On Calculation of Turbulent Free-surface Flows with Particle Method

Hiroshi Inokuma
Department of Civil Engineering
Kobe University
1-1 Rokkodai, Nada-ku, Kobe 657-8501 Japan
inoshishikuma@gmail.com

Akihiko Nakayama
Department of Civil Engineering
Kobe University
1-1 Rokkodai, Nada-ku, Kobe 657-8501 Japan
nakayama@kobe-u.ac.jp

Kenta Ikenaga
Toyota Technical Development Corp.
Toyoda Japan
ikenta21@gmail.com

ABSTRACT

The particle interpolation method such as the Smoothed Particle Hydrodynamics (SPH) for solving free surface flows has been reformulated so that the effects of turbulent fluctuations are reflected in the equations for particle interpolated quantities. Calculation of benchmark flow of collapsing water column indicates that the terms representing the dispersion of fluctuating particle paths need to be included in addition to added viscosity similar to the sub-grid eddy viscosity used in Large Eddy Simulation (LES). Preliminary calculation of flat-bed open channel flows also indicates that the present formulation is consistent with the conventional fixed-grid methods of computing turbulent flows.

INTRODUCTION

Calculation of flows with large deformation of the flow boundaries and interfaces such as flood flows and coastal flows with large wave motion can be better done by computing flow quantities defined at positions of particles moving with the flow rather than quantities at fixed grid points. Smoothed Particle Hydrodynamics (SPH) (Gingold and Monaghan, 1977, and Monaghan, 1992) and Moving Particle Implicit (MPS) (Koshizuka et al. 1994) are examples of the particle method and are recently been applied to various engineering and environmental problems with potentially great success. These Lagrangian methods do not have stability problems associated with the nonlinear convective terms that Eulerian method must solve. However, when they are applied to high Reynolds number flows, the effects of the turbulent fluctuations must necessarily be accounted for but they are often overlooked and necessary models have not received as much attention as in the fixed-grid counterpart of Reynolds Averaged Navier Stokes method or Large Eddy Simulation (LES) methods. Colagrossi & Landrini (2003) Violeau & Issa (2007) and Gotoh et al.(2001) interpreted the SPH or MPS methods just as the numerical discretization techniques of given partial differential equations and directly applied to the Reynolds-averaged equations or spatially-filtered equations of motion without particular attention to the smoothing of turbulent fluctuations. Holme (1999) on the other hand, rigorously derived equations of motion for averaged flows in Lagrangian coordinates and pointed out that there are differences between Eulerian and Lagrangian averages and that the Lagrangian averages (which are close to the particle averages) are influenced by the dispersion of particle paths.

In the present paper, we formulate the SPH equations with due attention to the turbulent fluctuations. We point out that the terms arise from the interaction of turbulent fluctuations of velocity and the particle positions and propose a method of taking into account of their effects. Then we apply the proposed model and the procedure to some benchmark flows.

PARTICLE INTERPOLATION FOR TURBULENT FLOWS

The basic SPH method first considers the kernel-integral representation A_I for flow quantity A at position $\boldsymbol{r}$ by averaging around $\boldsymbol{r}$ with weight function W with smoothing distance h

$$A_I(r) = \iiint W(r - r', h) A(r') dr' \tag{1}$$

The integral on the right hand side is then approximated by the sum over finite number of discrete points within distances about h

$$A_I(r) = \sum_b W(\boldsymbol{r} - \boldsymbol{r}_b, h) A(\boldsymbol{r}_b) \frac{m_b}{\rho_b} \quad (2)$$

where b is a point or the fluid particle at position $\boldsymbol{r}_b$, m_b and ρ_b are the mass and the density so that m_b/ρ_b is the volume of fluid represented by particle b. The next step is to replace the original value $A(\boldsymbol{r}_b)$ on the right hand side by the interpolated value A_I at r_b denoted by A_b, so that the particle interpolation representation of A at r_a , the position of particle a, is given by

$$A_a = \sum_b W(\boldsymbol{r} - \boldsymbol{r}_b, h) A_b \frac{m_b}{\rho_b} \quad (3)$$

The last step is often overlooked as just a method of description but is an approximation that is poor when the scales of spatial variations of A is smaller than h. This equation allows representation of any flow quantity at arbitrary positions and that is what is usually done in particle methods. However, this approximation is not good to apply to unsmoothed flow quantities in turbulent flows in which the scales of fluctuations are very small for large Reynolds numbers.

In the present work, with application to turbulent flow in mind, we like to avoid using (3) and apply Eq. (1) directly to the unsmoothed original flow quantities and use it as a way of spatial filtering of the flow. To this end, we apply the particle interpolation procedure (2) to all terms in the equations of motion and the continuity equation for (slightly) compressible flow written in the Eulerian coordinates

$$\frac{\partial v}{\partial t} + \nabla \cdot (v\,v) = g - \frac{1}{\rho} \nabla p + \nu \nabla^2 v \quad (4)$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0 \quad (5)$$

so that we obtain the equations for the particle-interpolated quantities. When applying the particle interpolation to all terms in Eqs.(4) and (5), it is noted that it commutes with the spatial differentiation so that

$$[\nabla A]_I = \nabla_r A_I(\boldsymbol{r}) \text{ and. } \left[\nabla^2 A\right]_I = \nabla_r{}^2 A_I(\boldsymbol{r}) \quad (6)$$

where subscript I indicates the particle interpolated value at the indicated position $\boldsymbol{r}(t)$ at time t, and ∇_r is used to stress the differentiation with respect to $\boldsymbol{r}$. Now, since the position where the interpolation is taken is moving, the time derivative does not commute with the interpolation. The particle interpolation of the time derivative of A is

$$\begin{aligned}
\left(\frac{\partial A}{\partial t}\right)_I &= \iiint \frac{\partial A(r',t)}{\partial t} W(r(t) - r',h)\; dr' \\
&= \iiint \left[\frac{\partial (A(r',t) W(r(t) - r',h))}{\partial t} - A(r',t) \frac{\partial}{\partial t} W(r(t) - r',h)\right] dr' \\
&= \frac{d}{dt} \iiint A(r',t) W(r(t) - r',h) dr' - \iiint A(r',t) \frac{dr(t)}{dt} \cdot \nabla_r W\, dr' \\
&= \frac{d}{dt} A_I(r(t),t) - \frac{dr(t)}{dt} \cdot \nabla_r \iiint A(r',t) W\, dr' \\
&= \frac{d}{dt} A_I(r(t),t) - \frac{dr(t)}{dt} \cdot \nabla A_I(r,t)
\end{aligned} \quad (7)$$

This means that the particle interpolation of the time derivative introduces additional term involving the product of the velocity of the position of the interpolating point and the gradient of the interpolant arises. Therefore, the particle interpolations of Eqs. (4) and (5) become

$$\frac{dv_I}{dt} - \frac{dr(t)}{dt} \cdot \nabla_r v_I + \nabla \cdot (vv)_I = g - \nabla \left(\frac{p}{\rho}\right)_I + \nu \nabla^2 v_I \quad (8)$$

$$\frac{d\rho_I}{dt} - \frac{dr(t)}{dt} \cdot \nabla_r \rho_I(r,t) + \nabla \cdot (\rho v)_I = 0 \quad (9)$$

If we take the position of interpolation $r(t)$ to be exactly equal to the path of the particle travelling at the interpolated velocity v_I,

$$\frac{dr(t)}{dt} = v_I \quad (10)$$

we have

$$\frac{dv_I}{dt} = g - \nabla \left(\frac{p}{\rho}\right)_I + \nu \nabla^2 v_I - \nabla \cdot (vv)_I + v_I \cdot \nabla v_I \quad (11)$$

and

$$\frac{d\rho_I}{dt} = -\nabla \cdot (\rho v)_I + v_I \nabla \cdot \rho_I \quad (12)$$

These equations may also be written as

$$\frac{dv_I}{dt} = g - \nabla \left(\frac{p}{\rho}\right)_I + \nu \nabla^2 v_I - \nabla \cdot \left((vv)_I - v_I v_I\right) - v_I \nabla \cdot v_I \quad (13)$$

and

$$\frac{d\rho_I}{dt} = -\nabla \cdot \left((\rho v)_I - \rho_I v_I\right) + \rho_I \nabla \cdot v_I \quad (14)$$

The last terms in these equations are due to the slight compressibility assumed in the SPH methods that relate the pressure to the density. Apart from these terms the equations contain terms that are not quite the interpolated values. They are

$$\tau = (vv)_I - v_I v_I \text{ and } R = (\rho v)_I - \rho_I v_I \quad (15)$$

that resemble the Reynolds stress and the turbulent density flux and we need to model these terms in terms of the particle interpolated values.

If these terms are modelled, with Eqs.(10), (13), (14) and a state equation for the pressure the flow can be solved. Since our main interest is the incompressible turbulent flows, we use the relation used by Monaghan (1994) and subsequent authors which is

$$p = B\left(\left(\frac{\rho}{\rho_0}\right)^{\gamma} - 1\right) \tag{16}$$

with the value of γ=7, the value of B is taken so that the speed of sound is about 0.1 of the flow speed.

τ resembles the sub-grid stress in LES while R is a new term representing the correlation between the density and the velocity. It should be noted that Eqs. (11) and (12) assume that Eq.(10) is accurately satisfied. In the present work, we try to extend the Smagorinsky model used in LES to model the term

$$\tau = 2(C_S h)^2 |S| S \tag{17}$$

where

$$S = \frac{1}{2}\left(\nabla v_I + (\nabla v_I)^T\right) \tag{18}$$

Following the LES of shear flows we use the value of C_S=0.1 (see for example Sagaut (2006)..

With regard to R, since the correlation between the density and the velocity is not expected to take significant values, we ignore this correlation for now and examine the results of calculation.

NUMERICAL CALCULATION METHOD

We first evaluate the right hand sides of Eqa.(13) and (14) by the particle summation formula Eq.(3). For the weight function W we use the cubic spline function

$$W_{ab} = W(\mathbf{r}_a - \mathbf{r}_b, h)$$
$$= \begin{cases} \frac{1}{6}\left[(2-r_{ab})^3 - 4(1-r_{ab})^3\right], & 0 \le r_{ab} < 1 \\ \frac{1}{6}(2-r_{ab})^3, & 1 \le r_{ab} < 2 \\ 0, & 2 \le r_{ab} \end{cases}, \tag{19}$$

The pressure gradient and the viscous terms in Eq.(13) are evaluated by the equation proposed by Morris et al. (1997). The time advancement of Eq.(13) and (14) is done by semi-implicit method. The pressure is obtained by explicit evaluation of the state equation Eq. (16). The rest of the details are very similar to the method used by Kajitar and Monaghan (2008) and we do not repeat here.

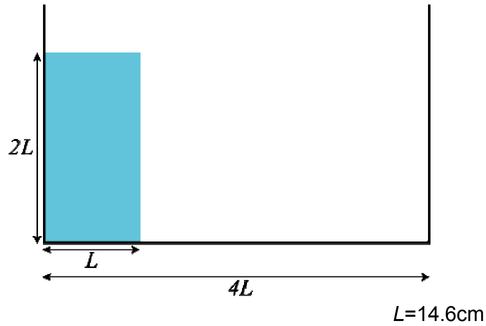

Figure 1. Initial state of water column in Koshizuka et al.'s (1995) experiment.

CALCULATION OF BENCHMARK FLOWS

First we apply our approach to the calculation of benchmark flow of collapsing water column, which is used as a validation case of many numerical methods. The experiment conducted by Koshizuka et al. (1995) used a rectangular tank of size 14.6cm long and about 10 cm high as shown in Figure 1. The water column of 29.2cm high and 14.6cm wide initially held at rest is made to fall freely within the chamber and the flow of this water column at subsequent times is observed. Figure2 shows the photographs of the collapsing water column at six instances. The maximum flow speed exceeds 2m/s and the depth is about 5cm. The Reynolds number based on these values is about 10^5. The flow when accelerating is smooth but is highly turbulent after colliding against the right wall. Therefore, calculation methods that are applicable to turbulent flows will have to reproduce these stages of t=0.5 and 1.0 sec.

Figure 3 is the results of Koshizuka et al.'s (1995) calculation using the MPS method. The method does not consider any turbulence effects except possibly adjusting the value of the effective viscosity. The calculation is done with relatively small number of particles and the results up to t=0.3 sec are seen to be close to the experiment but after t=0.3 sec the particles at the front tend to splash and scatter over large area. The results at t=1.0 sec do not show the free surface shape seen in the photograph. The value of the effective viscosity in this calculation is not known but the scattery results may be due to the lack of effective viscosity.

Figure 4 shows the results of calculation done by following Monaghan's (1994) original SPH method. The particles of 40x20x4 initially arranged in four planes in the direction normal to the vertical plane shown here. The viscosity is taken 100 times larger than the actual viscosity of the real water. If it is taken smaller, the calculation diverged, quicker for smaller values of viscosity. The results again up to t=0.3 sec are very good. The flow at subsequent time is seen to be slower and the height of the water column hitting the right wall does not rise as high as the experiment. At t=1.0 sec the water is seen to have slowed down too much and there is no violent waves like those seen in the photograph. These results imply that the turbulence effects are not correctly represented.

Figure 5 shows the results of Monaghan's(1994) improved method XSPH. In this method, the particle is moved with the

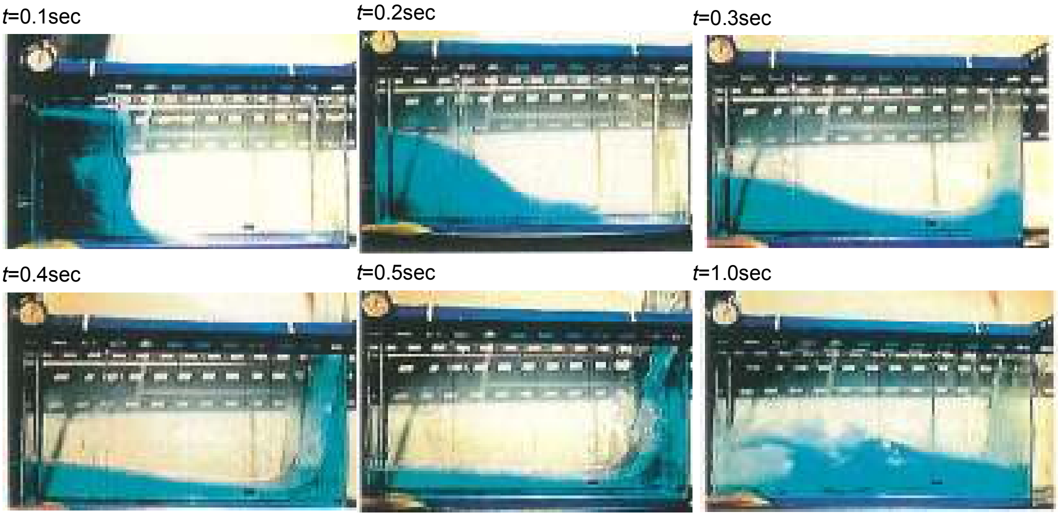

Figure 2. Experiments of Koshizuka et al. (1995) collapsing water column.

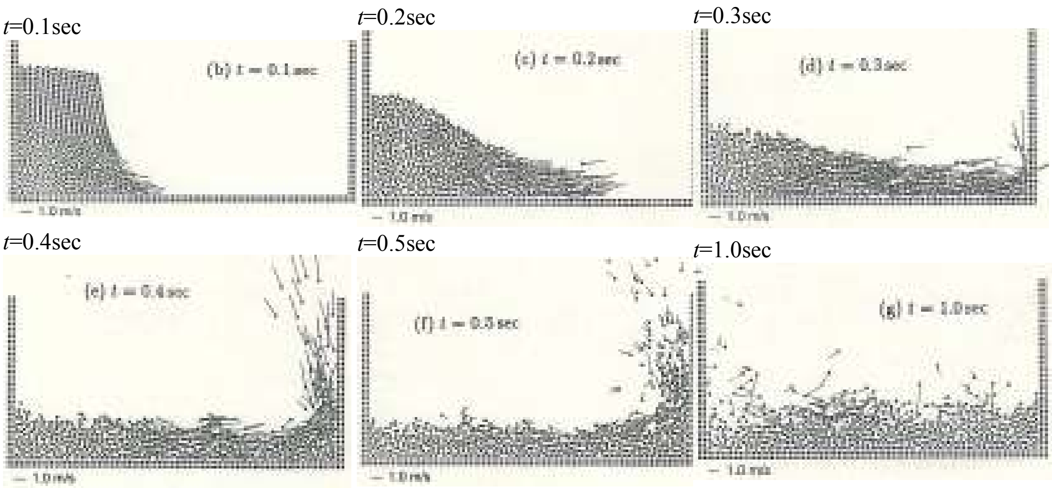

Figure 3. Calculation by Koshizula et al.(1995).

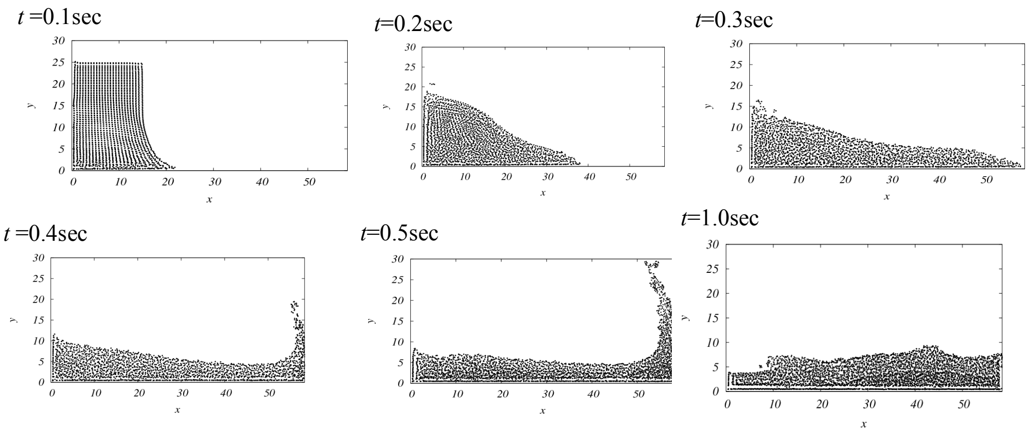

Figure 4. Calculation by Monahgan's (1994) orginal method.

speed averaged over nearby particles, Monaghan (1994) did not give theoretical justification for this model. The results indicate that the height of the water column hitting the right wall now rises higher and closer to the experiment and that at t=1.0 sec, the shape of the free surface is much closer to the experiment. Our reformulation implies that the extra term introduced by Monaghan (1994) is not quite consistent with the definition of the particle path.

Figure 6 are the results by the present method. The results, throughout the entire sequence and particularly at time t=1.0sec, of the motion and the shape of the free surface are very close to the experiment. The only disagreement is the slight shift in time so that according to the calculation, the water in the middle of the container is dropping at t=1.0, but in the experiment, the water is back and rising in the central region of the container. This is thought to be due to slight mismatch of the wall friction which would be important in the slight slowdown of the flow velocity.

CALCULATION OF FLOW IN OPEN CHANNEL

In the flow of collapsing water column, all particles stay within the region contained by the link cells in which the summation of effects from neighboring particles is taken. In many fluid flows, flow comes in and goes out of the computational region. In order to show that the present method works in such cases, we computed the steady (mean) flow in an open-channel with a flat bed. In this case we apply the periodic boundary condition in the streamwise direction and in the spanwise direction. Particles leaving the computational region are introduced at the corresponding position in the upstream inflow section. Interpolation near these boundaries assumes that there are particles outside the boundary same as the particles at the corresponding opposite side. Calculation has been conducted for two cases. The first one is a subcritical case with Froude number 0.35 and the Reynolds number 120,000. The second case is a supercritical case with Froude number 4.1 and the Reynolds number 20,000. The length and the width of the calculation region are 4.2 and 2.0 times the mean depth. 157,500 particles are placed at equal distances apart initially and calculation of subsequent positions along with the velocity, the density and the pressure of the particles are calculated. The standard log law is assumed for the mean velocity distribution and random fluctuation is superposed to the mean velocity.

The instantaneous distribution of particles obtained by the present calculation is shown in Figure 7. The color of the particles represents the velocity magnitude such that red color is fast flow and the blue slow. Although the figure shows the position and the velocity of particles at one instance, all particles are shown and they represent more like the spanwise average. The case of subcritical Froude number shown in Figure 7(a) shows very small and smooth free surface profile but the supercritical case of Figure 7(b) indicates that there are surface waves of sharper slope than the subcritical case.

CONCLUSIONS

The particle interpolation technique originated by Gingold and Monaghan (1977) has been reformulated to apply to turbulent flows with fluctuations of scales much smaller than the spacing of interpolating particles. The basic equations of motion for the particle-interpolated quantities have been obtained. They represent the large-scale flows of large since the process removes the small-scale turbulent fluctuations. We find that extra correlation terms appear that are similar to the sub-grid stress known in large eddy simulation method. If we take into account of these terms, the effective viscosity is increased where the turbulent stresses are large, resulting in stable solution but with appropriate fluctuating components.

The flow due to collapsing water column has been calculated and the results are in very good agreement with the experimental results and show an improvement over existing methods. The same method is applied to flows in straight open channel. Calculation results for both subcritical and supercritical flows are correctly calculated. However, detailed examination of the models of the extra terms is yet needed to improve the prediction in general cases.

REFERENCES

Colagrossi, A. and Landrini, M. (2003), Numerical simulation of interfacial flows by smoothed particle hydrodynamics, J. Comput. Phys. Vol.191, pp.448-475.

Gingold, R.A. and Monaghan, J.J. (1977), Smoothed particle hydrodynamics: theory and application to non-spherical stars. Mon. Not. R. Astron. Soc. Vol.181, pp. 375-89.

Gotoh, H., Shibahara, T. and Sakai, T. (2001), Sub-particle-scale turbulence model for the MPS method – Lagrangian flow model for hydraulic engineering, Computational Fluid Dynamics Journal 9(4) pp. 339-347.

Hockney, R.W. and Eastwood, J.W. (1988), Computer Simulation Using Particles, Hilger, Bristol .

Holm, D. D. (1999), Fluctuation effects on 3D Lagrangian mean and Eulerian mean fluid motion, Physica D, Vol. 133, pp.215-269.

Kajitar, J. and Monaghan, J.J. (2008), SPH simulation of swimming linked bodies, J. Comput. Phys., Vol. 227, pp.8569-8587.

Koshizuka, S., Tamako, H. and Oka, Y. (1995), Particle method for incompressible viscous flow with fluid gragmentation, Computational Fluid Dynamics Journal, Vol.4, 29-46.

Monaghan, J.J. (1994), Simulating Free Surface Flows with SPH, Journal of Computational Physics. 110, 399-406.

Monaghan, J.J. (2005), Smoothed particle hydrodynamics, Rep. Prog. Phys. Vol.68, pp.1703-1759.

Morris, J.P., Fox, P. and Zhu, Y. (1997), Modeling low Reynolds number incompressible flows using SPH, J. Comput. Phys. Vol. 136, pp. 214-226.

Sagaut, P. (2006), Large Eddy Simulation for Incompressible Flows, 3rd ed., Springer, Berlin, eidelberg, New York.

Violeau, D. and Issa, R. (2007), Numerical modelling of complex turbulent free-surface flows with the SPH model: an overview, Int. J. Numer. Meth. Flieds, Vol. 83, pp.277-304.

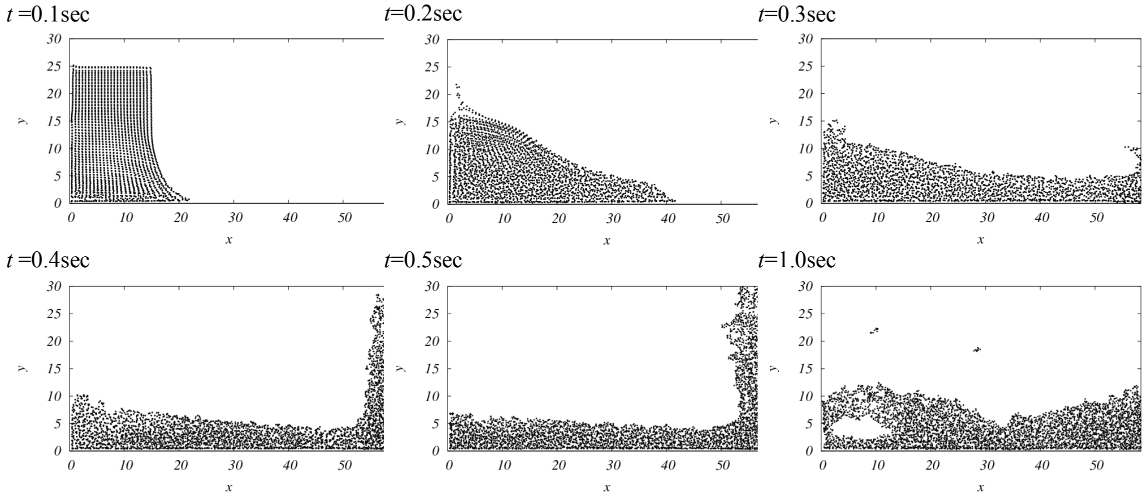

Figure 5 Calculation by Monahgan's (1994) XSPH method.

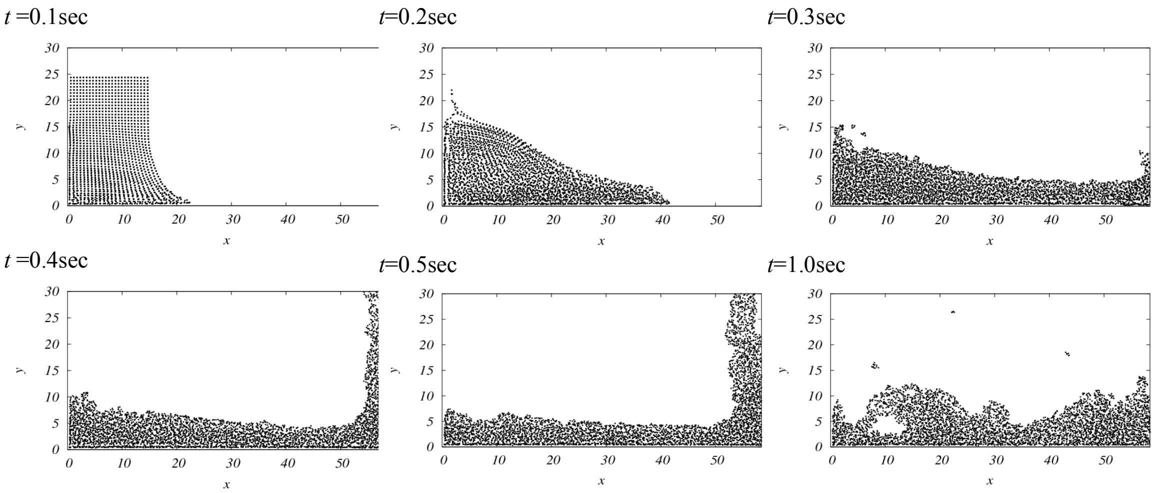

Figure 6 Calculation using present model.

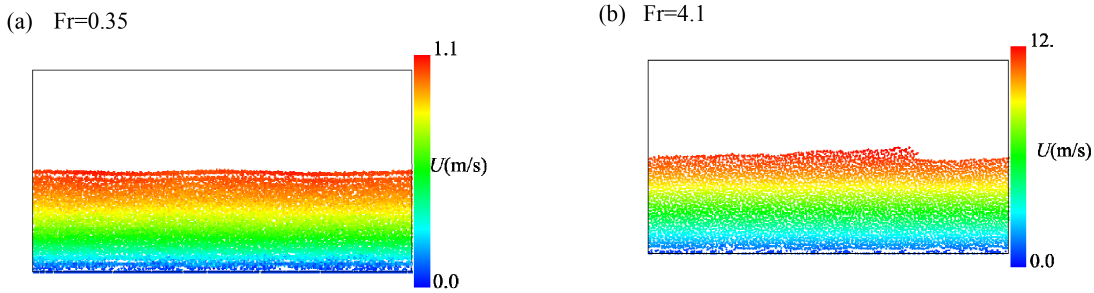

Figure 7. Calculation of open-channel flow.