INCOMPLETE SIMILARITY OF A PLANE TURBULENT WALL JET

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ABSTRACT

Incomplete similarity for a plane turbulent wall jet is reported in this paper. Typically, a wall jet is divided into two regions: an inner layer and an outer layer. The degree to which these two layers reach equilibrium with each other, and produce a self-similar velocity profile remains an open question. In this paper, the theory of Barenblatt et al. (2005), which proposes incomplete similarity, is used to analyse the mean velocity field of a turbulent wall jet based on a new set of laser Doppler anemometry (LDA) measurements. The experiments were carried out in a water tank. The fluid was discharged with a velocity of U_o = 1.21 m/s through a rectangular slot with a height of H = 6 mm. The Reynolds number of the wall jet, based on the jet exit velocity and the slot height, was approximately Re = 7500. The similarity in both the inner and outer regions of the wall jet flow was studied in the nominally fully developed region of the jet, i.e. x/H > 30. The outer and inner half-widths are shown to develop differently in the streamwise direction when scaled with the jet slot width. Using outer scales, a good collapse of the experimental data can be seen in the outer region, while the profiles deviate from each other near the wall. In contrast, using the inner scales collapses the mean velocity profile in the inner region of the wall jet, but not in the outer region.

INTRODUCTION

A turbulent wall jet is a “shear flow directed along a wall where, by virtue of an initially supplied momentum, at any downstream location the streamwise velocity over some region within the flow exceeds that in the external stream” (Launder and Rodi, 1981). A definition sketch of this flow is shown in Figure 1. In this figure, x and y denote the streamwise and wall-normal distances, respectively; U is the streamwise component of the mean velocity; U_o is the jet exit velocity; H is the slot height; U_m is the maximum velocity; y_m is the wall-normal location where U_m occurs; and $(y_{1/2})_{out}$ and $(y_{1/2})_{in}$ are, respectively, the wall-normal locations in the outer and inner regions of the wall jet where $U = U_m/2$ occurs. A wall jet flow is conventionally divided into two regions: the inner layer and outer layer. The inner layer, which has many of the characteristics of a turbulent boundary layer, extends from the wall up to $y = y_m$, while the outer region, which is structurally similar to a free jet, stretches from $y = y_m$ to the outer edge of the flow.

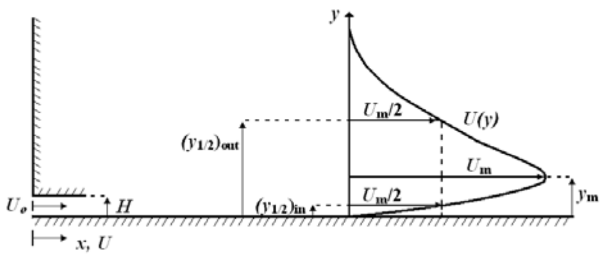

Figure 1. The schematic representation of the wall jet.

The interaction between these two layers makes the wall jet a complex and potentially insightful flow for turbulence research. The outer flow is characterized by large-scale turbulent mixing, and the inner flow is characterized by wall shear. Whether these layers reach complete equilibrium with a self-similar velocity profile to some degree remains an open question. Similarity is typically investigated with reference to a specific set of scaling laws.

Since wall jet flows frequently occur in environmental and industrial applications, they have been studied extensively. Applications of wall jets include separation control on airfoils, film cooling of the walls of combustion chambers in gas turbine engines, and space heating and air conditioning flows in buildings (Gad-el Hak and Bushnell, 1965; Hammond, 1982). In these applications, knowledge of the skin friction and the velocity and scalar fields enables one to calculate the friction forces as well as heat and mass transfer rates, and also to better understand mixing and dilution phenomena in these flows.

There have been many studies of plane wall jets on smooth surfaces. Hammond (1982) obtained a composite velocity profile for the inner and outer regions of the wall jet by coupling Spalding's formula for the inner layer with a sine function for the wake component. He also proposed an approximate skin friction formula based on the initial conditions of the wall jet. Wygnanski et al. (1992) studied a wall jet flow over a smooth surface using hot-wire anemometry (HWA) and analyzed the results using the momentum-viscosity scaling proposed by Narasimha et al. (1973). They used three different methods to estimate the skin friction. In particular, they found that use of a similarity profile in a momentum balance provided reasonable estimates of the local friction velocity. The scaling of Narasimha et al. (1973) implies that the flow itself would not retain any memory of the slot width or maximum velocity, but only the initial momentum flux. Eriksson et al. (1998) investigated both the mean and turbulent velocity fields of a wall jet on a smooth surface. They obtained very high quality experimental data using laser Doppler anemometry (LDA) with a relatively small measurement volume. In the region $40 < x/H < 150$, their data were found to be reasonably self-similar. George et al. (2000) proposed a theory based on the similarity analysis of the governing equations in a wall jet without an external stream. They compared their theoretical results with the experimental data of Eriksson et al. (1998) as well as those of Karlsson et al. (1988, 1992). They also presented a theoretical relation for the friction velocity. Afzal (2005) developed his own set of scaling laws for the velocity profile in the overlap region of a turbulent wall jet. Finally, Barenblatt et al. (2005) proposed a theory of incomplete similarity for the entire flow field of the turbulent wall jet, and evaluated their theoretical result using the high-quality experimental data of Karlsson et al. (1991). In contrast to George et al. (2000), they concluded that there is no single self-similar structure in the wall jet to which one can apply the scaling laws. Instead, they proposed a triple-layer structure for a wall jet flow consisting of two self-similar inner and outer regions separated by a mixing layer. In their theory, the inner and outer regions are characterised by separate scaling relations, which reflect the different mechanisms governing the inner and outer shear layers.

The main objective of this paper is to study the scaling laws for the mean velocity profile in a plane turbulent wall jet flow. More specifically, the incomplete similarity theory of Barenblatt et al. (2005) will be assessed based on a new set of measurements.

EXPERIMENTAL APPARATUS

The experiments were carried out in a water tank with a length, width and height of 4.16 m, 1.28 m and 1.7 m, respectively. The water flow was supplied by a pump which discharged through a rectangular slot at a jet exit velocity of approximately $U_o = 1.21$ m/s. The slot had a width of 750 mm and height of $H = 6$ mm, so that the jet can be considered two-dimensional. Velocity measurements were carried out at different streamwise positions measured from the jet exit up to $x = 80H$. The Reynolds number of the wall jet, based on the jet exit velocity (obtained from the integral of the velocity across the slot) and the slot height, was approximately $Re = 7500$. All measurements were made at a water temperature of 22°C. A glass plate was used for the smooth surface with a length of 1.01 m, width of 0.86 m, and thickness of 0.0127 m.

The velocity measurements were made using a two-component LDA system with a burst mode processor supplied by Dantec Inc. The LDA system was powered by a 750 − mW argon ion laser. The measurement volume sizes were 0.184 × 3.88 mm and 0.194 × 4.09 mm for the streamwise and wall-normal velocity components, respectively. Version 4.10 of the BSA Flow software was used for data collection and reduction. Hollow glass beads with an average diameter of 10 μm were used to uniformly seed the flow. One of the characteristics of the present wall jet apparatus is the use of a special nozzle configuration which produced a remarkably uniform velocity profile at the slot exit. This facilitated an accurate measurement of the exit momentum, M_o. The turbulence intensity in the central region of the jet at the exit plane was approximately 0.7%.

RESULTS AND DISCUSSION

In the present work, the similarity in both the inner and outer regions of the wall jet was studied based on a set of mean streamwise velocity measurements in the nominally fully developed region of the jet, i.e. $x/H > 30$. The scaling law for the mean velocity in the outer region typically uses the outer half-width as the length scale, i.e.

$$\frac{U}{U_m} = f\left(\frac{y}{(y_{1/2})_{out}}\right). \quad (1)$$

For example, Afzal (2005) proposed the following expression for the mean velocity profile based on a gradient transport model:

$$\frac{U}{U_m} = \mathrm{sech}^2\left(0.881\frac{y}{(y_{1/2})_{out}}\right). \quad (2)$$

The present experimental data and the data collected by Eriksson et al. (1998) are shown in Figure 2. In this figure, the mean velocity is normalized by the wall jet maximum velocity, while the outer half-width, $(y_{1/2})_{out}$, was selected as the length scale. According to the figure, there is good agreement between the measurements at different streamwise locations and the data of Eriksson et al. (1998) using the outer velocity and length scales. However, some discrepancies are observed at the edge of the jet, which is also a region of higher measurement uncertainty. Figure 2 also shows the similarity profile for the mean velocity profile in the outer region as proposed by Afzal (2005). The profile only matches the data over a narrow range of the outer region, i.e. from approximately $y/y_{1/2} \approx 0.5$ to $y/y_{1/2} \approx 1.1$.

In their incomplete similarity theory for a plane turbulent wall jet, Barenblatt et al. (2005) showed that each of the two self-similar layers was described by a different scaling law for the respective half-width based on the slot width H, i.e.

$$\frac{(y_{1/2})_{out}}{H} = A_o\left(\frac{x}{H}\right)^{\gamma_o} \quad (3)$$

and

$$\frac{(y_{1/2})_{in}}{H} = A_i\left(\frac{x}{H}\right)^{\gamma_i} \quad (4)$$

where A_o and γ_o are the power law constants for the outer layer of the wall jet, while A_i and γ_i pertain to the inner layer. Note that any deviation of the exponents γ_o and γ_i from unity implies a continuing dependence of the flow on the slot width.

Barenblatt et al. (2005) reported values of $\gamma_o = 0.93$ and $\gamma_i = 0.68$ based on the data of Karlsson et al. (1991), while in the present work values of 0.78 and 0.50 were obtained for γ_o and γ_i, respectively. The present data set is located closer to the slot exit and has a lower slot Reynolds number than the data used by Barenblatt et al. (2005) for their analysis.

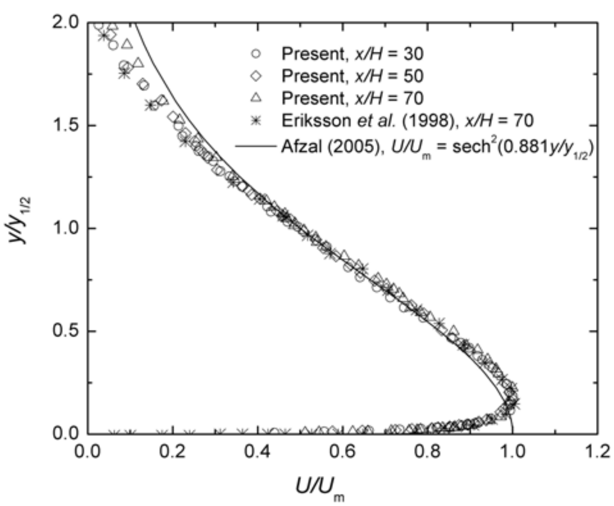

Figure 2. Mean flow velocity profile using outer length scales.

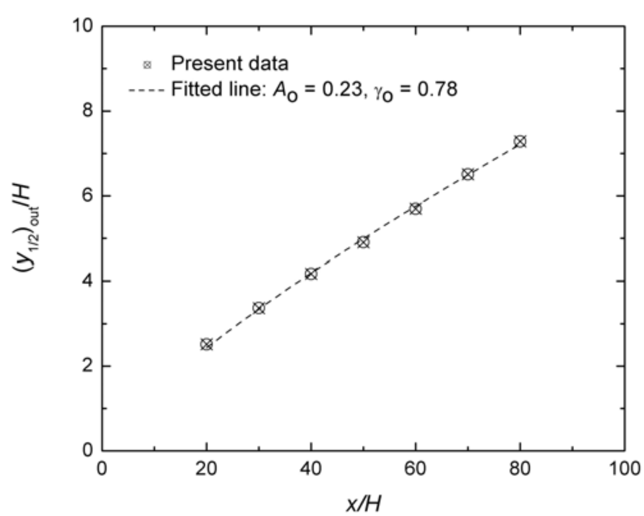

Figure 3. The wall jet spread rate in the outer region.

Figure 3 shows the streamwise variation of the jet half-width in the outer region, and gives the corresponding values of the power law constants obtained by fitting the experimental data to Equation 3. Note that both the outer half-width and streamwise distance are normalized by the slot width *H*. For each downstream section, the value of $(y_{1/2})_{out}$ was estimated directly from the mean velocity profile. The uncertainty in the estimation of the outer half-width was approximately ± 3%.

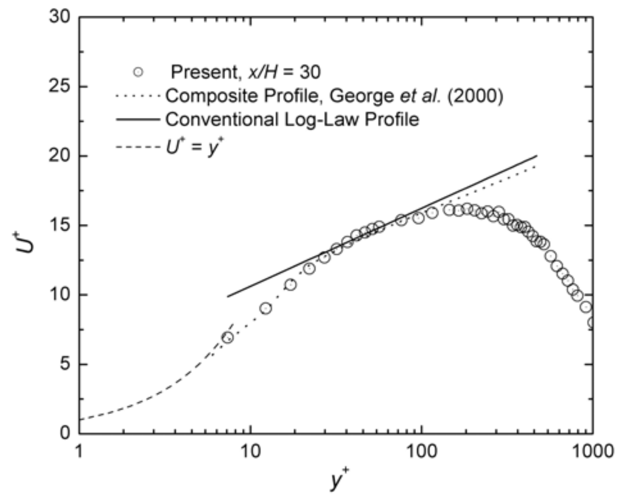

Figure 4. Mean velocity profile using inner coordinates.

Turning consideration now to the inner layer, for the present data set, the inner half-width was located very close to the wall, so that the size of the LDA measurement volume did not always allow for a direct measurement. Figure 4 shows a representative mean velocity profile at x/H = 30 using inner coordinates. In this case, the closest distance to the wall where the present LDA system was able to measure the velocity was approximately y^+ = 7, which was sufficient to enable the inner half-width to be resolved directly. However, the value of the inner half-width at three other sections (x/H = 20, 40, 70) could not be resolved directly, and instead was estimated from the experimental measurements by extrapolating the data to the wall. The extrapolation used for the velocity in the buffer layer of the jet was based on the composite profile proposed by George et al. (2000).

Figure 5 shows the variation of the inner half-width, $(y_{1/2})_{in}$, with respect to the downstream distance from the jet exit, when the slot width is used as the length scale. It also gives the corresponding values of the power law constants obtained by fitting the experimental data to Equation 4. The uncertainty in the estimation of the inner half-width was approximately ± 7%.

Figure 6 shows the mean velocity profile at three downstream locations (x/H = 30, 50 and 70) using the outer scaling, i.e. A_o and γ_o. In this figure the maximum velocity,

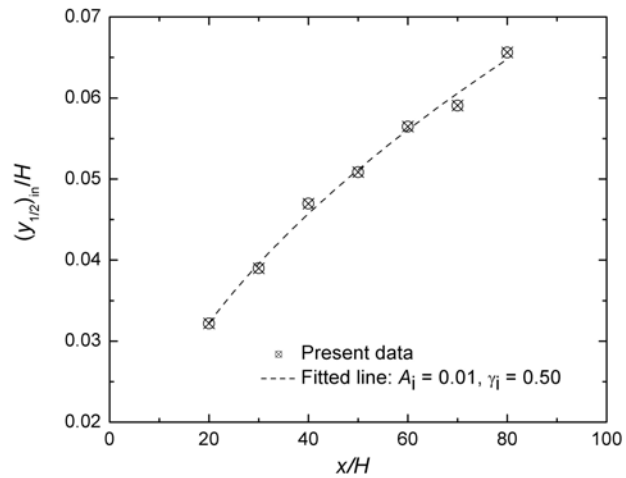

Figure 5. The wall jet spread rate in the inner region.

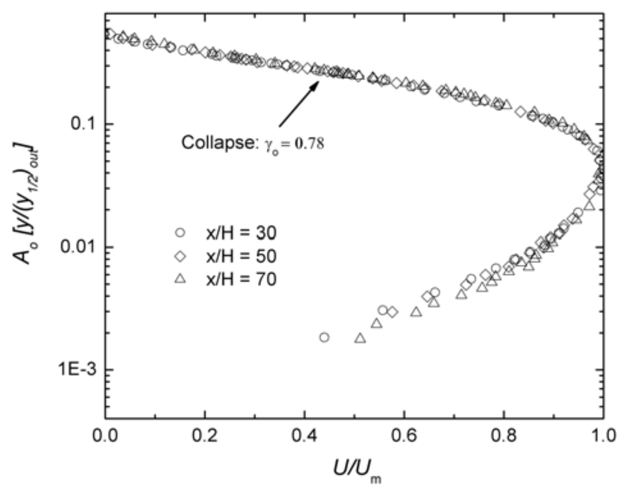

Figure 6. The mean velocity profiles of the turbulent wall jet using outer length scale.

U_m, was used to normalize the mean velocity and the wall jet outer half-width proposed by Barenblatt et al. (2005) was used as the length scale. A good collapse of the experimental data for all three sections is seen in the outer region of the jet, while the data sets begin to deviate from each other closer to the wall. This is consistent with the expectations of the theory of Barenblatt et al. (2005) who proposed the scaling used in this figure for the outer region. Note that within the inner region, the outer scaling fails. According to Figure 6, the present data collapse reasonably well in the outer region of the jet from the edge down to the location of $A_o(y/y_{1/2})_{out} \approx 0.05$.

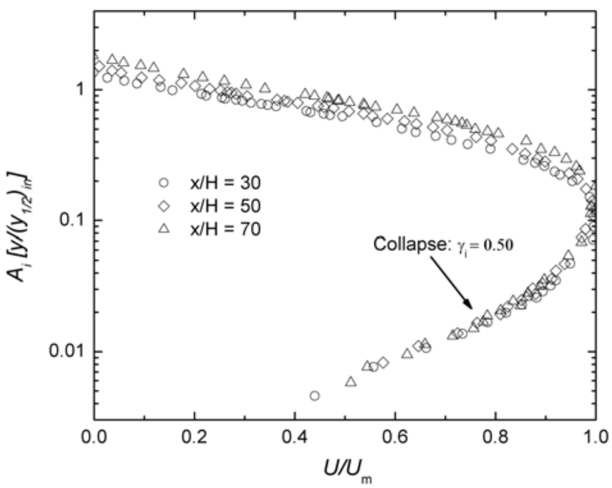

Figure 7. The mean velocity profiles of the turbulent wall jet using the inner length scale.

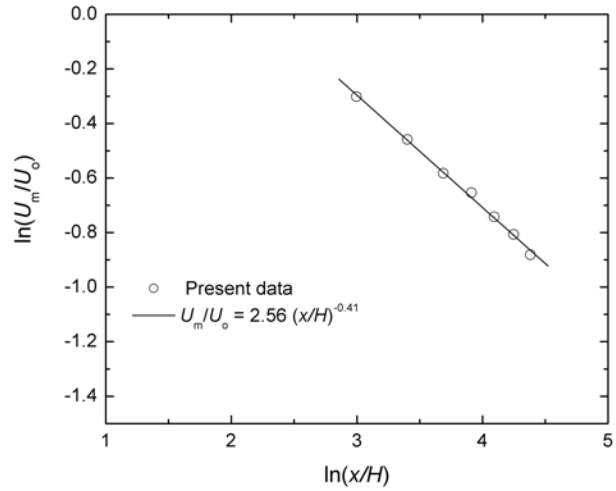

Figure 8. Decay of the maximum mean velocity.

Figure 7 shows the mean velocity profile for three downstream locations (x/H = 30, 50, and 70) using the inner scaling, i.e. A_i and γ_i . A good collapse of the mean velocity profiles for these three sections is obtained for the data closest to the wall, i.e. at the base of the inner region of the wall jet. Then, as y increases into the outer region, the data sets again begin to deviate from each other. In Figure 7, the three velocity profiles collapse reasonably well up to the location of $A_i(y/y_{1/2})_{in} \approx 0.02$. Use of the inner length scale clearly fails to collapse the data in the outer region of the wall jet.

The decay of the maximum velocity of the present wall jet is presented in Figure 8. The maximum velocity is shown to decay with respect to the distance from the jet exit in a power law form, when the slot width is used to normalise the streamwise distance from the exit. The decay rate of the jet is estimated to be -0.41, compared to the value of -0.6 obtained by Barenblatt et al. (2005) based on the data of Karlsson et al. (1991). Barenblatt et al. (2005) point out that the deviation of the decay rate from the value of -0.5 indicates that the parameter H cannot be neglected in scaling the mean velocity profile at different streamwise sections, which would be the approach implied by the assumption of complete similarity. Instead, the evidence is that the slot width as an initial condition continues to influence the streamwise development of the jet, at least for the extent of flow domain considered by the measurements.

CONCLUSIONS

An investigation of the theory of incomplete similarity in a turbulent plane wall jet on a smooth surface based on a new set of LDA measurements is reported in the present study. The present data were collected in the nominally fully developed region of the wall jet, i.e. $x/H \geq 30$. The Reynolds number of the wall jet was Re = 7500 based on the jet exit velocity and the slot height. In order to study the similarity of the wall jet, the scaling laws proposed by Barenblatt et al. (2005) were applied to the present data. The present results agree with their conclusions based on the data of Karlsson et al. (1991), i.e. that the outer and inner half-widths of a plane wall jet exhibit a separate and nonlinear dependence on the slot width H. The values of the present power law constants are different from those proposed by Barenblatt et al. (2005), indicating the sensitivity of the constants to the facility-dependent boundary conditions. The present results also showed that when scaled with the outer length scale (half-width), the mean velocity profiles collapse well in the outer region and deviate in the inner region of the wall jet close to the surface. On the other hand, the mean velocity profiles collapse well in the inner region when scaled using the inner length scale (half-width) and deviate from each other as the wall-normal distance increases into the outer layer. The present results support the observation of Barenblatt et al. (2005) that the slot width, although a small parameter, cannot be neglected in analyzing the similarity of the flow.

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