures results in a stronger cross wake momentum transfer and consequently, an increased lateral spreading. In addition, the instantaneous visualization of the braided structures reveals an increased streamwise alignment with increasing compressibility.

INTRODUCTION

The dependence of the spreading rate on the convective Mach number is a well-established characteristic of the high speed mixing layer [1–5]. The current consensus attributes the inhibited turbulence production to a reduction in the pressure-strain term [2]. An a priori analysis would suggest that these findings should also be applicable to the case of the planar wake. Yet the literature on the high speed wake does not seem to show a convincing spreading rate reduction with increasing Mach number. At first thought, it might be an expected result as the compressibility effects decay with the wake evolution; to this effect Clemens and Smith [6] noted many similarities between the supersonic and incompressible wakes in the far-field. But, on closer inspection, the possibility of a coupling between the compressibility and lateral spreading of the wake raises a fundamental inconsistency which has not yet been addressed. On the one hand, if there is a spreading reduction caused by compressibility effects, we must question the scaling of the wake half-width, as it appears rather invariant to the Mach number [7, 8]. On the other hand, if there is no spreading rate reduction, there must be a structural characteristic, which distinguishes it from the mixing layer. The essence of the present work is to understand the effect of compressibility in the planar wake by investigating the structures during transition.

The extensive studies on the compressibility effects in the mixing layer guide our investigation of the high speed wake. The two notable features of the mixing layer are an increased three-dimensionality and a reduction of the lateral spreading with increasing Mach. The three-dimensionality is related to the linear stability characteristics as primary instability mode changes from a two- to three-dimensional wave above the convective Mach number $Ma_c = 0.6$ [9]. The question of the reduced spreading has been the focus of many recent investigations on the compressible mixing layer [2,3,10]. It is generally accepted that the lateral spreading reduction is related to a reduction of the turbulent kinetic energy production as opposed to increased dilatational effects [11] or reduced linear instability mode growth. Vreman *et al.* [2] found that the pressure-strain term is reduced with increasing compressibility and developed a model to illustrate this finding. Similarly, Pantano and Sarkar [10] suggested the energy transfer from the streamwise to the cross-flow direction is inhibited by compressibility. Physically, they noted that the finite speed of sound adds a time delay in the pressure transmission, which effectively reduces the correlation of the developed eddies.

Despite many studies on the mixing layer, a clear and quantitative evaluation of the spreading rate reduction remains an open question [5] as the scatter in the available data remain quite large. That said, some workers have noted a better agreement when the spreading rate is scaled with different parameters [12]. Furthermore, nothing in these studies on the mixing layer suggest that the compressible wake should behave any differently; yet, a convincing demonstration of a spreading rate reduction has not yet been shown (despite a reduced exponential growth of the most unstable mode [14, 16]). As stated previously, this raises an important issue which puts into question to use of the wake half-width as the principal scaling parameter in the high speed wake.

NUMERICAL DETAILS

Numerical scheme

We developed and validated a predictor/corrector finite difference solver, which was used to compute the compressible Navier-Stokes equations. The spatial scheme is fourth-order accurate inside the domain with a biased third-order scheme at the finite boundaries. The high-order MacCormack-like spatial scheme was chosen as the biased stencil on the convective terms provides a robust and efficient method to deal with the high gradients appearing in the form of shocklets while offering adequate dispersion and dissipative qualities. The over-resolution needed for a good small-scale calculations compared to other higher-order schemes (spectral, Padé etc.) is offset by the computational efficiency, parallelisability and small memory footprint allowed. The time-dependent compressible Navier-Stokes equations are solved in conservative form with skew-symmetric convective terms for robustness and to reduce the aliasing errors. The time was advance using a second-order Runge-Kutta scheme in which the time-step was set by an imposed acoustic Courant number. The numerical code was extensively validated against the analytical solution of a viscous shock, Taylor-Green vortex, decaying compressible isotropic turbulence and mixing layer in addition to the incompressible wake.

Grid, boundary and initials conditions

A homogeneous grid was used in the stream- and spanwise direction. In the cross wake direction, the grid was clustered about the centerline using a hyperbolic tangent mapping. The grid resolution was chosen to resolve down to the order of the Kolmogorov scale. As the flow is temporally evolving, periodic boundary conditions were set in the streamwise (x) and spanwise (z) directions and a finite boundary in the normal direction (y). In the bounded direction, a non-reflecting boundary condition [15] is supplemented with a sponge layer in order to remove any numerical oscillations caused by the inviscid approximation at the boundary. The domain size for all cases was: 50 , 35, 12.5 in the streamwise, normal and spanwise directions. With the current Mach and Reynolds number the domain allows for the development of a minimal of 16 to 20 rollers in the streamwise direction.

We simulated the flow for a constant Reynolds number of 1500 at four different free stream Mach numbers: Ma_∞=0.3, 0.8, 1.2 and 2.0. A laminar initial velocity profile was used: $\langle u(y)\rangle = U_\infty - U_d \exp(-\log(2)y/b)^2$ where U_∞, U_d and b are respectively the free stream velocity, the initial deficit velocity and the initial wake half-width. The mean velocity and mean temperature fields are related through the Crocco-Busemann relationship. The mean wake profile is perturbed by broadband fluctuations in x- and y-directions with an *rms* value of 10% of the velocity deficit in order to break the symmetry about the centerline. The broadband perturbations adds generality to the obtained results at the cost of longer transitional time compared to specific mode forcing. The initial wake half-width was unity resulting in constant momentum flux deficit at $\dot{m} \approx 0.9$.

RESULTS

The transitional mechanism of the incompressible planar wake is well understood as the wake is inherently unstable because of an inflectional velocity profile. The principal instability modes, which generally agree with linear stability approximations, grow as energy is transferred from the mean to the fluctuating flow. As the spanwise coherent rollers develop, ribs (also called braids) appear between neighbouring structures along the separatrix of the flow. The streamwise inclined vorticity of the ribs represents the first sign of three-dimensionality in the wake. The rollers pair when the energy content of the principal instability modes becomes saturated which leads to the onset of full turbulence in the wake. The effect of compressibility on the transitional features of the wake is important although very subtle.

In order to assess the influence of the Mach number on the flow, we need to quantify the level of compressibility during the transition of the wake. The free stream Mach number, Ma_∞, is not a dynamically relevant velocity scale. Instead, the relation between the free stream and centerline Mach number is a more meaningful flow parameter as it defines the relative velocity of the large-scale eddies with respect to the free stream. Analogous to the convective Mach number in the mixing layer, the relative Mach number is: $Ma_r = Ma_\infty - Ma_o = \frac{U_\infty - U_0}{c_\infty}$. As the defect varies with the evolution of the flow, so does the relative Mach number. Figure 1 shows the monotonically decreasing relative Mach number; given a long enough domain, the relative Mach number asymptotically tends toward zero. During transition, which roughly corresponds to the time $t = [20:80]$, the relative Mach number decays at an increasing rate with the Mach number. The maximum slope of the relative Mach number is: -0.011, -0.0275, -0.0380 and -0.0532 for Ma_∞=0.3, 0.8, 1.2 and 2.0 respectively. In addition to the relative Mach number, we may also define the turbulent Mach number as: $Ma_t = \frac{\langle u' \rangle_{max}}{c_\infty}$. The maximum turbulent Mach number is reached during transition and corresponds to : 0.048, 0.128, 0.192 and 0.32 (respectively for cases Ma_∞=0.3, 0.8, 1.2 and 2.0). In the far-wake the turbulent Mach numbers fall to about a fourth of its maximal value.

In order to confirm the linear evolution in the pre-

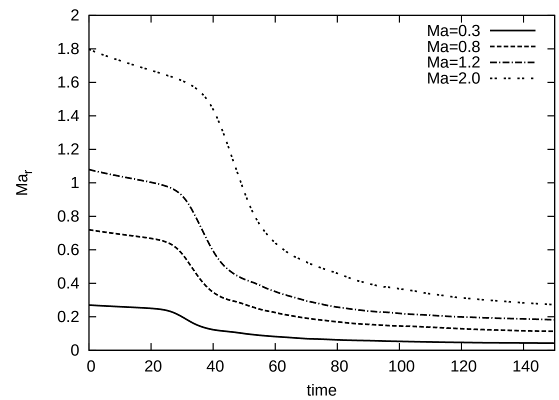

Figure 1. Evolution of relative Mach number in the flow.

transitional domain, we validate our results against results from an inviscid linear stability calculations for a compressible wake by [14] in figure 2. We calculate the time takes the flow to reach arbitrarily chosen fixed velocity fluctuation of $v_{rms}(t) = 0.2$ which is located within the linear transitional domain. Using the case of Mach 0.3 as the baseline, we compare the ratio of elapsed time with the ratio linear stability growth rates for different Mach numbers. As we are normalizing on the Mach 0.3 case we find a maximum difference under 1.5% for cases 0.8 and 1.2. For the case of $Ma = 2.0$, the variation between theoretical growth and computed was slightly larger at 3.5%. When we normalize the time by the growth rate of the lowest Mach number case, an excellent collapse is observed (apart from the case 2.0) which has a very slight offset in figure 3. The collapse between all the cases is even greater for the streamwise velocity fluctuations. Considering the underlying assumption of inviscid flow for the theoretical analysis, these are rather good comparison which allow us to gain confidence in our results. The larger discrepancy with the higher Mach number case will be discussed below.

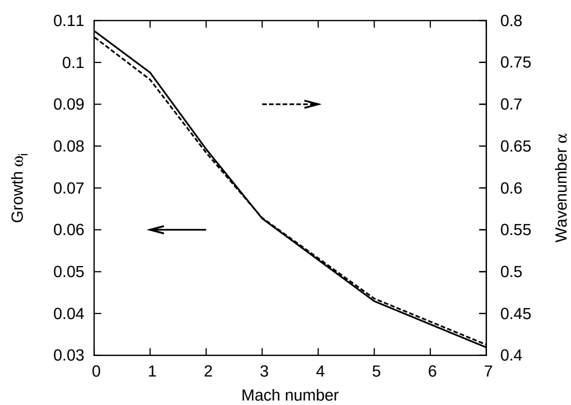

Figure 2. Growth rate and streamwise wavenumber of the most unstable mode as calculated by Chen et al., 1990.

Overlooked by most of the previous studies on the transitioning high speed wake is the influence of increased streamwise wavelength of the most unstable anti-symmetric mode with increasing Mach number, see figure 2. In order to evaluate the preferential modes in the transitional flow, the two-point correlation with streamwise separation of the cross-wake velocity fluctuation was found to be a very sensitive measure of the preferential wavelength in figure 4. The two-point correlation was measured along the plane with the highest vorticity magnitude, which roughly corresponds to the location at the wake half-width. During transition, the first positive peak of the two-point correlation represents the average spacing of the spanwise rollers and is directly proportional to the size of the coherent structures. Using this metric, we can follow the evolution of the size of the rollers in 6. A key feature in all wakes is the growth of the rollers in the pre-transitional region followed by a plateau. By comparing with other integral statistics, the plateau in the streamwise wavelength occurs over the same time frame as the very rapid decay of the centerline velocity (figure 1) and the rapid spreading of the wake half-width (figure 7). These features are attributable to the pairing of the main structures. The observed plateau (and even wavelength reduction) followed by a rapid increase in the diameter of the structure is an expected result during the vortex pairing [13]. The visualization of the idealized vortex pairing simulations (figures 3 (a) and (b) in [13]) reveal the formation of oval-shaped structure which, after pairing, eventually regains a circular shape after pairing. The oval shaped structure has a reduced streamwise extent when the principal axis of the paired structure is perpendicular to the plane of the shear layer. Once both vortex cores merge, the structure regains its circular shape and the diameter increases rapidly.

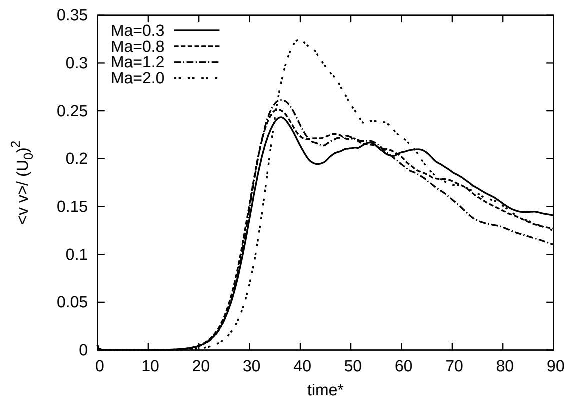

Figure 3. Evolution of the maximum cross-wake turbulent fluctuation (Favre-averaged). The time is normalized using the growth rate from linear stability.

The study of linear stability theory on the wake reveals that two-dimensional anti-symmetric (sinuous) modes have the highest growth rate. Chen *et al.* [14] noted that for Mach numbers below 1.2, the most unstable symmetric (varicose) mode is perfectly two-dimensional. At a Mach number above 1.0, the symmetric mode with the greatest growth is found at an angle of approximately $50-55^{\circ}$ from the streamwise direction. As the calculated growth rate of the anti-symmetric modes are about an order of magnitude greater than the symmetric modes, it is generally accepted that the wake maintains its two-dimensionality during the linear evolution. But by investigating the two-point correlation of the streamwise vortic-

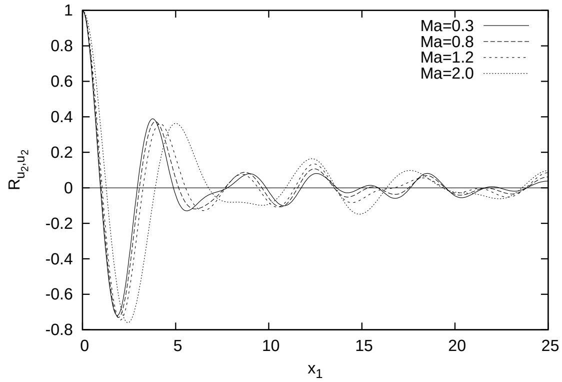

Figure 4. Two-point correlation of the cross-wake velocity (v) with a streamwise separation (x) along the plane of peak vorticity magnitude at time $t = 30$.

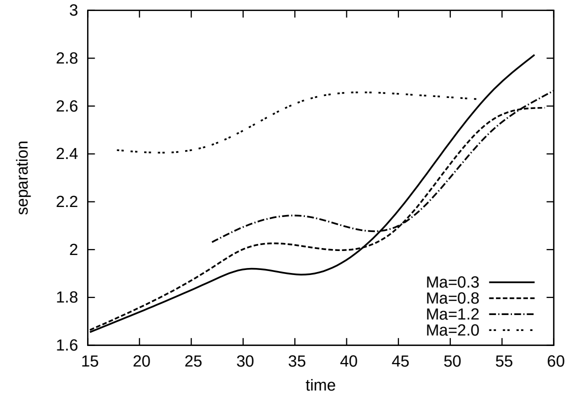

Figure 6. Evolution of the size of the roller as calculated by the first positive peak of the two-point correlation of the cross-wake velocity.

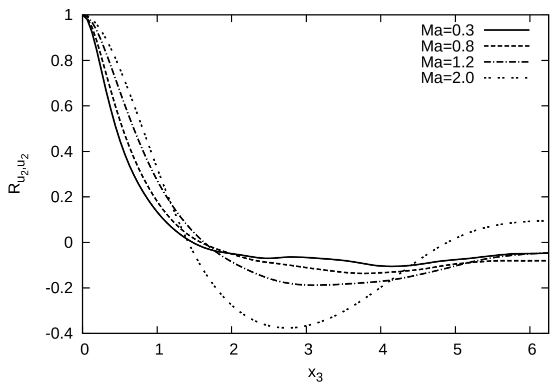

Figure 5. Two-point correlation of the streamwise vorticity (ω_x) with a spanwise separation (z) along the plane of peak vorticity magnitude at time $t = 30$.

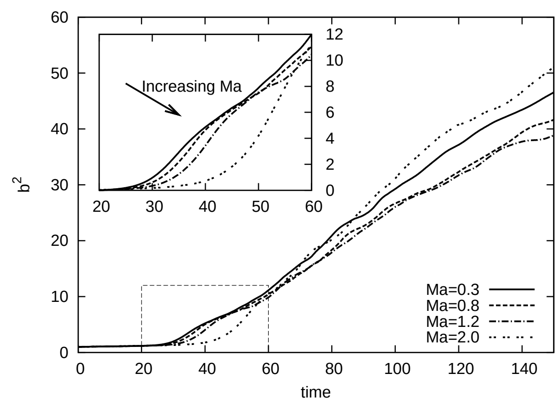

Figure 7. Evolution of the lateral spreading in the wake. Inset shows spreading during transition to turbulence.

ity with spanwise separation, figure 5, we note an increased three-dimensionality with the Mach number. Using the peaks in the two-point correlation in the stream-(4) and spanwise (5) directions, we infer an inclination of about 66°. Similar experimental work on transitional planar wakes for a freestream Mach number of 2.0 has also reported three-dimensional perturbations to be the most unstable Lysenko [22]. It was suggested that the receptivity of the wake to the varicose modes may result in an optimal growth for a three-dimensional disturbance.

As observed in the incompressible and compressible wakes [8, 17], the spreading (defined as db^2/dt) is constant in the intermediate and far-wake. Despite the modest scatter in the measured spreading rates, no distinctive trend is observed between different levels of compressibility, see figure 7. During transition, where the compressibility effects are the most prevalent, significant variation in the spreading rate is observed. As expected from linear stability theory, the increased Mach number stabilizes the flow which results in a delayed transition. But unlike the velocity perturbations, the lateral spreading does not scale with the theoretically calculated growth rate of the wake. A striking feature during transition is the higher spreading rate with increased Mach number which is shown in the inset of figure 7. As the pairing of vortical structures is the main mechanism responsible for the lateral spreading of the wake during transition [18, 19], the larger structures (resulting from the longer instability modes of the high Mach number) lead to a faster spreading of the wake.

Structural features

We visualize the flow using the λ_2-method by [20] in the transitional regime. Admittedly, the physical definition of the second eigenvalue of the velocity tensor is somewhat lost for dilatational flows. But using the assumption that the compressibility of the turbulence is negligible compared to the compressibility of the mean, the visualization of the structures in compressible case can be approximated by this method. To confirm that these structures are significant, we plotted the isosurface of the minimum pressure and found a general agreement with the λ_2 structures. Figure 8 presents the structures for the limiting cases of Ma =0.3 and 2.0 at a similar stage of transition based on the normalized time of $t \approx 36$ (based on the growth rate of the instability modes). The overall structures in the high-speed wake differ rather significantly from those observed in the temporally evolving plane mixing layer which show a very distinctive diamond pattern [21]. A clear difference can be seen between the essentially incompressible and supersonic cases; a close up on a single structure in figure 9 highlights the main features. The braids in the case at Ma =2.0 have a greater alignment ($\approx +/-25°$) in the streamwise direction, while the low Mach number case has a greater inclination ($\approx +/-45°$). The greater alignment of the braids for Mach 2.0 explains the observed negative correlation with spanwise separation in figure 5. It could be inferred from these figures that the longer wavelength, which results in an increased spacing between rollers, allows for a greater alignment of the secondary structures along the streamwise direction, although, a further study is necessary to confirm a causal link between the wavelength and angle of the braids. The phenomenological features of a three-dimensional instability mode appear in our present high-Mach number simulation, namely in the structures (fig. 8) and statistics (fig. 5). This is a rather surprising feature as both inviscid [14] and viscid [16] linear stability calculation clearly reveal a two-dimensional primary instability mode. The appearance of a three-dimensional mode at higher-Mach number might also explain the poorer scaling with the linear stability theory.

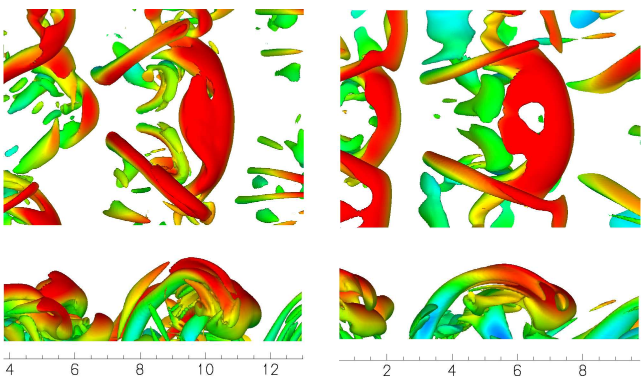

Figure 9. Close-up of one structure of the iso-surfaces of $\lambda = -0.10$ at normalized time $t \approx 36$. Mach number of 0.3 (left) and 2.0 (right). Top: $x-z$ plane. Bottom: $x-y$ plane. Only the top half of the wake is shown for clarity in all figures

CONCLUSION

We presented four temporally evolving direct numerical simulations of transitioning planar wakes. The free stream Mach numbers of 0.3, 0.8, 1.2 and 2.0 are chosen to cover a range from the incompressible limit well into the supersonic regime. The maximal growth rate of the inviscid linear stability theory collapses the turbulence statistics in the linear region of the flow of the low compressible flows. At Mach 2.0, the scaling does not agree as well with the linear evolution of the flow. The investigation of the primary wavelengths reveal an increased three-dimensionality with Mach number, a result is supported by experimental observations [22] but contradicts the linear stability results. A source of the three-dimensionality might be caused by the receptivity of the varicose mode which is unaccounted for the linear stability calculations. In opposition to the mixing layer, the high-speed wake undergoes a spreading rate increase with increased Mach number during transition as the larger structures, which are related to the longer wavelength of the principal instabilities, pair and effectively reduce the mean velocity gradients in the wake. The visualization of the braided structures reveals an increased streamwise alignment with increasing compressibility.

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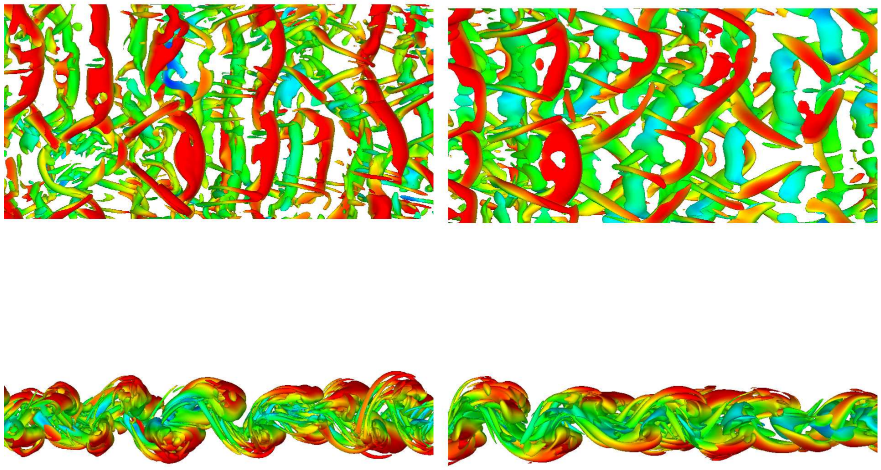

Figure 8. Iso-surfaces of $\lambda = -0.15$ at normalized time of $t \approx 36$ at freestream Mach number of 0.3 (left) and 2.0 (right) with *x*-*z* plane (top) and *x*-*y* plane (bottom). The flow is from left to right. The extent of the streamwise field represents half of the total computational domain.

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