INTERACTION BETWEEN TURBULENCE AND RADIATIVE HEAT TRANSFER IN SUPERSONIC CHANNEL FLOW

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ABSTRACT

The interaction between turbulence in a minimal supersonic channel and radiative heat transfer is studied using large-eddy simulation. The working fluid is pure water vapour with temperature-dependent specific heats and molecular transport coefficients. Its line spectra properties are represented with a Statistical Narrow Band correlated-k (SNB-cK) model. Computations with a fictitious gray gas resulting in a higher optical thickness are also performed. Results for the mean flow variables, Reynolds stresses and certain terms of their transport equations indicate that thermal radiation effects counteract compressibility effects.

Introduction

In turbulent gaseous combustion radiative heat transfer often plays an important role besides heat transfer by conduction and convection. Since thermal radiation is a phenomenon of much longer range than the other two, partial differential equations (the compressible Navier-Stokes equations) suffice to describe heat conduction and convection mechanisms while an integro-differential equation is needed to predict the directional dependence of the radiative intensity inside absorbing/emitting gases. A further complication arises from the fact that radiative properties of gases vary strongly with the wavenumber of radiation (Modest, 2003). Due to these difficulties which imply enormous costs of numerical simulations, effects of radiative heat transfer are frequently predicted by strongly simplified models or are neglected even in situations where they play a role. Coelho (2007) provides an extensive overview of current understanding of turbulence-radiation interaction (TRI) and its modelling in reactive low Mach number flows. While most work on TRI has been devoted to the influence of turbulence on radiation, little work has concentrated on the question of how radiation affects turbulence variables. It is the aim of this work to focus especially on the behaviour of the turbulence structure under the influence of thermal radiation in inert, compressible, channel flow. Large-eddy simulation of the flow field along with a Discrete Ordinates Method (DOM) for the radiation field is used as a compromise between computational cost and accuracy.

Mathematical model

The direct influence of radiation on the flow comes through a source term in the energy equation which reads:

$$\frac{\partial \rho E}{\partial t} + \frac{\partial \rho E u_j}{\partial x_j} - \frac{\partial}{\partial x_j}(-u_i p \delta_{ij} + u_i \tau_{ij} - q_j) = u_i f_i \quad (1)$$

Here ($\rho E = \rho(e + 0.5 u_i u_i)$) is the total energy and $q_j = q_{j,C} + q_{j,R}$ is the sum of heat fluxes due to conduction and radiation. f_i represents a body force which is needed to drive fully-developed channel flow with streamwise periodicity. The radiative source term, $\frac{\partial q_{j,R}}{\partial x_j}$ in the energy equation is obtained by integrating the radiative transfer equation (RTE) over all wavenumbers and directions. For an emitting-absorbing and non-scattering gas, the RTE reads (Modest, 2003):

$$\frac{dI_\eta}{ds} = \kappa_\eta I_{b\eta} - \kappa_\eta I_\eta \quad (2)$$

The spectral radiation intensity, I_η, depends on the wavenumber η, and on the direction $\vec{s}$ in which radiation propagates. Its time-dependency is neglected because the speed of light is much larger than any flow velocity. $\kappa_\eta, I_{b,\eta}$ denote the spectral absorption coefficient and Planck's black body radiation intensity, respectively. The first term on the right-hand-side of eq. (2) describes the gain of radiation intensity via emission and the second the loss by absorption. The radiative source term is obtained from (2) through integration over wavenumber η and solid angle Ω:

$$div\vec{q}_R = \int_0^\infty \kappa_\eta \left(4\pi I_{b\eta} - \int_{4\pi} I_\eta d\Omega\right) d\eta = \underbrace{4\kappa_P \sigma T^4}_{emission} - \underbrace{\int_0^\infty \int_{4\pi} \kappa_\eta I_\eta d\Omega d\eta}_{absorption} \qquad (3)$$

κ_P defines Planck's absorption coefficient which is an average over all wavenumbers:

$$\kappa_P = \frac{\int_0^\infty \kappa_\eta I_{b\eta} d\eta}{\int_0^\infty I_{b\eta} d\eta} = \frac{\pi}{\sigma T^4} \int_0^\infty \kappa_\eta I_{b\eta} d\eta \qquad (4)$$

σ is the Stefan-Boltzmann constant.

Numerical and computational details

The present configuration is that of a supersonic turbulent flow in a minimal channel with pure water vapour (being an important product of combustion, besides CO_2) as working fluid and black, no-slip surfaces kept at a constant temperature of $1000K$. To mimic a high-altitude supersonic flight situation and to keep the Reynolds number sufficiently low, the pressure level is kept at 0.05 bar. The compressible Navier-Stokes equations are solved numerically in a characteristics based form (Sesterhenn, 2001). While the molecular transport coefficients are computed efficiently using the code EGLib (Ern & Giovangigli, 1995), polynomial expressions are used to specify the temperature dependence of specific heats (Gardiner, 1984). The compact sixth-order central scheme (Lele, 1992) is chosen to discretize the convection and molecular transport terms, respectively. A third-order low-storage Runge-Kutta scheme (Williamson, 1980) advances the solution in time. The domain size in the streamwise (x), wall-normal (y) and spanwise (z) directions is $3.5H \times 2H \times 1.35H$ and it is discretized using $96 \times 141 \times 64$ grid points. Our LES approach is a variant of the approximate deconvolution method (ADM), namely the explicit filtering technique of Mathew *et al.* (2003). At each time step the discrete values of p, T, u_i defined on the Cartesian grid are filtered using a composite low-pass filter. We solve the RTE as in eq. (2), using the method of discrete ordinates (DOM), implemented in the code PRISSMA and the filtered pressure and temperature. The 3D RTE is solved on a Cartesian, structured mesh using finite volume discretization (Joseph *et al.*, 2005). A Statistical Narrow Band correlated-k (SNB-cK) model (Liu *et al.*, 2000) is mostly used here for spectral integration. The parameters for the Statistical Narrow Band model are described in Soufiani & Taine (1997). A gray gas model has also been used in one flow case to study effects of a higher optical thickness. The flow field and the radiation field are coupled using Message Passing Interface (MPI) such that PRISSMA gets the pressure and temperature fields at about every characteristic convective time interval and the compressible LES solver receives the radiative source term. The flow solver uses domain decomposition while the radiation solver employs parallelisation in wavenumbers while using the SNB-cK model and in directions while using the gray gas model. The channel walls are kept at a constant temperature of $1000K$ and are treated as black surfaces. Fully reflective boundary condition is used for radiation across the periodic planes.

Supersonic channel flow of water vapour with radiation using the SNB-cK model

The friction and bulk Reynolds and Mach numbers as defined below for this flow case are shown in Table 1.

$$Re_\tau = \bar{\rho}_w u_\tau H/\mu_w, \qquad M_\tau = u_\tau/\sqrt{\gamma R T_w}$$
$$Re_m = \bar{\rho}_m \bar{u}_m H/\mu_w, \qquad M_m = \bar{u}_m/\sqrt{\gamma R T_w}$$

The last column contains values of the optical thickness computed via the statistically averaged Planck mean absorption coefficient (eq. (4)):

$$\tau_H = \int_0^H \bar{\kappa}_P(y) dy$$

For very small values of τ_H, a linear relation holds between the fraction of radiated energy that is absorbed and the optical thickness. $\tau_H = 0.006$ hence indicates that about 0.6% of the energy emitted at the channel centre is absorbed by water vapour within the optical path travelled by photons from the channel centre to the wall. A way to increase the low optical thickness of the sytem would be to increase the channel height (H). This would lead to even higher Reynolds numbers which are not achievable using wall-resolved LES. Due to low optical thickness, the effects of radiation in this flow are expected to be weak. From the mean total energy equation, integrated in the wall-normal direction, we found that the radiative heat flux, in this case, is about 4% of the total heat flux at the wall. It is interesting to look at the instantaneous emission and absorption integrals (eq. (3)) in the channel. Fig. 1 (left) shows that emission is dominant in this flow and we are able to make out certain characteristic flow structures in the emission and temperature fields. We also note that the amount of absorption in the channel is, on an average, not more than 8% of emission at wall temperature. Figure 2 (left) shows that radiation leads to a decrease in the mean temperature in the channel core. The near-wall rise in mean temperature from $1000K$ at the wall to $1190K$ in the core region is due to kinetic energy dissipation. As a result of radiation, the temperature in the core is reduced by about $20K$. Consequently, the mean density (Figure 2 (right)) falls off more gently from its wall value, has higher values in the core region and varies inversely with the mean temperature (mean pressure is nearly constant in the wall-normal direction). The Van Driest transformed velocity in the log-layer moves towards the incompressible log-law (fig. 3 (left)) due to radiation. Since the mean temperature gradients are smaller when radiation is taken into account we see a decrease (about 25% in the log-layer) in RMS temperature fluctuations in this case (Fig. 3 (right)).

Re_τ	M_τ	Re_m	M_m	τ_H
1026 (1041)	0.0681 (0.068)	16477 (17059)	1.26(1.32)	0.006

Table 1. Flow parameters for supersonic turbulent channel flow of water vapour (values in brackets correspond to the case without radiation).

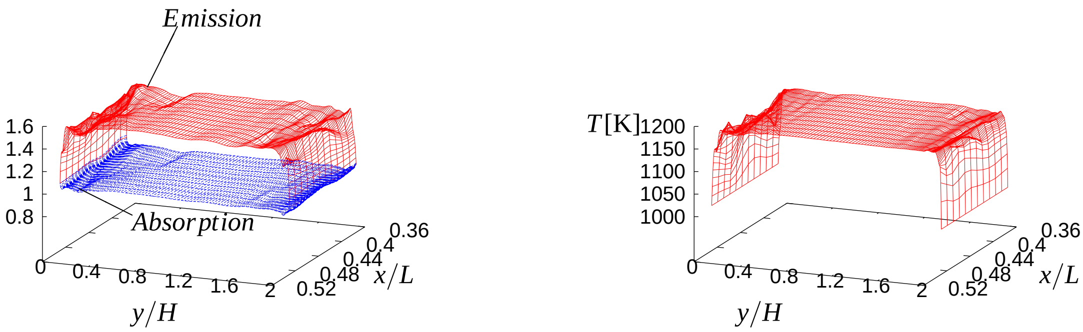

Figure 1. Instantaneous emission and absorption integrals (eq. 3), normalized with emission at wall temperature (left) and instantaneous temperaure (right) in the 1/8th of the wall-normal plane at $z/H = 0.675$.

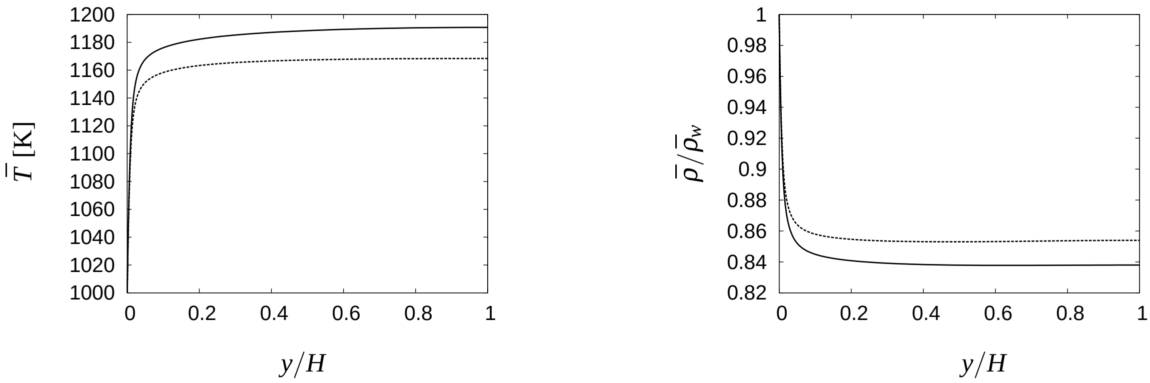

Figure 2. Mean temperature (left) and mean density distribution (right). Dashed line: with radiation, solid line: without radiation

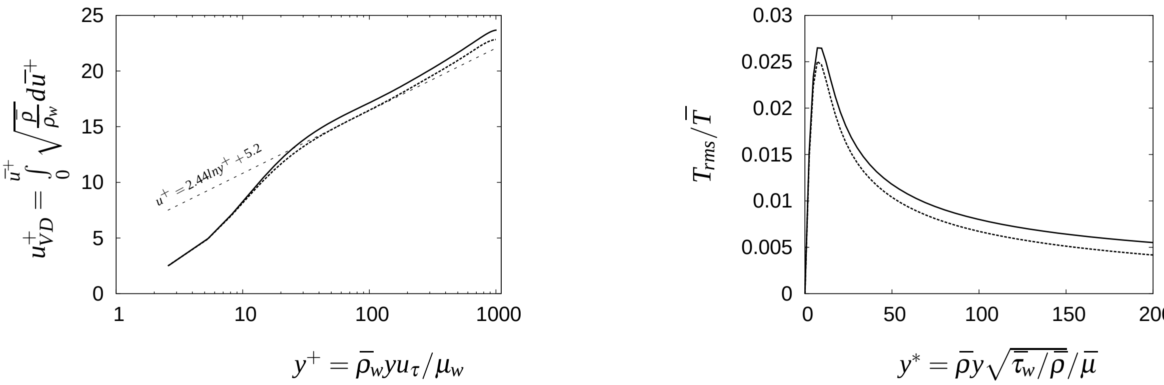

Figure 3. Van Driest transformed velocity (left) and RMS temperature fluctuations (right). Dashed line: with radiation, solid line: without radiation

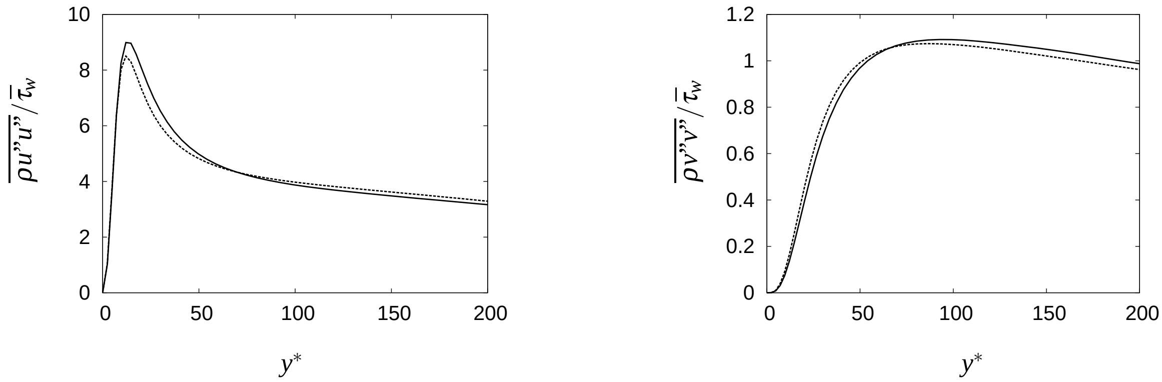

Figure 4. Streamwise (left) and wall-normal Reynolds stresses (right). Dashed line: with radiation, solid line: without radiation

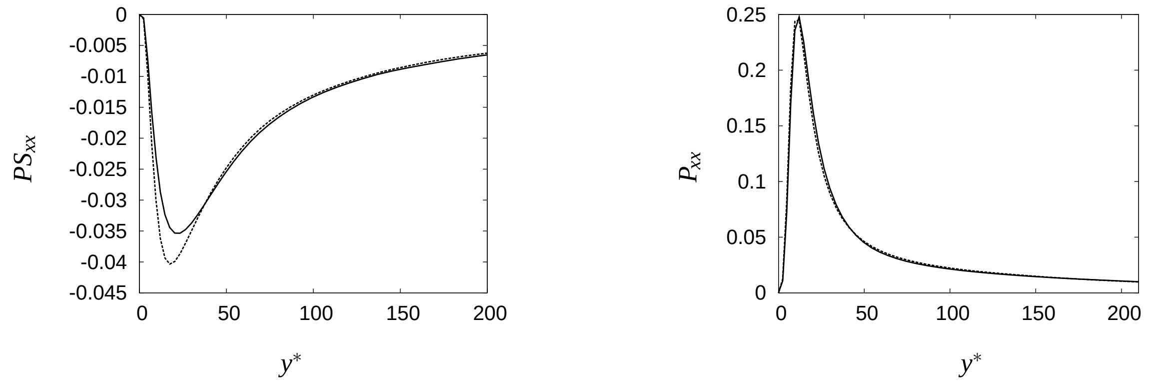

Figure 5. Streamwise pressure-strain correlation (left) and production (right) terms normalized with $\bar{\tau}_w^2/\bar{\mu}$.Dashed line: with radiation, solid line: without radiation

It has been shown in supersonic channel flow (Foysi *et al.*, 2004), that an increase in Mach number leads to an increase in the streamwise Reynolds stress and a decrease in the other components in the near-wall region. Plots of streamwise and wall-normal Reynolds stresses (Fig. 4) reveal that radiation counteracts the abovementioned compressibility effect. In Foysi *et al.* (2004), it was also shown that the increase in Reynolds stress anisotropy at higher Mach numbers is caused by a decrease in pressure-strain correlations. The reason for this decrease lies in the decrease of mean density from its wall value at supersonic Mach numbers. When radiation effects are included, the decrease in mean density ratio is less (fig. 2 (right)), which leads to an increase of the near-wall pressure-strain correlation (fig. 5 (left)). The streamwise production term is hardly affected by radiation (fig. 5 (right)). This means that radiation leads to an enhancement of redistribution of the Reynolds stresses by the pressure-strain correlations in the near-wall region.

Supersonic channel flow of a fictitious gray gas

The previous flow case has revealed weak effects of radiation on the statistics of the velocity vector and the state variables ρ, T etc. as a result of the very low optical thickness of the gas layer between the channel walls. In this section, we look at the flow of a fictitious, gray gas having a 'tunable', temperature dependent absorption coefficient. We increase the absorption coefficient, so that the optical thickness now has a value 0.04. The Planck mean absorption coefficient is given by the following form:

$$\kappa_P = C_k \Big[c_o + c_1 (A/T) + c_2 (A/T)^2 + c_3 (A/T)^3 + c_4 (A/T)^4 + c_5 (A/T)^5 \Big] \quad (5)$$

The coefficients c_o - c_5 and A were taken from a radiation model suggested for water vapor (*Sandia National Laboratories Combustion Research Facility, International Workshop on Measurements and Computation of Turbulent Nonpremixed Flames, 2002*). The coefficient C_k allows for a variation of the optical thickness independent of the other parameters. We have chosen $C_k = 0.1$ for this flow. We also adjust the mass flux of the flow without radiation so that it matches with that under the effect of radiation. The flow parameters are specified in Table 2. Although the optical thickness is still fairly low (4%), the radiative heat flux accounts for nearly 35% of the total heat flux at the wall (assessed from the mean total energy equation integrated in wall-normal direction) and the effect of radiation on the state variables is very noticeable. From Figure 6 (left) we conclude that the temperature increase in the channel core (and in turn the density decrease) due to energy dissipation at supersonic speeds, is to a large

Flow	Re_τ	M_τ	Re_m	M_m	τ_H
Radiation turned off	982	0.07	16700	1.29	–
Gray gas	977	0.071	16620	1.28	0.04

Table 2. Flow parameters for supersonic channel without radiation and with radiation using a fictitious gray gas

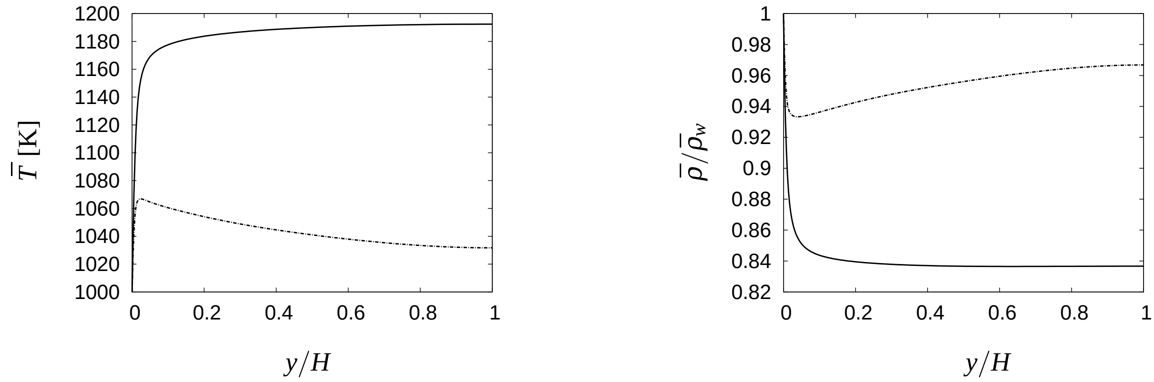

Figure 6. Mean temperature (left) and mean density distribution (right). Dash-dotted line: gray gas, $\tau_H = 0.04$, solid line: without radiation

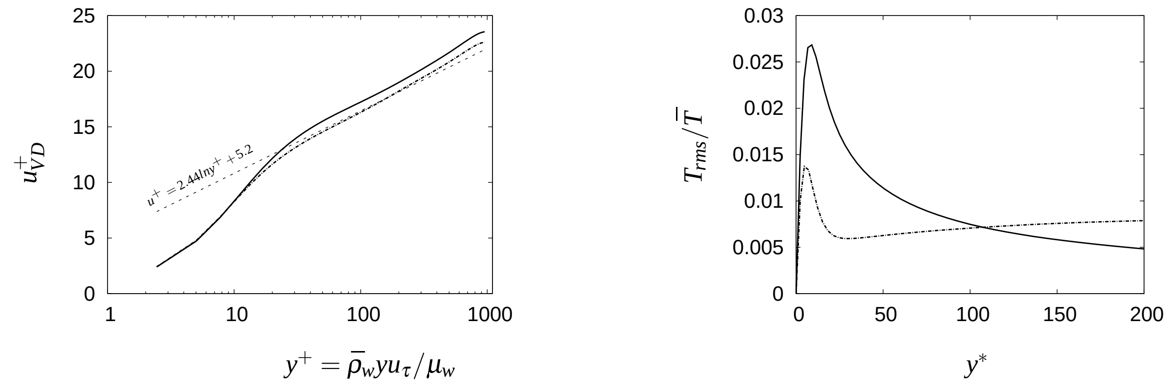

Figure 7. Van Driest transformed velocity (left) and RMS temperature fluctuations (right). Dash-dotted line: gray gas, solid line: without radiation

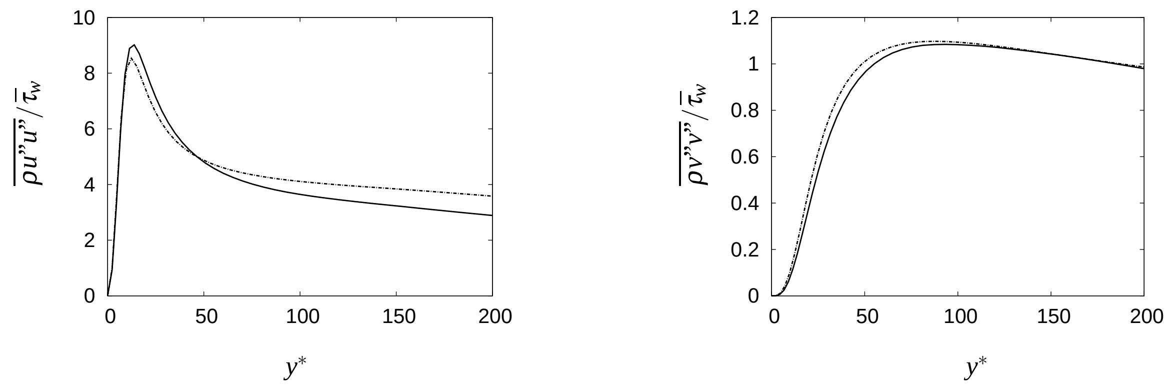

Figure 8. Streamwise (left) and wall-normal Reynolds stresses (right). Dash-dotted line: gray gas, solid line: without radiation

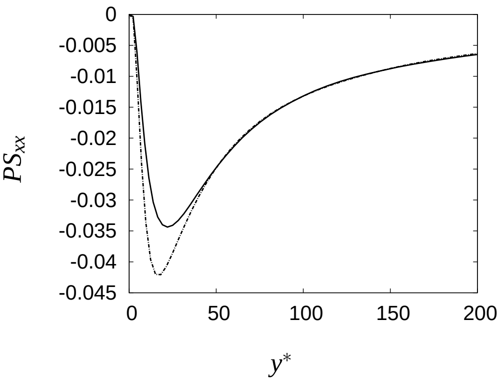

Figure 9. Streamwise pressure-strain correlation normalized with $\overline{\tau}_w^2/\overline{\mu}$. Dash-dotted line: gray gas , solid line: without radiation

extent removed through radiative energy transfer towards the wall. Effects, similar in strength to those of the mean temperature are also observed in the temperature fluctuations, see Figure 7 (right). We observe a reduction by a factor of nearly two in the wall layer and an enhancement of comparable order of magnitude away from the wall. The shift of the mean Van Driest transformed velocity towards the logarithmic law in the log-region, shown in the previous flow case (fig. 3 (left)), is confirmed in Figure 7 (left) . Figure 8 shows the streamwise and wall-normal Reynolds stresses. When we compare the present effect of radiation with that at lower optical thickness, we note that the previous trends are confirmed, i.e. we observe a decrease in the streamwise stress (fig. 8 (left)) and an increase in the wall-normal (fig. 8 (right)) and spanwise (not shown here) stresses in the wall layer. The effect that radiation counteracts compressibility is even slightly enhanced in this case. The streamwise pressure-strain correlation in Figure 9 shows larger deviations, in the near-wall region, from those where radiation is turned-off, than in the case of very low absorption coefficient (Figure 5). Thus, an increase in the absorption coefficient enhances the effect of radiation on the pressure-strain correlations and confirms the previously found tendencies.

Conclusions

The present study of flow of radiating gases in supersonic minimal channels shows that radiation produces noticeable effects in non-reacting flows, provided that the optical thickness is not too low. Both flow cases demonstrate that effects of radiation counteract effects of compressibility. While non-radiating fully-developed supersonic channel flow shows an increase in the normalized streamwise Reynolds stress and reductions in the other stress components compared to incompressible flow, thermal radiation dampens the streamwise stress and enhances the other stresses in the wall layer ($0 \leq y^* \leq 50$). This behaviour is a result of the decrease (Mach number effect), or increase (radiation effect) of the corresponding pressure-strain correlations in the wall layer. Radiation effects on the energy balance will be discussed during the oral presentation.

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