GEOMETRIC NATURE OF PARTICLE TRAJECTORIES IN ISOTROPIC TURBULENCE

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ABSTRACT

Characteristics of curvature and torsion of particle trajectories are investigated by using direct numerical simulations. Curvature, torsion and their ratio are expected to have crucial roles in identification of the vortical structures. Curvature and torsion are highly intermittent and their PDFs show a rigorous slope in log-log scale which indicates self-similar power-law behavior. It is well known that extremely high curvature comes from nearly zero-velocity event, but for the case of torsion it is not clealy explained. We found that the magnitude of torsion is determined mainly by the magnitude of $|\dot{a}|/|a_n||u|$, but detailed physical interpretation of $|\dot{a}|/|a_n||u|$ is required. Enstrophy-conditional PDF and various Stokes number cases for heavy particles are also investigated, but they give almost the same shape of PDF, and that self-similar characteristics of curvature and torsion are not related with turbulent structures.

BASIC AND DEFINITION

We investigate the geometric nature of trajectories of Lagrangian fluid and heavy particles in isotropic turbulence. The shape of a trajectory can be characterized by three basic vectors which are the tangent vector **t**, the normal vector **n,** and the binormal vector **b**. In terms of Frenet-Serret formula, the time derivatives of each vector are described by a combination of curvature and torsion. (Millman and Parker, 1977),

$$\frac{d}{ds}\begin{pmatrix}\mathbf{t}\\ \mathbf{n}\\ \mathbf{b}\end{pmatrix}=\begin{pmatrix}0 & \kappa & 0\\ -\kappa & 0 & \tau\\ 0 & -\tau & 0\end{pmatrix}\begin{pmatrix}\mathbf{t}\\ \mathbf{n}\\ \mathbf{b}\end{pmatrix} \tag{1}$$

Here, **s** is a line element of a trajectory and κ is curvature and τ is torsion. Eq. (1) represents that curvature is the rate of change of the tangential vector to the normal direction, and torsion is the rate of change of the binormal vector to the normal direction. Literal meaning of curvature is the ratio of change in the angle of tangent that moves over a given arc to the length of the arc. The meaning of torsion is the degree of departure of a curve from a plane. Thus curvature makes curved shape of trajectories, and torsion makes 3-dimensional motion of trajectories. For some special cases, constant curvature and zero torsion draw a circle in a plane, and constant curvature and constant torsion make a helix shape of trajectories. Curvature and torsion can be described by velocity and acceleration vectors as written in Eq. (2) and (3).

$$\kappa=\frac{|\mathbf{u}\times\mathbf{a}|}{|\mathbf{u}|^3}=\frac{|\mathbf{a}_n|}{|\mathbf{u}|^2} \tag{2}$$

$$\tau=\frac{\mathbf{u}\cdot\left(\mathbf{a}\times\dot{\mathbf{a}}\right)}{\left(\mathbf{u}\cdot\mathbf{u}\right)^3\kappa^2} \tag{3}$$

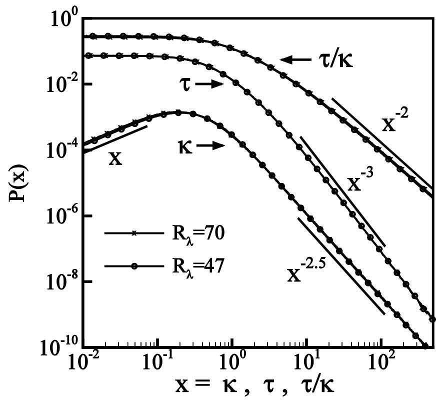

Figure 1. Probability Density functions of curvature, torsion and their ratio (Choi et al, 2010).

For the direct numerical simulation of isotropic turbulence, a pseudo-spectral method is used for the spatial discretization, and a 3rd order Runge-Kutta method is used for the time advancement. To maintain a stationary turbulent flow, a random forcing process is used based on the method proposed by Eswaran and Pope (1988). For the interpolation for the Lagrangian particle, a 4-point 4th order Hermite interpolation scheme is used, which guarantees a high accuracy with reasonable computational cost (Choi et al, 2004). Detailed simulation parameters are listed in a recent paper of Choi et al. (2010).

PDF OF CURVATURE AND TORSION

Most distinguishable characteristics of curvature and torsion are self-similar behavior. The probability density function of curvature and torsion shows a strong exponent tail as shown in Figure 1 (cited from Choi et al, 2010). Each plot is normalized by their mean absolute value (plot of torsion is shifted by 1/10, and that of curvature is shifted by 1/1000 for clarity). Exponent of -5/2 of curvature is observed both numerically and experimentally (Braun et al, 2006; Xu et al, 2007; Scagliarini, 2009). An order of -2 exponent is observed in a forced two-dimensional simulation (Kadoch et al, 2010). Normalized PDF of geometric variables are not sensitive to the Reynolds number although their mean values differ substantially as shown in Table. 1. Fig. 1 shows two Reynolds number cases of numerical simulation, but they are nearly identical. From this result we can infer that the exponential behaviour of curvature and torsion is not coming from turbulent characteristics. Previous study showed that extremely high curvature (larger than hundreds times of the mean value) is caused by nearly zero-velocity event. With the assumption of a Gaussian distribution of velocity components, -5/2 slope of curvature is clearly explained. However, for the case of torsion, it is difficult to tell clearly which statistics is responsible for the high or low torsion. Thus we investigate the correlation of the magnitude of torsion and other statistics.

Table 1. Numerical parameters and mean values

Re_λ	$k_{max}\eta$	$\langle\kappa\rangle$	$\langle\tau\rangle$	$\langle\tau/\kappa\rangle$
47	1.24	2.21	5.08	31.7
70	1.5	2.70	6.41	39.0

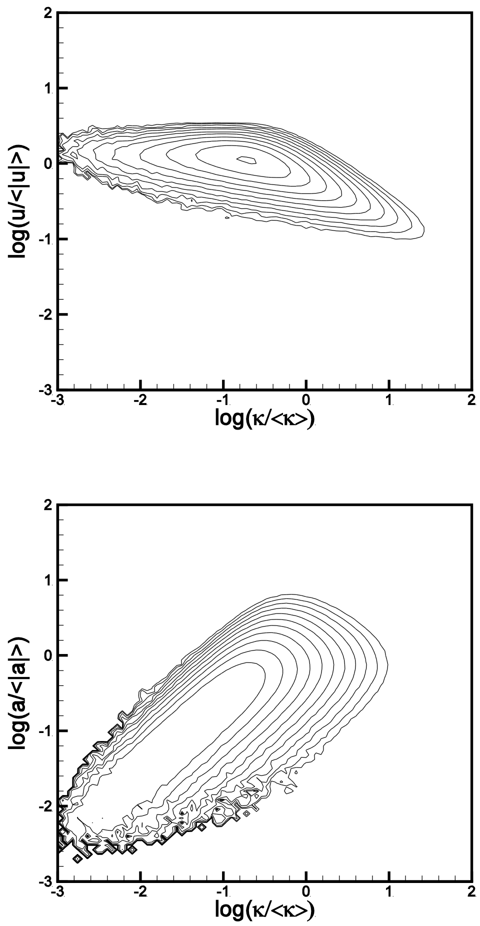

Figure 2. Joint PDF of curvature and velocity(upper) , curvature and normal acceleration(bottom).

JOINT PDF OF TORSION AND OTHER STATISTICS

As written above, high curvature is originated by small magnitude of velocity, which indicates change of the direction of a particle path. At that point, acceleration remains at finite value, and distribution of curvature is determined by velocity only. In Fig. 2, we can observe a certain negative slope at high

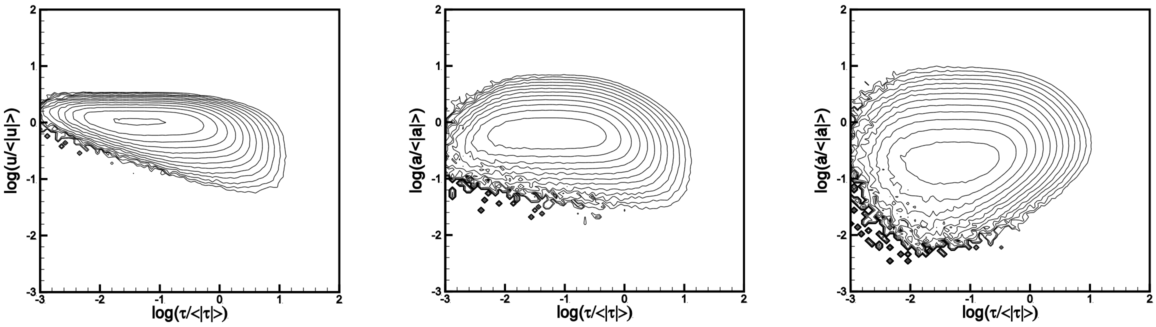

Figure 3. Joint PDF of torsion and velocity, torsion and acceleration, torsion and derivation of acceleration.

curvature. For low curvature case, acceleration must be small and distribution of curvature is determined by acceleration only. It is obvious that the normal acceleration and curvature are positively correlated with each other when the curvature is low. We can easily observe the exponential relation among curvature, velocity, and acceleration in the joint PDF in log-log scale. Likewise, we investigated joint PDF of torsion and the other statistics. As shown in Eq. 2, torsion is determined by the combination of velocity, acceleration, material derivative of acceleration, and curvature. Fig. 3 shows joint PDF of torsion and other statistics. Unlike the plot of curvature, an exponential relation is not observed in the joint PDF of torsion especially at high value of torsion. These results imply that the magnitude of torsion is not simply determined by just one variable. Thus we rearrange Eq. 2 as Eq. 4, in order to find out more detailed characteristics of torsion.

$$\tau = \frac{\mathbf{u}\cdot(\mathbf{a}\times\dot{\mathbf{a}})}{(\mathbf{u}\cdot\mathbf{u})^3\kappa^2} = \frac{\mathbf{u}}{|\mathbf{u}|}\cdot\left(\frac{\mathbf{a}}{|\mathbf{a}_n|}\times\frac{\dot{\mathbf{a}}}{|\mathbf{a}_n||\mathbf{u}|}\right) \qquad (4)$$

We assume that $|a|/|a_n| \sim 1$, then the magnitude of torsion is reduced to Eq. 5 where θ is the angle between velocity vector and $(a \times \dot{a})$ vector, and ϕ is the angle between a and $\dot{a}$.

$$|\tau| \sim \frac{|\dot{\mathbf{a}}|}{|\mathbf{a}_n||\mathbf{u}|}\cos\theta\sin\phi \qquad (5)$$

If the angles between the vectors are randomly distributed, then the magnitude of torsion is mainly determined by $|\dot{a}|/|a_n||u|$. Thus we investigate the correlation between $|\dot{a}|/|a_n||u|$ and torsion. Unlike the other variables, $|\dot{a}|/|a_n||u|$ possesses a strong correlation with torsion. Fig. 4 is the joint PDF between torsion and $|\dot{a}|/|a_n||u|$ in log-log scale. It clearly shows a positive correlation between torsion and $|\dot{a}|/|a_n||u|$. Further we investigate the PDF of $|\dot{a}|/|a_n||u|$ and

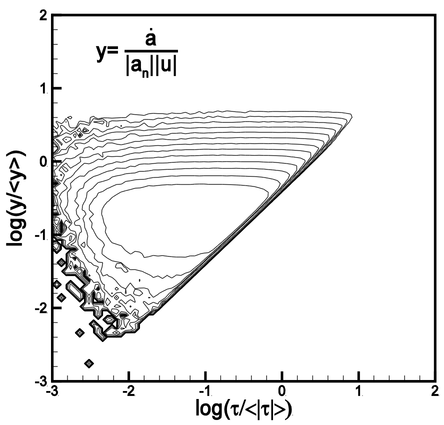

Figure 4. Joint PDF between torsion and $|\dot{\mathbf{a}}|/|\mathbf{a}_n||\mathbf{u}|$ in log-log scale.

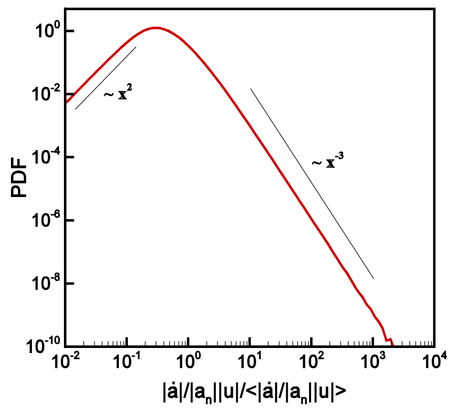

Figure 5. Probability density function of $|\dot{\mathbf{a}}|/|\mathbf{a}_n||\mathbf{u}|$.

the result is shown in Fig. 5, indicating that PDF of $|\dot{\mathbf{a}}|/|\mathbf{a}_\mathbf{n}||\mathbf{u}|$ possesses a tail with a slope of -3 in log-log plot, and it corresponds to the slope of PDF of torsion. From Fig. 4 and Fig. 5, it can be concluded that the behaviour of torsion is determined by $|\dot{\mathbf{a}}|/|\mathbf{a}_\mathbf{n}||\mathbf{u}|$. Although the newly defined variable $|\dot{\mathbf{a}}|/|\mathbf{a}_\mathbf{n}||\mathbf{u}|$ can explain the characteristic of torsion very well, $|\dot{\mathbf{a}}|/|\mathbf{a}_\mathbf{n}||\mathbf{u}|$ has no (known) physical meaning at all. Further investigation is required to find out the physical interpretation of $|\dot{\mathbf{a}}|/|\mathbf{a}_\mathbf{n}||\mathbf{u}|$.

ENSTROPHY CONDITIONAL STATISTICS

Geometric nature of Lagrangian particle trajectory is intuitively thought to be associated with rotation of fluid. Thus the enstrophy-conditioned statistics are investigated as shown in Fig. 7. Enstrophy is defined as $\frac{1}{2}\omega_i\omega_i$ and it represents the rotation of fluid. Conditional PDFs are obtained when local enstrophy is greater than 1, 3, 5 and 10 factors of the mean enstrophy for curvature and torsion. Even when the conditional factor is 10, the slope of PDF is not changed. Oscillatory behaviour of PDF conditional on $\Omega > 10\langle\Omega\rangle$ is just due to the lack of data (less than 0.3% of total event). It means that a self-similar power law still governs the behaviour of geometric variables, and it is strong evidence supporting that the exponent decay of PDF for curvature and torsion is coming from Gaussian statistics. This phenomenon is also observed in two-dimensional simulation (Kadoch et al., 2010).

However, in physical sense, curvature must contain the structure information because it represents the rotation of particle trajectories. Since curvature is only defined by velocity and acceleration, it can be easily obtained in the Eulerian frame. In Fig. 6, colour contour on iso-surfaces represent curvature when its value is around the peak of the PDF. Compared to the black surfaces which represent vortical structure, they seem to have some correlation with each other. As we can see from Fig. 7, the peak of PDF of curvature is shifted to right with conditional factor increasing. It is obvious that extremely high curvature is originated by Gaussian characteristics but there is some chance that the peak event of curvature is related with turbulent structures. Intermittency of acceleration near the structure is well known (Lee C. et al, 2005; Lee S. et al, 2005), thus it is expected that acceleration and curvature have some relation.

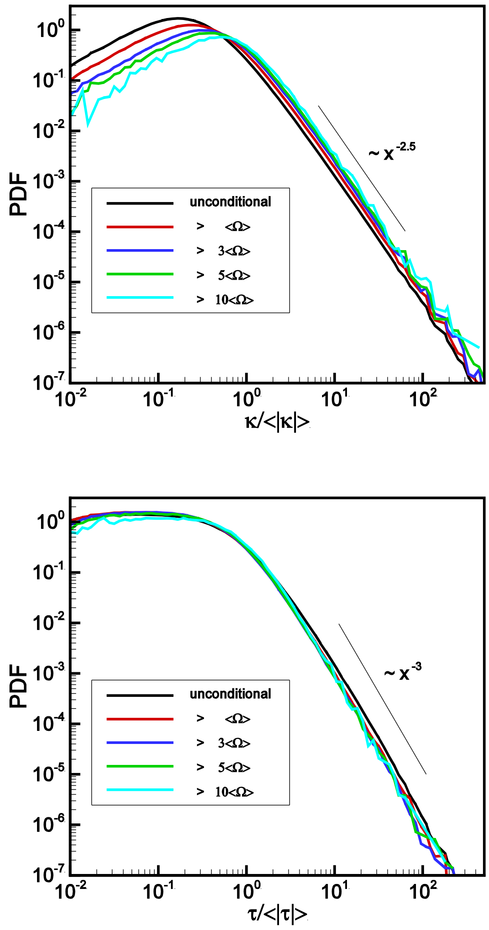

Figure 7. Conditional PDF of curvature (upper) and torsion (bottom) where Ω is enstrophy.

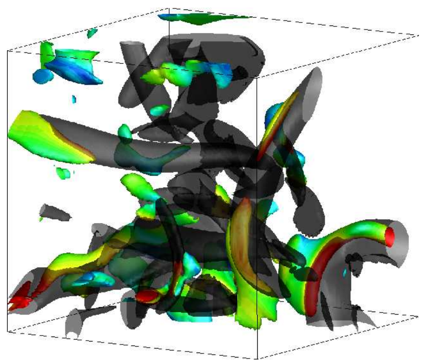

Figure 6. Vortical structure (black) and iso-surface of curvature (color). Color contour represents the magnitude of enstrophy on an iso-curvature surface.

CURVATURE AND TORSION OF HEAVY PARTICLE

Simulations of turbulence with heavy particles are also performed to investigate the inertial effect. Inertia of a particle is characterized by the Stokes number which is defined as the ratio of characteristic time scale of a particle and that of fluid. Typically Kolmogorov's time scale is used for the characteristic time scale of fluid. Characteristic time

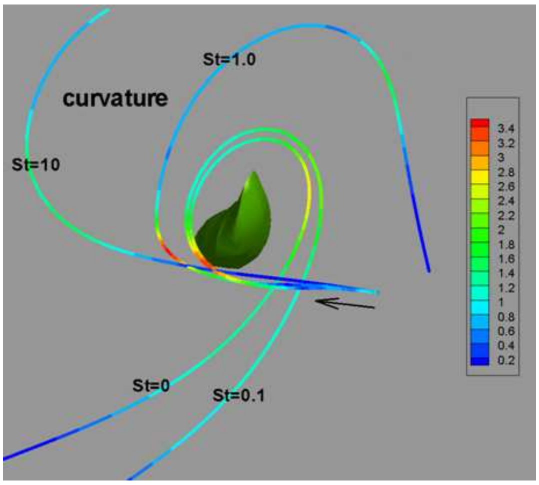

Figure 8. Vortical structure (green) and trajectories of heavy particle for Stokes number 0 (fluid) to 10.

Table 2. Mean value of curvature and torsion for various Stokes number

St	<|κ|>	<|τ|>
0(fluid)	3.54	1.90
0.1	3.51	1.85
1	3.33	1.66
10	2.42	2.12

scale for a heavy particle consists of the density of the particle, radius of the particle and fluid viscosity. Thus, the Stokes number is defined as below.

$$St \equiv \frac{\tau_p}{\tau_\eta} = \frac{2\rho_p a^2}{9\mu}\frac{1}{\tau_\eta} \qquad (6)$$

Here ρ_p is the density of a solid particle, a is radius of a particle and μ is the viscosity of the fluid. Small Stokes number particles have small inertia and they easily follow the motion of fluid. But high Stokes number particles hardly change their motion due to high inertia (Jung et al. 2008). Sample trajectories near a vortical structure for various Stokes numbers are presented in Fig. 8. Stokes number 0 represents fluid tracer, and as mentioned above it can be observed that a small Stokes number (St=0.1) particle follows the motion of fluid while a high Stokes (St=10) particle does not. Thus it is obvious that different Stokes number has different geometric characteristics. High curvature represents a steep change of the direction of particle motion, and it is clearly shown in Fig. 8. As the Stokes number increases, the mean value of curvature decreases (Table 2) because high Stokes number particles change their direction very slowly. Interestingly enough, normalized PDFs of various Stokes numbers give almost the same distribution as shown in Fig. 9. As we can see in Fig. 8, trajectories of heavy particles are clearly different, and normalizing factor is also different, but the normalized PDF is not distinguishable. It can be evidence supporting that self-similarity of curvature and torsion is coming from Gaussian characteristics, not from turbulent gradient.

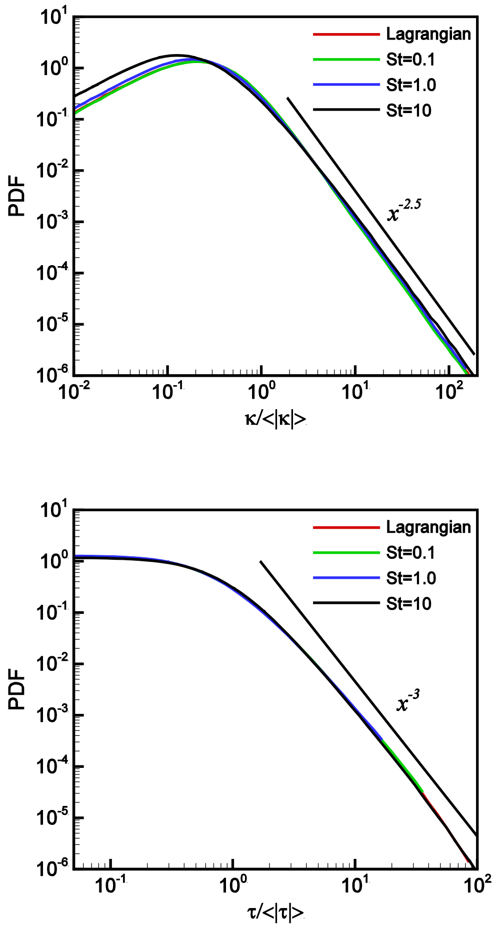

Figure 9. PDF of curvature (upper) and torsion (bottom) for variou Stokes numbers.

SUMMARY AND CONCLUSION

Characteristics of curvature and torsion are investigated by using direct numerical simulation of particle-laden isotropic turbulence. PDF of curvature and torsion show rigorous exponential tail which indicate self similar behavior. Using a Gaussian approximation of the velocity components, -2.5 slope of curvature can be explained, but there is no plausable explanation for torsion's behavior. Thus we focus on the analysis of torsion, and find that the magnitude of torsion is mainly determined by the magnitude of $|\dot{a}|/|a_n||u|$. But this analysis still defies a physical interpretation of torsion. Enstrophy conditional PDF and various Stokes number cases are also investigated, but they give almost the same result as unconditional and Lagrangian fluid cases, and that self-similar characteristics of curvature and torsion are not directly related with turbulent structures. For heavy particle simulations, we obtained the same shape of PDF although normalization factors are changed with the Stokes number. From these results we can infer that the rigorous exponent behavior of curvature and torsion is coming from Gaussian characteristics of velocity component.

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