TURBULENT COHERENT STRUCTURES IN A THERMALLY STABLE BOUNDARY LAYER

Owen Williams and Alexander J. Smits
Department of Mechanical and Aerospace Engineering
Princeton University
Princeton, NJ 08540
owilliam@princeton.edu

ABSTRACT

The effects of thermal stability on coherent structures in turbulent flat plate boundary layers are examined experimentally. Thermocouple and DPIV measurements are reported over a Richardson number range $0 < Ri_\delta < 0.2$. The reduction in wall shear and the damping of the turbulent stresses with increasing stability are qualitatively similar to that found by Ohya et al. (1996) including the major changes observed when the flow enters the strongly stable regime. In contrast, a critical bulk Richardson number of 0.05 is observed,which is much lower than the value of 0.25 found in this earlier study. In the weakly stable regime, hairpin vortices are seen to continue to populate the near-wall region and are elongated in the streamwise direction creating a smaller angle of inclination to the wall. With increasing stability, the angle of these structures continues to decrease and they are confined closer to the wall. In our experiments, the strongly stable flows show no evidence of large scale structures, or the presence of gravity waves.

INTRODUCTION

Thermally stable boundary layers are commonly found in arctic regions above the ice pack where the ice is typically at a lower temperature than the air flowing over it. Thermal stability causes a severe reduction in the turbulent fluxes and the heat transfer from the surface. Current General Circulation Models (GCM) are usually based on a form of Monin-Obukhov similarity theory, where the atmospheric surface layer is assumed to have either a constant vertical heat flux, or a modified form called local-scaling that uses a local heat flux. The vertical extent over which these theories are valid shrinks with increasing stability such that parameterizations based on them need to be significantly modified at stronger stratifications (Mahrt, 1998). For example, King et al. (2001) compared four such parameterizations in a coarse mesh simulation of the atmosphere over the Antarctic. They found a total surface heat flux variation of over 20 W/m^2 among the models, corresponding to surface average temperature differences of greater than $10°C$, indicating that there are still significant gaps in our understanding of these flows.

Stable boundary layers are generally classified as either weakly stable, corresponding to a nocturnal atmospheric boundary layer at moderate latitudes for which Monin-Obukhov similarity is valid, or strongly stable, represented by the arctic boundary layer for which current models are insufficient. Mahrt (1998, 1999) describes some of the important differences between these two regimes, including the increasing prominence of gravity waves, meandering motions, intermittency, increased anisotropy and the possible detachment of turbulence from the surface with intermittent recoupling. Gravity waves are believed to explain the existence of a counter-gradient flux sometimes observed at higher stratifications (Thorpe, 1972). Mahrt (1998) notes that a single definition of the strongly stable regime remains controversial and elusive since all of these phenomena are rarely observed within the same study. A better understanding of the strongly stable regime also hampered by measurement difficulties because small fluxes necessitate better instrumentation and significantly longer averaging times.

Of particular interest is the possible existence of a critical stratification that describes the transition between the strongly and weakly stable regimes. There are many parameters that are used to describe the extent of thermal stratification but it is currently unclear which parameter is the most appropriate. Apart from the Monin-Obukhov length, the most commonly cited parameter is the gradient Richardson number:

$$Ri = \frac{g}{\Theta}\frac{\partial\Theta/\partial z}{(\partial U/\partial z)^2} \qquad (1)$$

which describes the relative influence of the stabilizing effect of buoyancy and the destabilizing effect of shear. Here, Θ is the potential temperature, U is the mean velocity, z is the wall-normal distance and g is the gravitational constant.

It has been shown that turbulent statistics such as streamwise intensity and Reynolds shear stress correlate well with this quantity (Arya 1974; Ohya et al. 1996). In addition, it was established by Miles (1961) and Howard (1961) that a

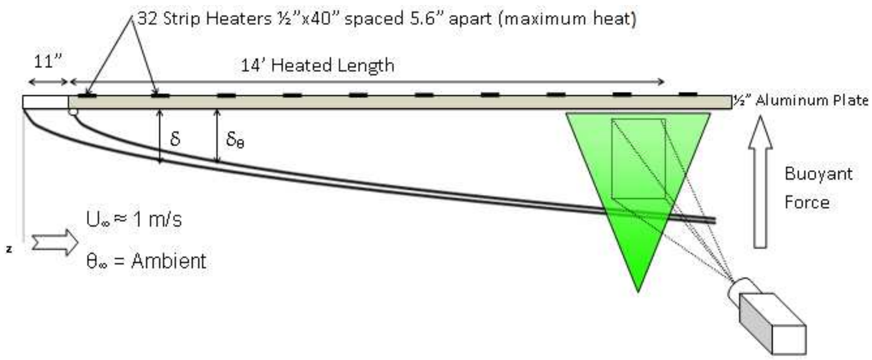


Figure 1. PIV setup to measure turbulent statistics of a thermally stable boundary layer developing on the underside of a heated plate

laminar, steady, inviscid flow will remain stable to small perturbations if $Ri > 0.25$ everywhere. This is a sufficient condition that was first predicted by Taylor (1931) and later verified experimentally by Scotti and Corcos (1971). This criterion has since been extended to compressible flows by Chimonas (1970) giving the same result. It should be noted, however, that unsteadiness in these flows has been shown cause instability at Richardson numbers greater than 0.25 (Majda and Shefter, 1998), possibly helping to explain some of the variability in the atmospheric data due to its natural transience. Here, due to the limits of our experiment, we will primarily consider the bulk Richardson number,

$$Ri_\delta = \frac{g\delta\overline{\Theta}_\infty}{T_o\overline{U}_\infty^2} \tag{2}$$

which is similar to the gradient Richardson number but where the gradients are evaluated across the entire layer. $\overline{\theta}_\infty$ is the temperature difference across the layer, T_o is the average absolute temperature, U_∞ is the freestream velocity, and δ is the boundary layer thickness.

Although the condition $Ri > 0.25$ has been shown to be a sufficient condition for the maintenance of laminar flow under certain conditions, it does not necessarily apply to the cessation of turbulence within an already turbulent flow. The first analysis of turbulent stratified flows by Richardson (1920) predicted that for $Ri > 1$ no turbulence would survive. Viscous dissipation was neglected in this analysis and this has since been found to be important for strongly stable flows. Recent experimental and observational studies have indicated, however, that this criterion is actually more robust than initially anticipated because turbulence actually has been observed to exist for $Ri >> 1$ (Galperin et al. 2007). Additionally, models of stratified turbulence that use a critical Richardson number as a threshold for the extinction of turbulence have been found to have insufficient mixing if the critical Richardson number $Ri_c < 1$ (see Galperin et al. 2007 for discussion). Recent work by Canuto (2001) showed that the presence of radiative losses and internal gravity waves acts to reduce stratification, further increasing the Richardson number required for the suppression of turbulent mixing. Strong stratification has also been observed to increase anisotropy and horizontal mixing even when vertical mixing has been largely suppressed. This observation leads Galperin et al. (2007) to conclude that a single critical Richardson number for the suppression of turbulence does not not exist.

Other works have used a flux Richardson number (Ri_f), defined as the ratio of work done against buoyant forces to the production of turbulent, two terms in the turbulent kinetic energy equation. That is,

$$Ri_f = \frac{g\frac{\overline{\theta w}}{\overline{\Theta}}}{\overline{uw}\frac{\partial U}{\partial z}} \tag{3}$$

Here, $\overline{\theta w}$ is the turbulent heat flux, $\overline{\Theta}$ is the local average temperature and $\overline{uw}$ is the Reynolds stress.

While the full problem of reverse transition due to stratification is presently intractable, simplified analyses based on equations of turbulent kinetic energy, mean square temperature fluctuations, and turbulent heat flux have been developed. Ellison (1957) first used this approach, modeling the dissipation terms as the ratio of the particular quantity to its decay time. Defining the critical stratification as that corresponding to a condition where continuous turbulence cannot be maintained, he arrived at a critical Richardson number of $Ri_f = 0.15$. A following study by Townsend (1958) based his model on an expected variation in turbulent Prandtl number, and suggested the threshold $Ri_f = 0.5$. Ayra (1972) improved on this approach with measured values, and found a critical value $Ri_f = 0.15 - 0.25$. These analyses allow for the fact that above this critical value intermittent turbulence can occur: the flux Richardson number is a local quantity that for a given flow can fluctuate above and below the critical stratification level.

It is difficult to match these critical Richardson number estimates with atmospheric observations as they are local quantities and the definitions of weak and strong stability are more macroscopic in nature. Additionally, it is unclear whether alternative global parameters such as the bulk Richardson number are sufficient to characterize the differences between these weakly and strongly stable flows.

There are only a limited number of previous laboratory experiments that have examined the effects of thermal stability on turbulence. It was found by Ayra (1974) ($Ri_\delta < 0.1$)

Experimental Case	NV4	T1V4	T2V4	T3V4	T4V4	T5V4	T6V4	T7V4
$\Delta\Theta_s$	0	19.7	42.8	60.4	76.3	100.0	115.5	128.9
Re_θ	1032	986	1048	990	901	907	813	773
Ri_δ	0	0.0087	0.035	0.048	0.060	0.077	0.088	0.097
u_τ/U_∞	0.048	-	-	-	-	-	-	-

Table 1. Flow properties with varying wall temperature but constant velocity of $U_\infty = 1.44m/s$

Experimental Case	T3V1	T3V2	T3V3	T3V4	T3V5	T3V6	T3V7	T3V8	T3V9
$U_\infty(m/s)$	0.96	1.12	1.28	1.43	1.61	1.78	2.08	2.42	2.63
Re_θ	696	774.8	841	990	1169	1370	1533	1803	2040
Ri_δ	0.11	0.079	0.061	0.048	0.038	0.031	0.023	0.017	0.014

Table 2. Flow properties with varying velocity but constant $\Delta\Theta_s = 60$

and also Ogawa et al. (1985) $(Ri_\delta < 0.25)$ that, unlike unstable flows, the mean velocity profile shows significant deviations from the neutral case even at moderate stratifications. They also observed a marked reduction in turbulent intensity and fluxes with increasing Richardson number. Further investigations by Ohya et al. (1996) $(Ri_\delta < 1.33)$ found only gradual deviation of turbulent quantities from the neutral case for weak stratification, but significant reductions in turbulent intensity were found at stronger stratifications. Furthermore, the near-wall turbulence peak moved away from the wall with increasing stability. A critical stratification, $Ri_\delta = 0.25$, was found to separate these two regimes, a value that agrees with the analysis of Miles-Howard theory for laminar flow or Arya (1972) for the onset of intermittancy.

These experiments were limited to single point measurements of velocity and temperature, and little information on the behavior of the turbulent structure is currently available. Here, we investigate the differences between weakly and strongly stable flows, and examine the changes in the coherent structures, such as the nature of the hairpin vortices, with increasing stability. Strongly stable flows were also examined for the presence buoyantly driven structures such as gravity waves.

EXPERIMENTAL APPARATUS

The experiments were conducted in 5 m long, 1.2 m by 0.9 m cross-section, open-return wind tunnel that was modified to study thermally stratified flows. The tunnel was operated at freestream velocities between $0.8 \leq U_e \leq 2.5$m/s. The upper surface of the tunnel was replaced with a 12.7 mm thick aluminum plate backed with strips of heating tape allowing the plate to be heated isothermally. Eight thermocouples were mounted on the centerline of the plate to ensure that this condition was maintained. The freestream temperature was also measured using a thermocouple. The flow was tripped using a 6.35 mm rod mounted to the leading edge of the plate, just after the convergent section of the tunnel. The experimental apparatus is shown in Figure 1.

The experiment was conducted at nine velocities (V1-V9) for each of eight wall temperatures. Including the neutrally stable case, they were labelled N, S1–S7. The temperature difference between the wall and freestream, $\Delta\Theta_s$ varied between zero (N) and $130°C$ (S7). The corresponding Richardson number and Reynolds number ranges were $0 \leq Ri_\delta \leq 0.2$ and $600 \leq Re_\theta \leq 2050$.

Particle Image Velocimetry (PIV) was used determine the velocity field in a plane containing the wall-normal and streamwise directions. A New Wave Tempest and Gemini dual head ND:YAG laser system was used as the laser source. Each laser delivers 100 mJ energy per pulse at a wavelength of 532 nm. The flow was imaged with a PCO.1600 Camera with an interframe time of $300\mu s$. Seeding was generated using an MGD Max 3000 APS mineral oil based fog generator. It was injected into a large enclosure attached to the inlet of the tunnel allowing the particles to be well-mixed with the incoming air before entering the tunnel inlet.

The PIV images were processed using the a modified WIDIM code detailed in Scarano and Riethmuller (2000). It is an adaptive multigrid scheme that uses iterative image deformation to enhance correlation and reduce peak-locking. The final window size was 32×32 pixels with 50% window overlap. The regression filter was set to 2. The internal signal to noise filter was disabled because it was found to have negligible impact on statistical resuls while requiring a large proportion of vectors to be interpolated. In some higher stability cases, near-wall seeding was found to be insufficient due to the strong local density gradient and low levels of mixing. These regions were cropped from the images and results.

The field of view of PIV measurements was approximately half the boundary layer height so the remaining mean velocity profile was measured using a Pitot tube. The static pressure was measured using a static pressure probe mounted in the freestream. An Omega PX653-0.05BD5V high accuracy, pressure transducer was used. Using these profiles, the boundary layer thickness and freestream velocity could be estimated. Due to the low dynamic pressures involved and the variation in density across the layer, the Pitot tube profiles measured at higher wall temperatures were found to be unreliable. Therefore, the boundary layer thicknesses and freestream velocities found in the neutral case were used to non-dimensionalize the data for the stable cases, as well.

RESULTS AND DISCUSSION

As data was taken varying both temperature and velocity, two sets of statistics will be shown, keeping one of these variables constant. The case with constant velocity enables us to examine the statistics with the smallest Reynolds number variation. Other data sets show very similar trends and are not included. Tables 1 and 2 list the global properties of each of these flows.

The mean velocity profiles, shown in Figure 2(a), show a strong reduction in wall shear as the increasing level of stability decreases turbulent mixing. The strongest stability cases are almost laminar in nature. These profiles are qualitatively very similar to those shown by Ayra (1974) and Ohya et al. (1996) .

The damping of turbulence is clearly seen in Figures 2(b) and 2(c), and the data are in good qualitative agreement with the results obtained by Ohya et al. (1996) . As with previous studies, the profiles can be divided into two regimes: the weakly stable, with minor reductions in turbulence intensity and shear stress, and the strongly stable where the turbulent stresses are significantly damped. The strongly stable stable profiles are also observed have a fundamentally different shape, with the peak in turbulence intensity moving away from the wall. This phenomena was also observed in Ohya et al. (1996). Case T3V4, common to both figures, appears to represent a transitional state between these two regimes and we will refer to it as the critical case. One of the most interesting aspects of our results was the critical Richardson number. It was found to be $Ri_\delta = 0.05$, which is much lower than the critical values measured by Ohya et al. (1996) or predicted by Miles-Howard theory (Miles, 1961) or Arya (1972).

To examine whether this discrepancy is a Reynolds number effect, Figure 3 plots Re_θ against Ri_δ for all the cases studied in our experiment. The data were divided into weakly and strongly stable categories based on the behavior of the turbulent intensity profiles. While the Reynolds number range near the critical value is small in extent, it can be seen that the value of 0.05 defines a clear threshold above which the flow becomes strongly stable. Although Ohya et al (1996) do not quote momentum thickness Reynolds numbers, they were estimated from mean velocity profiles to be in the range $2500 \leq Re_\theta \leq 5000$ and thus it seems unlikely that the differences between these two critical Richardson numbers can be ascribed to Reynolds number differences.

The source of the discrepancy in critical Richardson number is unclear. The statistics show a finite turbulence intensity within the strongly stable regime whereas the Reynolds stress is almost identically zero. It is possible that the remaining turbulence is uncorrelated noise and the

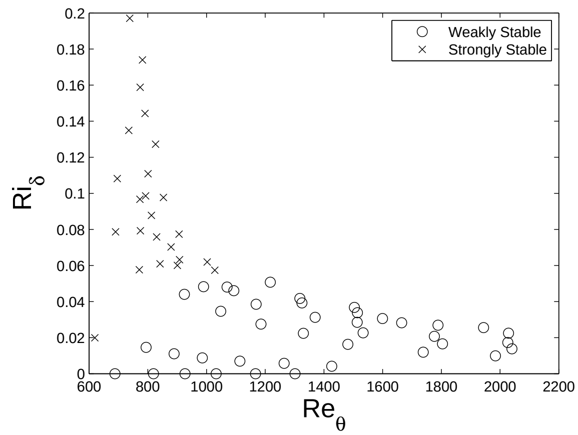


Figure 3. Reynolds and Richardson numbers for all cases studied here.

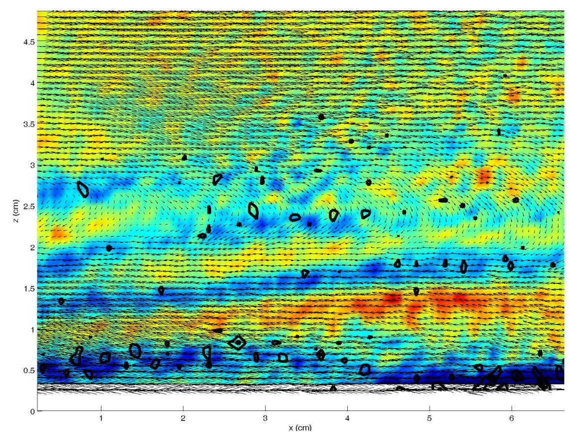


Figure 4. Structure of boundary layer for T3V4 ($Ri_\delta = 0.048$). Symbols as in Figure .

flow is in fact fully relaminarized. Alternatively, the peak in Reynolds stress observed by Ohya et al. (1996) occurred within the outer layer of the boundary layer and it is possible that this was missed in our study due to the restricted field of view. In addition, it is possible that these flows are more sensitive to initial conditions than previously thought since $Ri < 0.25$ is not large enough to force the laminar flow to remain laminar when tripped. Further studies will investigate the effect of trip wire size and wall roughness on critical Richardson number.

The PIV data were then used to examine the changes in turbulent structure between the weakly and strongly stable cases. Structure was examined by plotting velocity vectors and contours of constant swirl criteria over vorticity fields, as recommended by Adrian et al. (2000).

Representative results for the neutral, weakly stable and strongly stable regimes are shown in Figure 5 . Hairpin vortices with an inclination angle of approximately 45° were seen in the neutrally stable case and what appeared to be "older" hairpins were observed further away from the wall, in line with the model proposed by Adrian (2007). These older structures were not observed in the weakly stable regime, and the near-wall hairpins were stretched in the downstream direction, apparently in response to the additional work necessary to overcome the vertical density gradient. This characteristic structure angle was observed to continue to decrease with Ri_{δ}.

Figure 5(c) shows that once the flow had progressed to the strongly stable state, all large scale turbulent structure has been suppressed. Additionally, no gravity waves were observed. This is consistent with the observed reduction in turbulence intensity.

A snapshot of the critical case (T3V4) is shown in Figure 4. The structure is a combination of the weakly and strongly stable regimes, with stretched hairpin vortices of significantly reduced strength appearing intermittently. This intermittency is most likely connected with fluctuations in the local Richardson number variation. Conventionally, it is thought that as turbulence is reduced, shear begins to build up due to increased stratification. This should then trigger a shear instability such as Kelvin-Helmoltz waves which increase mixing, reducing stratification until they are then damped out. No strong gravity waves were observed within this experiment, possibly increasing the sharpness of the transition between weakly and strongly stable flows.

CONCLUSIONS

Mean and fluctuating turbulent statistics were measured within a thermally stable boundary layer using PIV. The wall shear was found to reduce significantly with increasing stability and mean velocity profiles approached the laminar case. Turbulent intensities and stresses could be separated into weakly stable and strongly stable regimes. These results were found to be qualitatively similar to the studies of Arya (1974) and Ohya et al. (1996), although the critical stratification between the two regimes was $Ri_{\delta} = 0.05$, which is significantly lower than that observed by Ohya et al. (1996). Within the weakly stable regime hairpin structures were observed to remain confined to the near-wall region and were elongated in the streamwise direction when compared with the neutral case. The angle of these structure was observed to continue to decrease with increasing stratification. Large-scale structure was found to have been damped within the strongly stable regime and no gravity waves were observed. As gravity waves are one mechanism that can increase local mixing, it is thought that their absence helped contribute to the sharpness of the observed transition to a strongly stable state. At critical stratification, hairpin vortices with a shallow angle to the freestream were intermittently observed in a flow that was otherwise strongly stable in nature.

ACKNOWLEDGEMENTS

We would like to thank Princeton University's Grand Challenges–Energy program and the Thomas and Stacey Siebel Foundation for funding this research.

REFERENCES

Adrian, R.J., Christensen, K.T. and Liu, Z.-C., 2000, "Analysis and Interpretation of instantaneous turbulent velocity fields", Experiments in Fluids, Vol. 29, pp. 275–290.

Adrian, R.J., 2007, "Hairpin vortex organization in wall turbulence", Physics of Fluids, Vol. 19, 041301.

Arya, S., 1972, "The critical condition for the maintenance of turbulence in stratified flows", Quarterly Journal of the Royal Meteorological Society, Vol. 98, pp. 224–235.

Arya, S., 1974, "Buoyancy effects in a horizontal flat plate boundary layer", J. of Fluid Mech., Vol. 68, pp. 321–343.

Canuto, V., 2002, "Critical Richardson numbers and gravity waves", Astronomy and Astrophysics Journal, Vol. 384, pp. 1119–1123.

Chimonas, G., 1970, "The extension of the Miles-Howard theorem to compressible fluids", Journal of Fluid Mechanics, Vol. 43, pp. 833–836.

Ellison, T., 1957, "Turbulent transport of heat and momentum from an infinite rough plane", Journal of Fluids Mechanics, Vol. 2, pp. 456–466.

Galperin, B., Sukoriansky, S. and Anderson, P., 2007, "On the critical Richardson number in stably stratified turbulence", Atmospheric Science Letters, Vol. 8, pp. 65–69.

Howard, L.N., 1961, "Note on a paper of John W. Miles", Journal of Fluid Mechanics, Vol. 10, pp. 433.

King, J., Connolley, W. and Derbyshire, S., 2001, "Sensitivity of modelled Antarctic climate to surface and boundary-layer flux parameterizations", Quart. J. Roy. Met. Soc., Vol. 11, pp. 263–279.

Mahrt, L., 1998, "Stratified atmospheric boundary layers and breakdown of models", Theoretical and Computational Fluid Dynamics, Vol. 127, pp. 119–194.

Mahrt, L., 1999, "Stratified atmospheric boundary layers", Boundary-Layer Meteorology, Vol. 90, pp. 375–396.

Majda, A.J. and Shefter, M., 1998, "The instability of stratified flows at large Richardson numbers", Proceedings of the National Academy of Sciences, Vol. 95, pp. 7850–7853.

Miles, J., 1961, "On the stability of heterogeneous shear flows", Journal of Fluid Mechanics, Vol. 10, pp. 496.

Ogawa, Y., Diosey, K., Uehara, K and Ueda, H., 1985, "Wind tunnel observation of flow and diffusion under stable stratification", Atmospheric Environment, Vol. 19, pp. 65–74.

Ohya, Y., Neff, D. and Meroney, R., 1996, "Turbulence structure in a stratified boundary layer under stable conditions", Boundary-Layer Meteorology, Vol. 83, pp. 139–161.

Richardson, L., 1920, "The supply of energy from and to atmospheric eddies", Proceedings of the Royal Society A, Vol. 97, pp. 354–373.

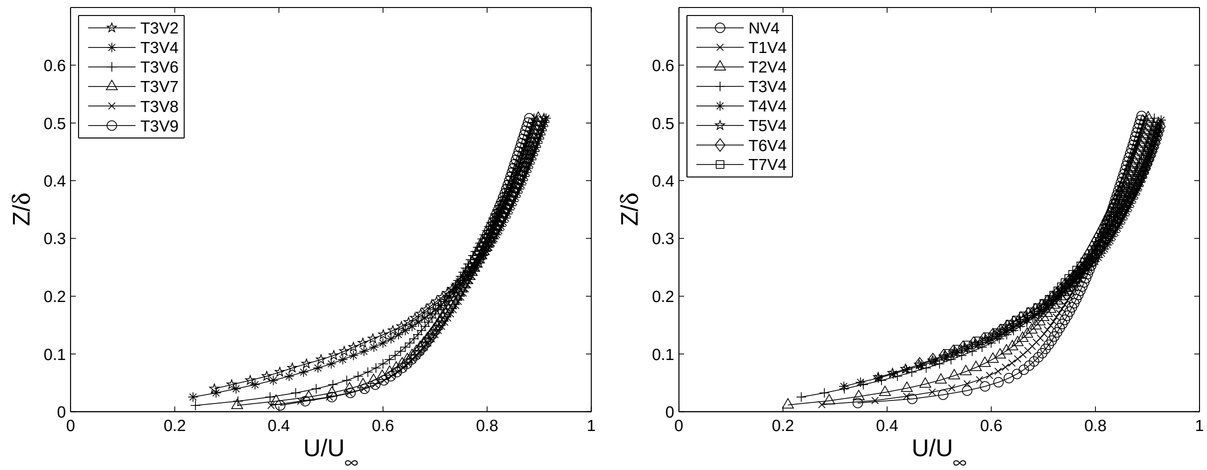
Scarano, F. and Riethmuller, M. L., 2000, "Advances in iterative multigrid PIV image processing", Experiments in Fluids, Vol. 29, pp. S051-S060.

Scotti, R.S. and Corcos, G.M., 1971, "An experiment on the stabililty of small disturbances in a stratified free shear layer", Journal of Fluid Mechanics, Vol. 53, pp. 499–528.

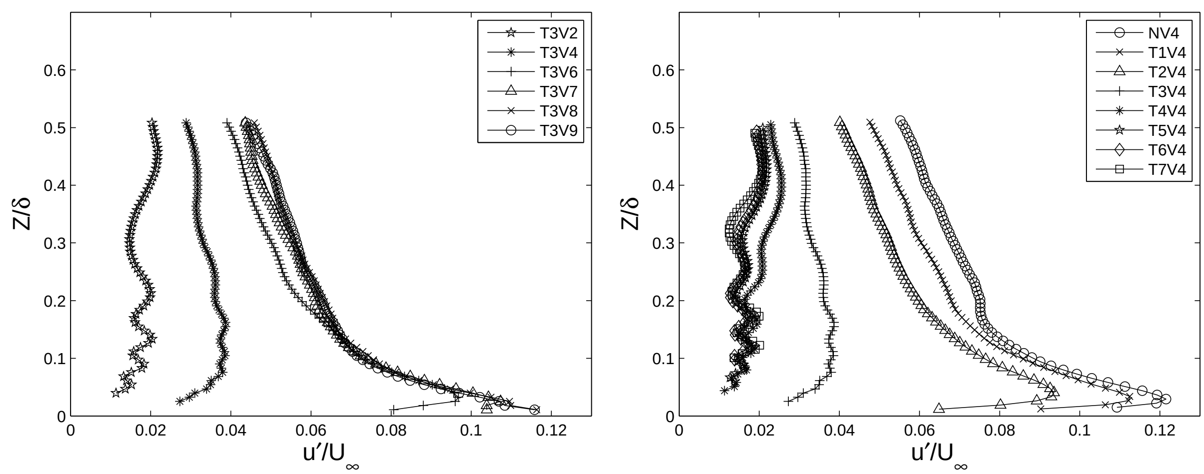
Taylor, G.I., 1931, "Effect of varation in density on teh stability of superimposed streams of fluid", Proceedings of the Royal Society A, Vol. 132, pp. 499

Thorpe, S., 1972, "Turbulence in stably stratified fluids: A review of laboratory experiments", Boundary-Layer Meteorology, Vol. 5, pp. 95–119.

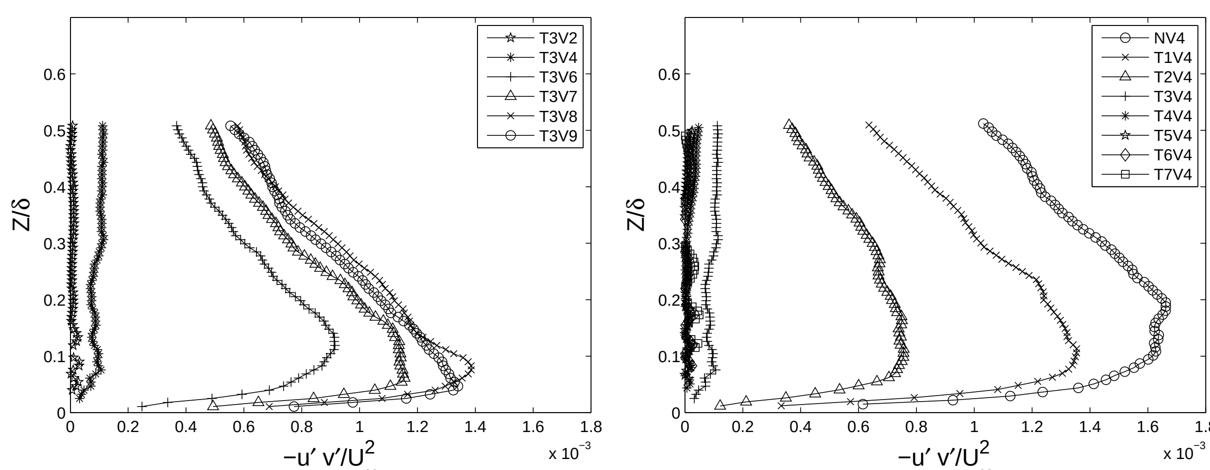
Townsend, A., 1958, "Turbulent flow in a stably stratified atmosphere", Journal of Fluid Mechanics, Vol. 3, pp. 361–372.



(a) Mean velocity profiles for varying freestream velocities (left), and varying wall temperatures (right)

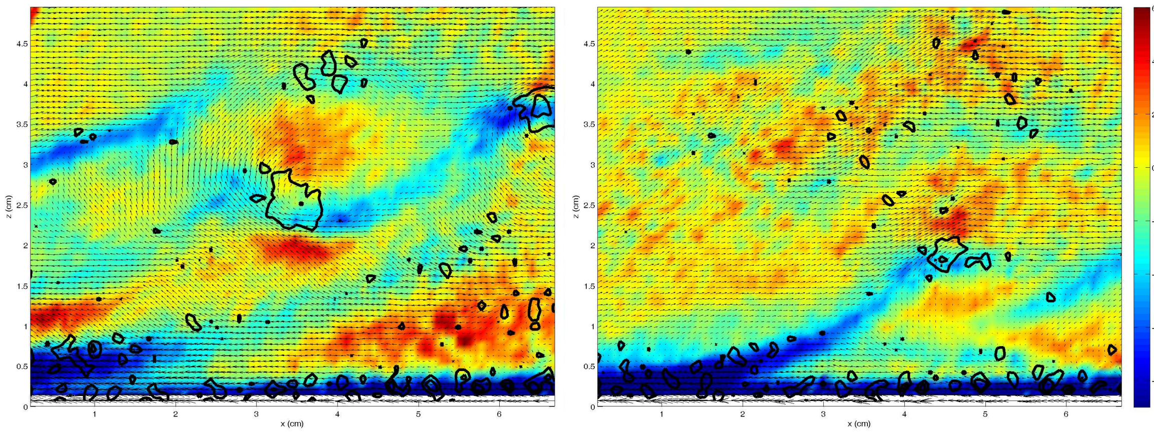


(b) Streamwise turbulent intensity profiles with varying velocity (left) and varying wall temperature (right)

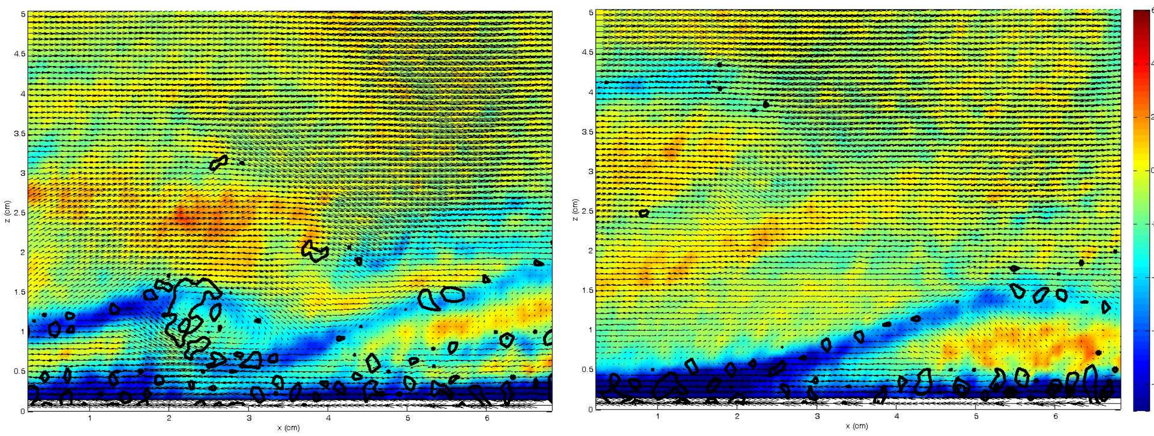


(c) Reynolds stress profiles with varying velocity (left) and varying wall temperature (right)

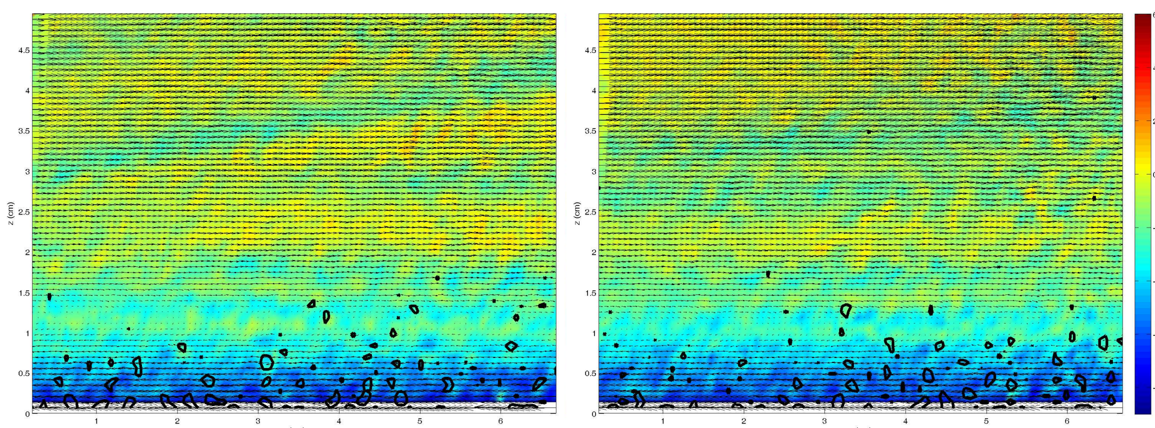
Figure 2. Variation in turbulent statistics with velocity (left) and wall temperature (right).



(a) Neutrally stable boundary layer



(b) Weakly stable (Ri_δ = 0.014)



(c) Strongly stable (Ri_δ = 0.08)

Figure 5. Turbulent structures within stratified and neutrally stable boundary layers. These are visualized using a combination of vorticity contours, swirl strength and a Galilean transform of the velocity field.