STOCHASTIC MODEL OF SPRAY FORMATION CLOSELY TO AIR-BLAST INJECTOR: LES-SIM (STOCHASTIC INTERFACE MODEL) APPROACH

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ABSTRACT

Targeting on primary air-blast modeling, the paper proposed a new approach, in which an unsteady, incompressible, immiscible gas-liquid system closely to injector is simulated, as a porous medium implemented in the framework of standard LES approach. Such porosity is characterized by an immersed boundary force, depending on probability of the free interface location, on the outward normal to this interface, and on the rate of velocity change of the relative gas-to-liquid motion. All these parameters are modeled in the paper. Presuming the radius distribution, those parameters give the position and the pinch off direction of produced droplets around the liquid core. In the framework of this approach, the simulation of the gas flow was done, and the spray, formed closely to the air-blast injector, was assessed.

I. INTRODUCTION

In air-craft and rocket engines, the liquid jet is atomized by a coaxial high-speed jet of the gas. Such a type of breakup is referred to as air-blast atomization. If the produced spray is not well-atomized, the combustion process will be uncompleted. Therefore numerical prediction of flow and spray formation in close to injector region is of interest to engineers. The momentum balance equation governing the motion of an unsteady, incompressible, immiscible, gas-liquid system is the Navier-Stokes equation:

$$\frac{\partial u_i}{\partial t} = -u_j \frac{\partial u_i}{\partial x_j} - \frac{1}{\rho}\frac{\partial p}{\partial x_i} + \frac{1}{\rho}\frac{\partial \tau_{ij}}{\partial x_j} \qquad (1)$$

where u_i ($i = 1,2,3$) are instantaneous velocity components in a given point, p is the pressure (here the surface tension force we included in the pressure gradient), and τ_{ij} are components of the viscous stress tensor. Hereafter a summation over the repeated indices is implied. If the considered point is in the liquid, then the molecular viscosity ν, in the viscous stress tensor, and the density ρ are attributed to the liquid, ν_l and ρ_l, otherwise those parameters are prescribed to the gas, ν_g and ρ_g. Considering subregions associated with the gas only, we have:

$$\frac{\partial u_i^g}{\partial t} = -u_j^g \frac{\partial u_i^g}{\partial x_j} - \frac{1}{\rho_g}\frac{\partial p}{\partial x_i} + \nu_g \frac{\partial^2 u_i^g}{\partial x_j \partial x_j} + F_{\mathrm{lg}} e_i \delta\left(x_i - x_{fi}\right) \qquad (2)$$

$$\frac{\partial u_j^g}{\partial x_j} = 0 \qquad (3)$$

where $F_{\mathrm{lg}} e_i$ denote components of the boundary force per unit surface of the liquid/gas interface. The last one is identified in (2) by its location $x_{f,i}(t)$ and its unit outward

normal $\mathbf{e}$; here $\delta\left(x_i - x_{fi}\right)$ is the Dirack delta function. The source term $F_{\mathrm{lg}} e_i$ depends on the rate of velocity change of the relative gas-to-liquid motion, and on the surface tension effects. This term is non-zero only on the free interface. In the atomization process, when a continuous liquid jet disintegrates into filaments and drops due to interaction with the gas flow, the free surface geometry $\mathbf{x}_f(t)$ is unknown. The problem is that when the Reynolds number is high, $\mathbf{x}_f(t)$ may behave randomly, often burst-likely, with strong fluctuations. Its deterministic prediction becomes a difficult task. In this paper, we propose a simple toy model, referred to as LES-SIM (Stochastic Interface Model) approach, with primary atomization modeling on a target. In the framework of this approach, the objective in this paper is to assess both, the simulation of gas flow and the spray formation model, closely to the air-blast injector and in the far-field of spray.

II. LES-SIM APPROACH

Filtering of (2) and (3) gives

$$\frac{\partial \overline{u_i^g}}{\partial t} = -\frac{\partial \overline{u_i^g}\,\overline{u_j^g}}{\partial x_j} - \frac{1}{\rho_g}\frac{\partial \overline{p}}{\partial x_i} + \frac{\partial\left(2\left(\nu+\nu_t\right)\overline{S}_{ij}^g\right)}{\partial x_j} + \overline{F_{\mathrm{lg}} e_i \delta\left(x_i - x_{fi}\right)} \tag{4}$$

$$\frac{\partial \overline{u_j^g}}{\partial x_j} = 0 \tag{5}$$

where $\overline{S_{ij}^g} = \frac{1}{2}\left(\frac{\partial \overline{u_i^g}}{\partial x_j} + \frac{\partial \overline{u_j^g}}{\partial x_i}\right)$, ν_t is the eddy-viscosity given by the Smagorinsky model: $\nu_t = \left(C_S \Delta\right)^2 \overline{S}$, Δ is the width of filter, and $\overline{S} = \left(2\overline{S}_{ij}^g \overline{S}_{ij}^g\right)^{1/2}$ is a characteristic filtered strain rate. Three main assumptions in our approach are as follows. The first proposal is to replace the source term in (4) by

$$\overline{F_{\mathrm{lg}} e_i \delta\left(x_i - x_{fi}\right)} = P_l\left(x_{fi}; x_i, t\right) \dot{\mathrm{v}}_{\mathrm{S}}\, n_i \tag{6}$$

where $P_l\left(x_{fi}; x_i, t\right)$ is the modeled probability to have interface in a given point, n_i is the modeled stochastically outward component in this point, $\dot{\mathrm{v}}_{\mathrm{S}}$ denotes acceleration on the interface, which also should be modelled. The second proposal is to decompose (4) into:

$$\frac{\partial \overline{u_i^g}}{\partial t} = \begin{cases} -\frac{\partial \overline{u_i^g}\,\overline{u_j^g}}{\partial x_j} - \frac{1}{\rho_g}\frac{\partial \overline{p}}{\partial x_i} + \frac{\partial\left(2\left(\nu+\nu_t\right)\overline{S}_{ij}^g\right)}{\partial x_j}, \ \textit{if } P_l = 0 \\ P_l\, \dot{\mathrm{v}}_{\mathrm{S}} n_i, \ \textit{if } P_l \neq 0 \end{cases} \tag{7}$$

To this end, equation (4) can be rewritten by finite-differences:

$$\frac{\overline{u_i^g}^{n+1} - \overline{u_i^g}^{n}}{\Delta t} = \overline{RHS_i} + \overline{F_i} \tag{8}$$

where

$$\overline{RHS_i} = -\frac{\partial \overline{u_i^g}\,\overline{u_j^g}}{\partial x_j} - \frac{1}{\rho_g}\frac{\partial \overline{p}}{\partial x_i} + \frac{\partial\left(2\left(\nu+\nu_t\right)\overline{S}_{ij}^g\right)}{\partial x_j}$$

$$\overline{F_i} = \begin{cases} 0, \ \textit{if } P_l = 0 \\ -\overline{RHS_i} + P_l\, \dot{\mathrm{v}}_{\mathrm{S}}\, n_i, \ \textit{if } P_l \neq 0 \end{cases} \tag{9}$$

If $\dot{\mathrm{v}}_{\mathrm{S}} n_i = \frac{u_S^{n+1} n_i - \overline{u_i^g}^{n}}{\Delta t}$, where $u_S^{n+1} n_i$ denotes the interface velocity, equations (8), (9) are reduced to $\overline{u_i^g}^{n+1} = \left(1 - P_l\right)\overline{u_i^g}^{n} + P_l u_S^{n+1} n_i$ in subregions with $P_l \neq 0$. In this paper, the interface velocity is taken constant and equal to the convection velocity $u_S = \left(\sqrt{\rho_g}\, u_{g,0} + \sqrt{\rho_l}\, u_{l,0}\right) / \left(\sqrt{\rho_g} + \sqrt{\rho_l}\right)$, where $u_{g,0}$ is the inlet gas velocity, $u_{l,0}$ is the inlet liquid velocity. The expression of the convection velocity is well-known [7]; it comes from equality of dynamic pressures on the interface: $\rho_g\left(u_{g,0} - u_S\right)^2 = \rho_l\left(u_S - u_{l,0}\right)^2$. The third proposal is to produce primary blobs in the region $0 < P_l < 1$, with presumed distribution of size, with modeled n_i and with location defined by P_l.

III. STOCHASTIC MODEL OF LIQUID INTAKE CORE

We consider a liquid jet atomized by a high-speed co-flowing jet of the gas. A simplest view on this jet is to represent it as a central liquid intact core surrounded by detached from this core liquid blobs. In the model, we assumed that the liquid core has random geometrical configuration; its 3D statistics is simulated by spatial trajectories of specific stochastic particles injected simultaneously with LES in the gas flow. Each trajectory represents one realization of the liquid/gas interface. The stochastic process prescribed to such a particle is based on assumption that exiting continuous liquid jet is depleted in the framework of statistical universalities of a fragmentation under scaling symmetry [1]. Two parameters of the stochastic process have been determined according to experimental observations [2]; they are the ratio of typical lengths in

Kelvin-Helmholtz and Rayleigh-Taylor instabilities, respectively, and the gas-to-liquid initial momentum ratio. Details of such simulation can be found in [3, 4]. The spray around the non-depleted liquid core is assumed to be thin; thereby the computed probability P_l is attributed to spatial distribution of all the liquid in the close to injector region. Fig 1 shows an example of such a distribution. Concerning each stochastic particle, in difference with [3], this paper prescribes also to the random outward normal components $n_i(t)$. The stochastic evolution of $n_i(t)$ is emulated by Brownian random walk on the surface of a unit radius sphere, where the diffusion coefficient is inversely proportional to the stochastic particle life-time: $T^{-1} = D_l^{-1}\sqrt{\left|\rho_g u_{g,0}^2 - \rho_l u_{l,0}^2\right|/2\rho_l}$, where D_l is the diameter of the liquid jet orifice. In the Cartesian coordinate system, the Langevin equation for direction components $n_i(t)$ can be derived in the following form [5], [6]:

$$n_i(x_i + u_i\,dt) - n_i(x_i) = -2\,T^{-1} n_i\,dt + \left(\delta_{ij} - n_i n_j\right)\sqrt{2\,T^{-1}}\,dW_{\mathbf{x}j} \tag{10}$$

where $W_{\mathbf{x}j}$ represent independent components of Brownian vector process $\mathbf{W}_\mathbf{x}$ at spatial point $\mathbf{x}$, and δ_{ij} is the Kronecker delta. It follows from (10) that norm of the vector $n_i(t)$ is conserved: $n_i n_i = 1$, and the correlation of n_i is an exponential function. Fig.2 represents a sample path of one stochastic particle indicating the instantaneous angle from $\alpha = \arctan\dfrac{u_* n_y}{u_S}$, where u_* is the scale of entrainment velocity direction $u_* = \sqrt{\rho_g/\rho_l}\,u_{g,0}$ [7], and n_y is the projection of the normal on vertical plane. This angle is used further in simulation of the spray angle. In the region $0 < P_l < 1$, the primary blobs around the liquid core are produced prescribing sampled n_i and sampled radius from the presumed exponential distribution: $\mathrm{f}(r) = \lambda_{RT}^{-1}\exp(-r/\lambda_{RT})$, in which the expression for λ_{RT} is proposed in [3]. This sampling procedure is organized in such a way that the injected liquid mass is continuously conserved in the produced drops. The produced drops are dragged by resolved velocity field from (7), (5), and are subjected to the secondary fragmentation [8]. Their dynamics in $0 < P_l < 1$ is simulated in the framework of the Lagrangian procedure. Three different approaches in the Lagrangian tracking have been compared. The first one is the standard approach, in which the droplets velocity $\mathbf{u_p}$ is given by

$$\frac{d\,\mathbf{u_p}}{dt} = \frac{\overline{\mathbf{u^g}} - \mathbf{u_p}}{\tau_{St}} \tag{11}$$

where the Stokes time is given by $\tau_{St} = \dfrac{\rho_p d_p^2}{18\rho_g \nu_g}\dfrac{1}{1+0.15\,\mathrm{Re}_p^{0.687}}$, and the Reynolds number is $\mathrm{Re}_p = d_p\left|\overline{\mathbf{u^g}} - \mathbf{u_p}\right|/\nu_g$, d_p is the droplet diameter. The second approach used the new definition of the Reynolds number:

$$\mathrm{Re}_p = \frac{\varepsilon^{1/6}\,\tau_{St}^{1/2}}{d_p^{1/3}} \tag{12}$$

where ε is the local turbulent dissipation rate. The third approach is based on the "smoothed" trajectory approach, developed in [9]:

$$\frac{d\,\mathbf{u_p}}{dt} = \frac{\overline{\mathbf{u^g}} - \mathbf{u_p}}{\tau_{St}} - \nabla\overline{\mathbf{u}_P'^2} \tag{13}$$

in which $\overline{\mathbf{u}_P'^2}$ is coupled with subgrid kinetic turbulent energy.

IV. SOME RESULTS OF COMPUTATION AND DISCUSSIONS

As an example of present simulations, two snapshots of the filtered velocity field in the gas flow computed by and the droplet size distribution are shown in Fig.3. The impact of fluctuating liquid core on the gas flow, the recirculation zone in front of the liquid core, and a large spectrum of produced droplets, from 20μm to 200μm, at each spray position are seen in this figure. Such properties were emphasized in experimental study in [2]. Fig.4 compares the kinetic energy of droplets with measurements [2] at different height along the spray. It is seen that while at $\dfrac{y}{D_l} = 0.5$, the numerical prediction is close to measurements, at $\dfrac{y}{D_l} = 0.75$ and $\dfrac{y}{D_l} = 1$, the computation overpredicts the experimental data, with still explicit advantage when the smoothed trajectory approach is used. Fig.5 shows this comparison for the mean Sauter diameter of produced numerically droplets. It can be seen that although the model is very simple, and no adjustable constants have been introduced, the predicted diameters at $\dfrac{y}{D_l} = 0.75$

and $\frac{y}{D_l}=1$, are relatively well predicted; at $\frac{y}{D_l}=0.5$ it is not the case.

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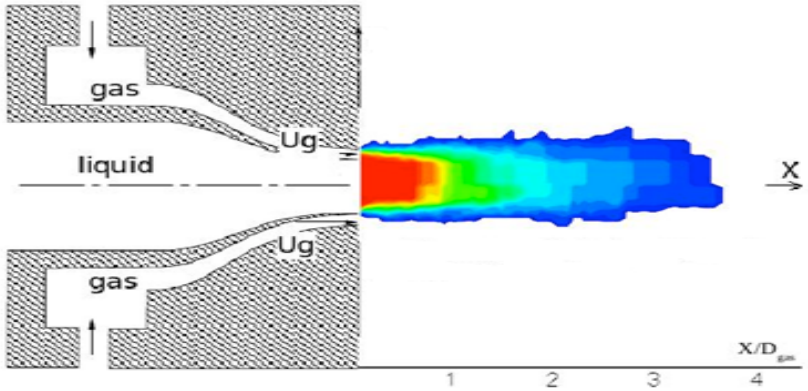

Fig.1 : Simulation of spatial probability distribution of liquid in the close to injector region. Inlet parameters: $u_{g,0}=60\,m/s$; $u_{l,0}=0.33\,m/s$

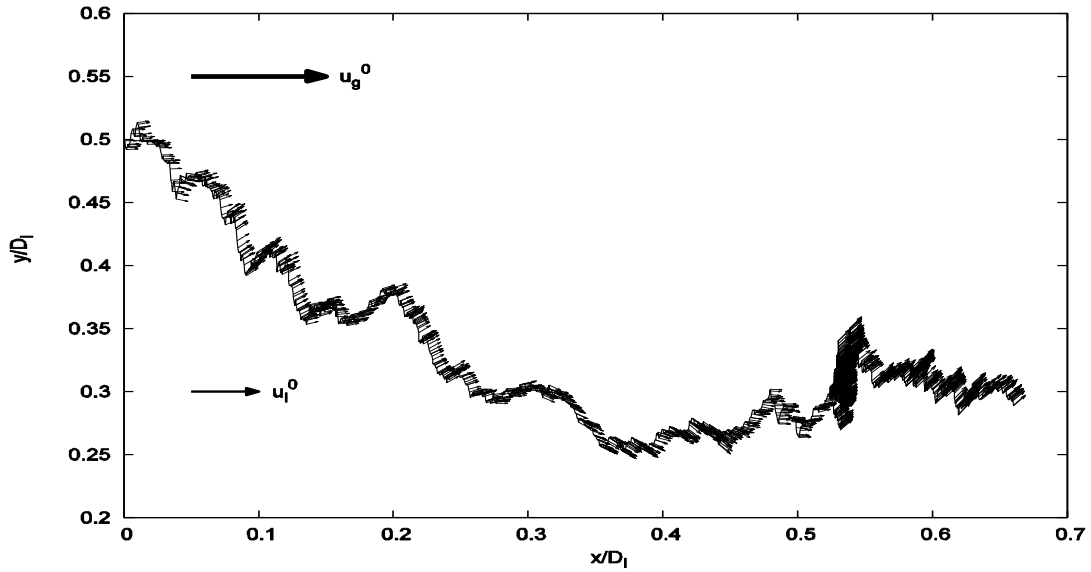

Fig.2 : Single sample path of stochastic particle; its trajectory defines instantaneous liquid core boundary; its angle defines instantaneous outward normal; Inlet parameters: $u_{g,0}=60\,m/s$; $u_{l,0}=0.33\,m/s$

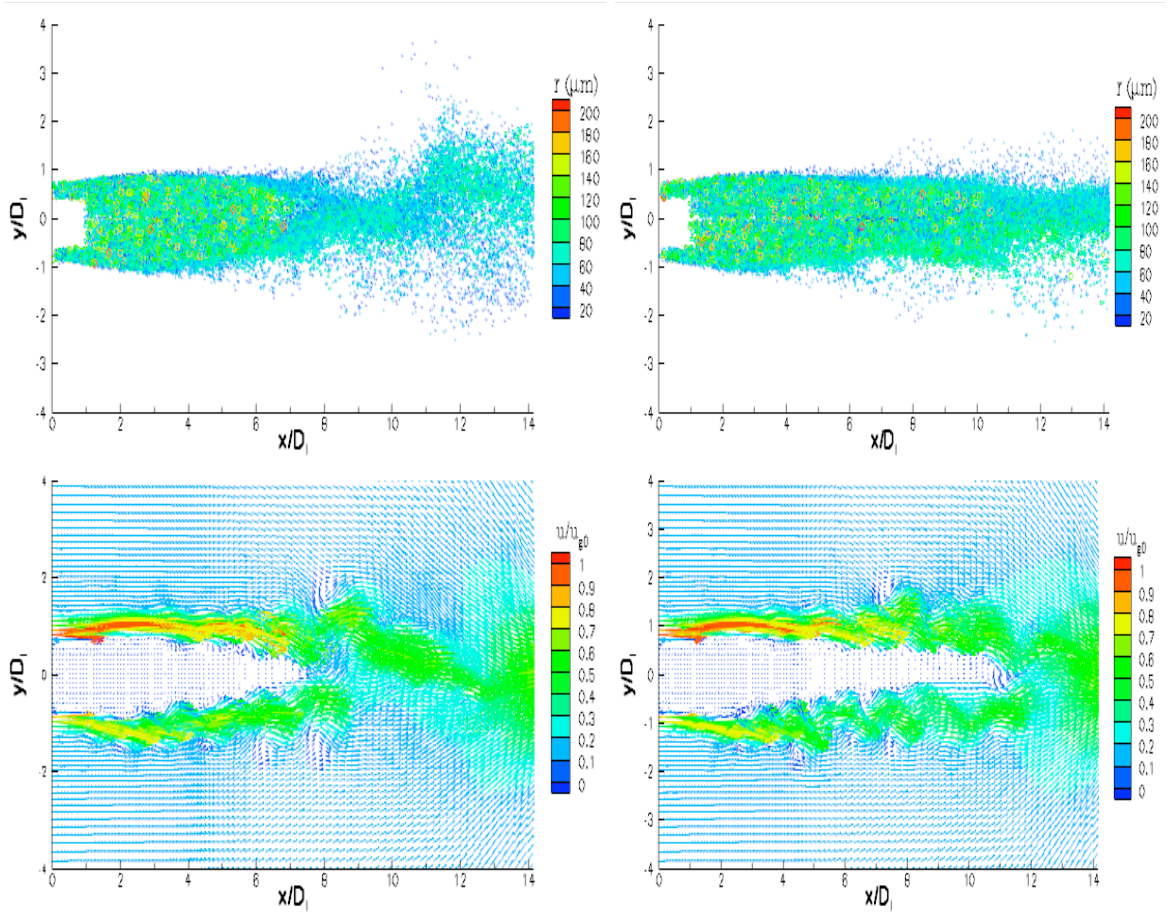

Fig.3 : Two snapshot of the filtered velocity field in the gas flow and the droplet size distribution. Inlet parameters: U_g=60m/s, U_{liq}=0.33m/s

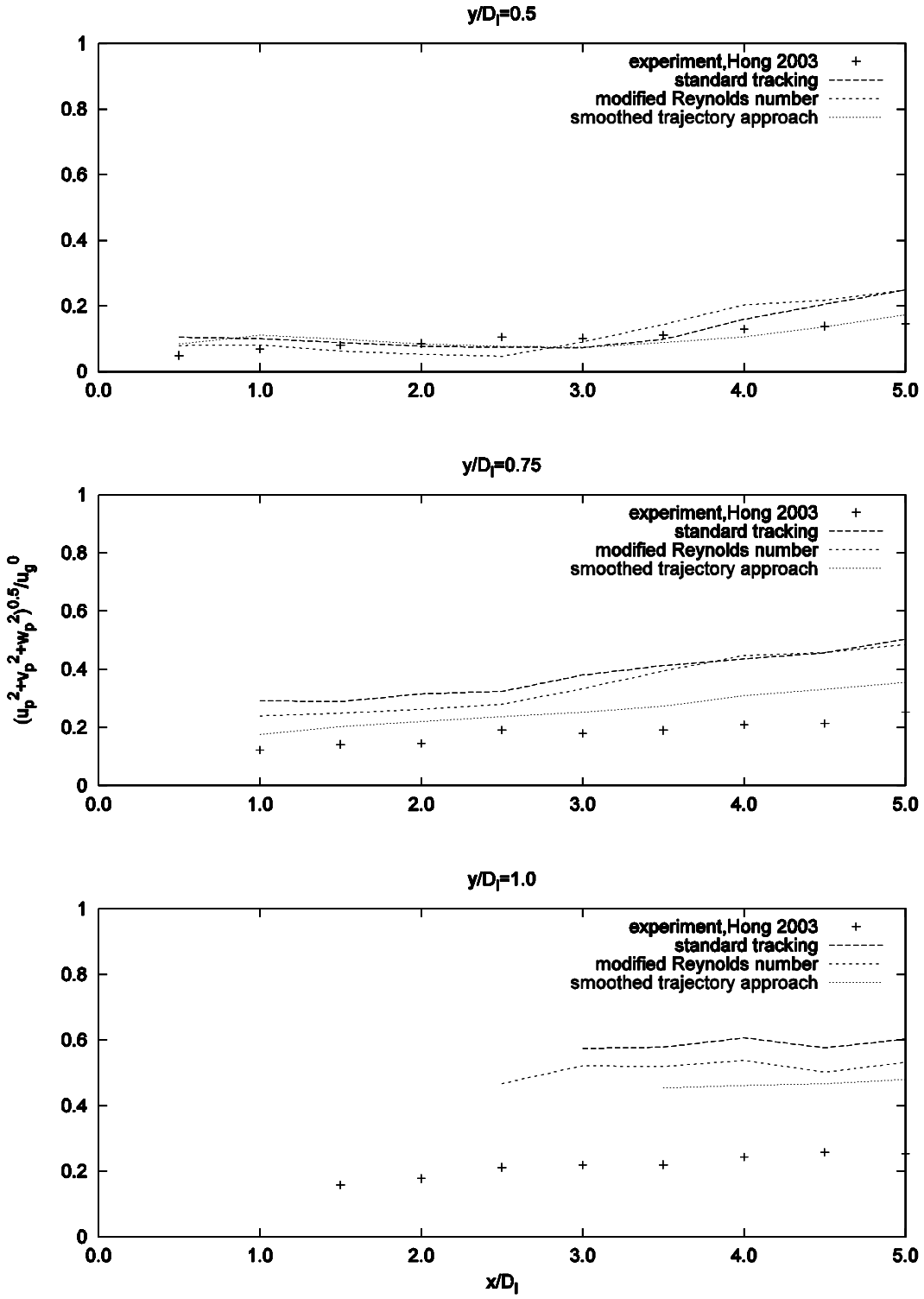

Fig.4 : Comparison of kinetic energy of droplets with measurements [2] at different height along the spray. Inlet parameters: U_g=60m/s, U_{liq}=0.33m/s

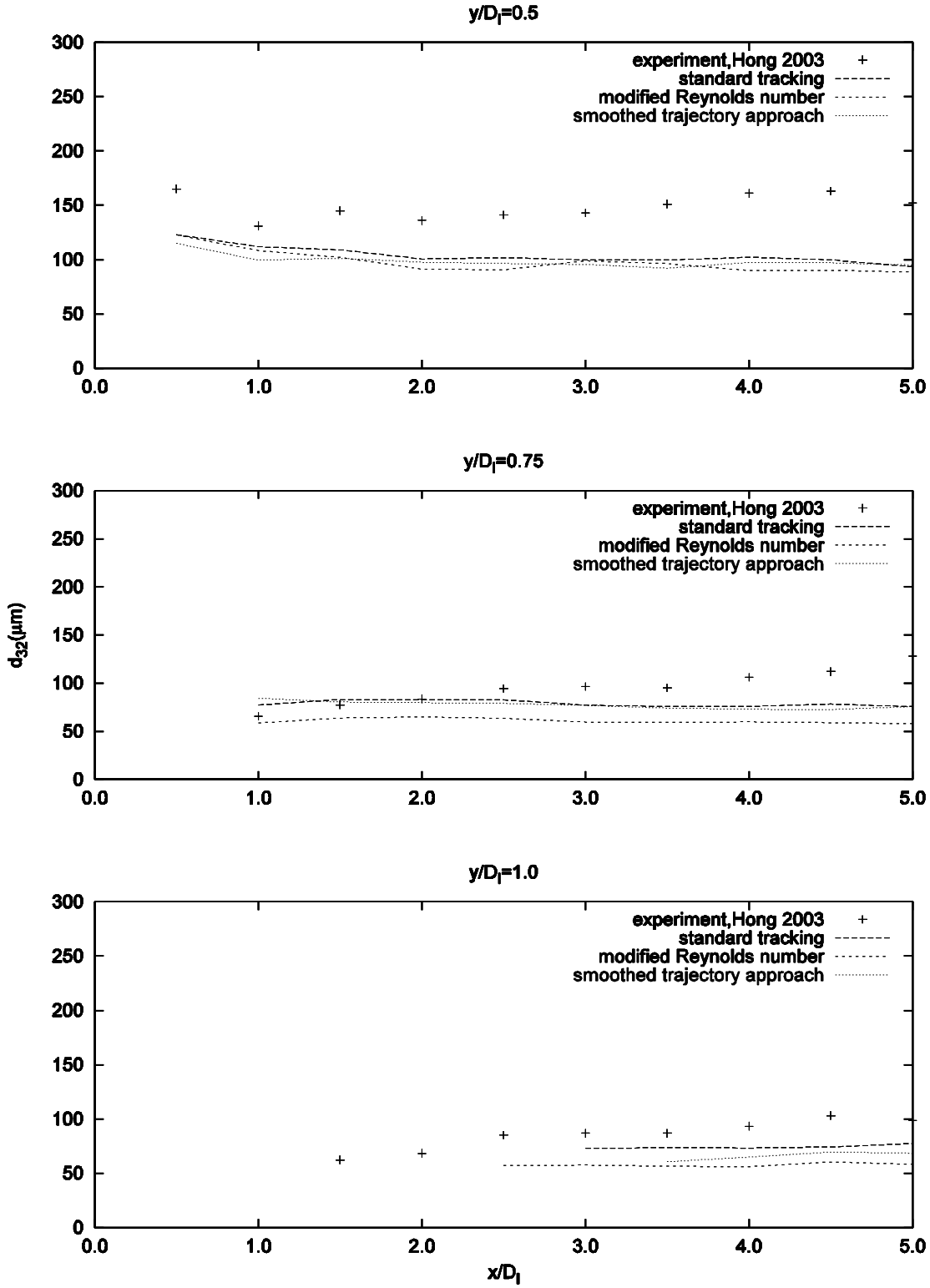

Fig.5 : Comparison of mean Sauter diameter of produced droplets with measurements [2] at different height along the spray. Inlet parameters: U_g=60m/s, U_{liq}=0.33m/s