APPLICATIONS OF FULLY-RESOLVED LARGE EDDY SIMULATION TO UNSTEADY FLUID FLOW AND AEROACOUSTICS PREDICTIONS

Chisachi Kato
Institute of Industrial Science
The University of Tokyo
4-6-1 Komaba, Meguro-ku, Tokyo, 153-8505 Japan
ckato@iis.u-tokyo.ac.jp

ABSTRACT

With tremendous speed-up of high-end computers, applications of fully-resolved LES, which directly computes the streamwise vortices in a turbulent boundary layer, will become feasible for engineering flows with a bulk-flow Reynolds number up to several million in about 5 years when the number of computational grids will probably reach 100 billion. A project that is aimed at providing industries with baseline software capable of performing a large-scale engineering LES and acoustical computations are being undertaken. Dynamic Smagorinky Model (DSM) is adopted for LES while Helmholtz equation is solved for the acoustical pressure by using Lighthill's acoustical tensor computed by the incompressible LES. Both solvers are designed such that they are to speed up to one million processing cores, and at this moment their parallel scalability has been confirmed up to 8,192 processing cores in various platforms. Results of extensive validation studies for basic flows show that the fully-resolved LES predicts fluid flow with an expected level of accuracy. In particular, it predicts frequency spectra of the fluctuating velocities and pressure that are almost identical to the measurements. Sound pressure spectra radiated from a small industrial fan was accurately predicted by a combined use of the flow and acoustical solvers.

INTRODUCTION

Performance of high-end computers is steadily increasing with a considerable rate of speed-up for the past several decades. In fact, since 1990s it has become ten times faster for three years and a thousand times for a decade. This trend of speed up is expected to continue at lease for the coming decade as well. Within 2011 or 2012, the performance of the world's fastest computers is expected to reach 10 Peta Flops (10^{16} floating point operations per second).

As is well known to the turbulence community, the scale of the smallest active eddies in a turbulent boundary layer is approximately inversely proportional to the Reynolds number of the bulk flow. This has been limiting applications of fully-resolved LES, which directly computes motions of the smallest active eddies, to simple research flows with a low and moderate Reynolds number. For engineering applications of LES, near-wall small structures of a turbulent boundary layer have been modelled such as in detached eddy simulation (DES).

With the tremendous increase in computer performance mentioned above, applications of fully-resolved large eddy will probably become feasible in the design and testing phases of various flow-related industrial products, including automobiles, high-speed trains and turbomachinery such as pumps and turbines, if the Reynolds number of the bulk flow does not go beyond in the order of million.

Since fully-resolved LES models only conversion process of kinetic energy contained in the smallest active eddies being dissipated to heat, it gives most accurate solutions to the Navier-Stokes equations, except for Direct Numerical Simulation (DNS). In fact, it is generally believed that fully-resolved LES gives almost as accurate results as DNS does with at least one tenth lower computational cost than that of DNS. This reduction in the computational cost is primarily attributed to the difference in the scales of the active eddies and eddies responsible to dissipation.

In the design of various flow-related products, prediction and reduction of the aerodynamic sound, radiated by motions of turbulent eddies, are also one of the crucial issues and in many cases, they are as important as reduction of aerodynamic drag of a vehicle or improvement in the performance of turbomachinery. This is because the magnitude of the aerodynamic sound is theoretically proportional to the 5th to 8th power of the bulk flow velocity (Lighthill, 1952; Curle, 1955), and therefore a small increase in the bulk flow velocity may results in a drastic increase in the aerodynamic sound radiated from the flow. Because it is generated by temporal deformation of vortices in the flow, aerodynamic sound can also be computed accurately provided that fully-resolved LES provide accurate information regarding vortical motions in the flow.

With all these situations in mind, the ultimate goal of this research project is to provide industries with a set of baseline flow and acoustical solvers that is capable of handling a very large scale computational mesh with up to 100 billion grids and of predicting fluid flow and aeroacoustics problems encountered in engineering with virtually the same level of accuracy as DNS does. This paper will give a brief introduction to the project and present some of the results obtained so far.

FORECAST FOR ENGINEERING LES

Within this year (2011), the world's fastest computer will accommodate approximately 1 million cores with a theoretical performance of 10 billion floating point operations per second (10 GFLOPS). In general, researchers and engineers in industry will have an access to a computer with a peak performance one-tenth smaller than the world's fastest computer at date in about three years. Therefore, in 2014 a computer with 100 thousands processing cores will generally be available in industries. The computational size of LES that is feasible with this number of processing cores depends on the numerical method adopted for LES and its implementation to the flow solver. But, our estimation as well as experience tells that a maximum of one million grids can be assigned to one processing core. Hence, engineering LES with 100 billion (1×10^{11}) grids will become feasible in 2014.

To adequately resolve active near-wall structures in a turbulent boundary layer by LES, a computational grid with $\Delta x^+ = 40$, $\Delta y^+_{min} = 2$, $\Delta z^+ = 10$ is needed where Δx^+, Δy^+_{min} and Δz^+ respectively denote streamwise, wall normal, and spanwise resolutions expressed in the wall unit. The length scale l^+ in the wall unit can be converted to that of the bulk flow, l, with the following relation:

$$l^+ \equiv \frac{u_\tau l}{\nu} = \frac{UL}{\nu} \times \frac{u_\tau}{U} \times \frac{l}{L} = R_e \times \sqrt{\frac{C_f}{2}} \times \frac{l}{L} \tag{1}$$

or,

$$\frac{l}{L} = l^+ \frac{1}{R_e\sqrt{\frac{C_f}{2}}} \tag{2}$$

where R_e and C_f are Reynolds number of the bulk flow and local skin friction coefficient respectively, defined as follows:

$$R_e \equiv \frac{UL}{\nu} \tag{3}$$

$$C_f \equiv \frac{\tau_w}{\frac{1}{2}\rho U^2} \tag{4}$$

By assuming that the skin friction coefficient takes the flat plate values, the near-wall resolutions and number of computational grids required for performing a fully-resolved LES are estimated, for typical Reynolds numbers, as in Tables 1 and 2. We assumed that the extent of the boundary layer is L both in the streamwise and spanwise directions and that flow field of engineering applications is typically composed of a maximum of 10 such boundary layers. The boundary layers encountered in practical applications are more likely to be decelerating than they are accelerating. This estimation therefore gives a slight over-prediction for the necessary resolutions in general. In any case, engineering applications of LES will become feasible for flows with a Reynolds number up to several million in about five years when the maximum number of computational grids will probably reach 100 billion. Expected applications of LES are listed in Table. 3.

Table 1. Estimated near-wall grid resolutions needed for LES for typical Reynolds numbers.

R_e	C_f	$\frac{\Delta x}{L}$	$\frac{\Delta y_{min}}{L}$	$\frac{\Delta z}{L}$
2×10^4	1.1×10^{-2}	2.7×10^{-2}	1.3×10^{-3}	6.7×10^{-3}
2×10^5	6.2×10^{-3}	3.6×10^{-3}	1.8×10^{-4}	9.0×10^{-4}
2×10^6	3.9×10^{-3}	4.5×10^{-4}	2.3×10^{-5}	1.1×10^{-4}

Table 2. Estimated numbers of computational grids needed for engineering LES at typical Reynolds numbers.

R_e	n_x	n_y	n_z	$n = n_x \times n_y \times n_z$	N=10n (total number of grids)
2×10^4	3.7×10^1	2.5×10^1	1.5×10^2	1.4×10^5	1.4×10^6
2×10^5	2.8×10^2	1.0×10^2	1.1×10^3	3.0×10^7	3.0×10^8
2×10^6	2.2×10^3	4.0×10^2	9.0×10^3	7.9×10^9	7.9×10^{10}

Table 3. Engineering applications of LES expected in 2015.

products	specifications	R_e	N/n	N
automobile	L=1 m, U=28 m/s (100 km/h)	1.9×10^6	10	7.9×10^{10}
model ship	L= 5m (1/50 scale model), U=1.0 m/s	4.6×10^6	1.2	8.9×10^{10}
model pump	D_2=300 mm, 1500 rpm, L=0.15 m, U=24 m/s	3.6×10^6	12	3.9×10^{11}
wind turbine	D_2=40 m, L=0.4 m, U=64 m/s	2.5×10^6	3	4.0×10^{10}
axial-flow fan	D_2=600 mm, 1800 rpm, L=0.2 m, U=56 m/s	7.5×10^5	12	8.6×10^9
propeller fan	D_2=500 mm, 600 rpm, L=0.2 m, U=16 m/s	2.0×10^5	3	1.0×10^8
small cooling fan	D_2=80 mm, 3400 rpm, L=0.02 m, U=14 m/s	1.9×10^4	7	1.0×10^6

SOLUTION METHOD AND GOVERNING EQUATIONS

Aerodynamic sound is generated from change in the *force* in the flow that is caused by temporal deformation of vortices (Lighthill 1952; Powell, 1964; Howe 2001). It propagates to the far field by the speed of sound. Since its generation and propagation is also governed by the compressible Navier-Stokes equations, aerodynamic sound can be directly computed by solving them. But, for a relatively low Mach number flows, often times encountered in engineering, source vortices are much smaller than the waves of sound that such vortices generate. Hence, direct computation of aerodynamic sound is not feasible at least for engineering purposes.

We therefore adopt a so-called two-step approach for predicting aerodynamic sound by assuming that the feedback effects of the sound onto the source flow field are negligible. The source fluctuations in the flow are first computed by a Large Eddy Simulation (LES) of incompressible fluid with Dynamic Smagorinsky Model (Smagorinky, 1963; Germano et al., 1991; Lilly, 1992) and they are fed to the subsequent acoustical computation as input data.

Fully-resolved LES generally gives source fluctuations over a wide range of frequency and at high-frequencies sound can no longer be regarded as *compact*. At high frequencies it is important to take the effects of scattering of incident sound on solid walls where acoustical boundary condition of zero particle velocity must be satisfied. Note that fluctuations in the hydraulic pressure computed by incompressible LES do not necessarily satisfy the above-mentioned acoustical boundary condition. In this study, the scattering of incident sound on the solid walls are exactly computed by solving Lighthill equations (Lighthill, 1952) given below:

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2}p_a - \frac{\partial^2}{\partial x_i \partial x_j}p_a = \frac{\partial^2}{\partial x_i \partial x_j}T_{ij} \qquad (5)$$

$$T_{ij} = \rho u_i u_j + \delta_{ij}(p_a - c^2\rho_a) + \tau_{ij} \qquad (6)$$

where $p_a$is the acoustic pressure to be solved, c is the speed of sound in the fluid and T_{ij} is Lighthill tensor. For sound generated from flows with a low Mach number and a relatively high Reynolds number, the second and third terms of Lighthill tensor, which respectively denotes entropy production and forces due to viscous stresses, can usually be neglected. In this study, only the first term of the Lighthill tensor is considered. This term can be computed by incompressible LES and denotes change in momentum due to temporal deformation of vortices.

NUMERICAL METHOD AND ITS IMPLEMENTATION

For incompressible LES, fractional step (FS) method is implemented to solve the pressure Poisson equation, combined with Crank-Nicolson implicit time integration. To handle complicated geometries often encountered in engineering problems, the Finite Element Method (FEM) is adopted for the spatial discretization. The moving boundary interface that appears in applications of LES to turbomachinery flows is treated with overset grids from multiple dynamic frames of reference (Kato et al., 2003; Kato et al., 2007). The global linear system of equations that results from the implicit time integration as well as the pressure Poisson equation is solved by Bi-CGSTAB method (Vorst, 2002) combined with Residual Cutting Method (RCM) as its outer loop (Tamura, 1997).

For the acoustical computations, Lighthill equation is transformed into the frequency domain and Helmholtz equation is instead solved for the acoustical pressure. FEM is also used for solving Helmholtz equations. The global linear system of equations that results from discretizing Helmholtz equation may become very stiff. A matrix solver based on the conjugate gradient (CG) methods often times does not converge within a practical number of iterations (several thousands to several tenth of thousands). We therefore implemented a matrix solver based on the induced dimension reduction (IDR) method, which gives much faster (generally by a factor of 5 or more) rate of convergence, compared to that based on CG methods.

The flow and acoustical solvers run in massively parallel by the domain decomposition method. All the parts of these solvers are completely parallelized. By carefully estimating and minimizing the load imbalance and communication overhead, both solvers are designed such that a parallel efficiency of 50% is kept at least up to one million processing cores. The kernel routine that performs matrix-vector operations has been carefully tuned such that occurrence of the cache miss be minimized and a minimum of 5 % of theoretical performance be sustained in practical applications. Figure 1 shows some of the benchmark results of the flow solver. With increasing number of the processing cores, the solver perfectly speeds up. In these benchmark tests, 5 to 10 % of the theoretical performance is sustained at the maximum number (8,192) of the processing cores.

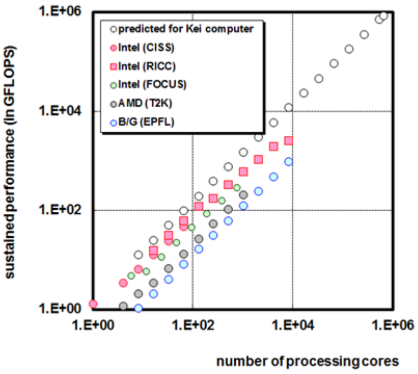


Figure 1. Benchmark tests for flow solver.

In engineering applications of computational fluid dynamics (CFD), it is crucial that computational mesh can be generated within reasonable time (several hours to a couple of days at longest) from the CAD (Computer Aided Design) data. But, is it essentially impossible to generate a computational mesh composed of hundreds billion grids by hand. The project provides two go-arounds against this obstacle: automatic grid refiner and voxel mesh generator. The solvers running in massively parallel can call (by the request from the user) an automatic grid refiner and then the original mesh will be refined in run time in order to achieve a desired level of local grid resolutions. On the other hand, the voxel mesh generator runs in a massively parallel environment and hands the generated mesh over to the flow or acoustical solvers. In this way, the user is able to handle a computational mesh as large as composed of 1,000 billion grids.

VALIDATION STUDIES

Before the developed solvers are applied to practical engineering problems, extensive validation studies have been made for a number of basic fluid-flow and acoustical problems. Among them, some of the results that most reflect accuracy to be obtained when these solvers are applied to practical engineering problems will be presented here.

Figure 2 shows an instantaneous distribution of streamwise vorticity computed by fully-resolved LES for a flow around a square cylinder at a Reynolds number of 40,000. The spanwise extent of the cylinder was 4 D with D denoting side of the cylinder. At this Reynolds number laminar separations take place at the front corner edges. The boundary layer begins to be accelerated from the stagnation point until they reach the corner edges where sudden acceleration followed by deceleration takes place. The thickness of the separating shear layers is very thin. For this flow it is crucial to accurately capture convective disturbances in these thin shear layers that eventually leads to transition to turbulence.

Figure 3 compares frequency spectra of the fluctuating velocity computed by LES with various grid resolutions with experimental data measured by a hot wire anemometer at two typical points in the flow. LES with 55 million grids gives virtually the same spectra as measured by the hot wire.

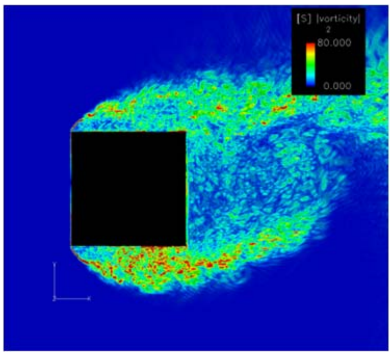
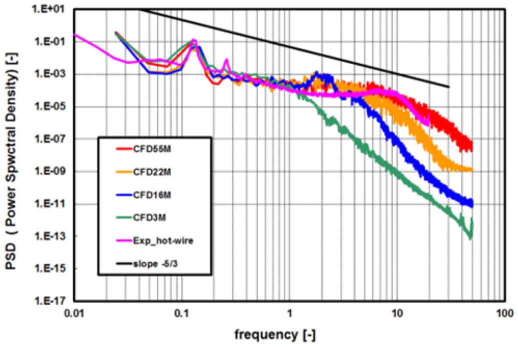
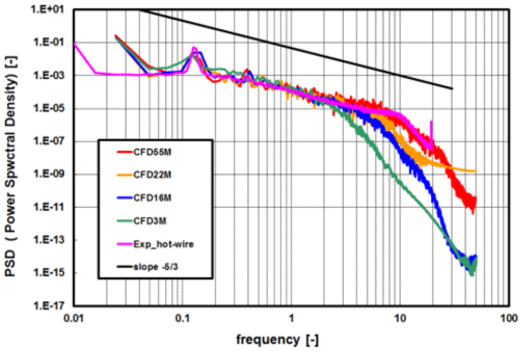


Figure 2. Streamwise vorticity computed by LES with 55 million grids for flow around a square cyliner at *R*e=40,000.



(a) 0.25 D downstream from the edge



(b) x = 3D (near wake)

Figure 3. Evolution of the power spctra of fluctuating velocity.

Flow around a NACA0012 aerofoil with an open tip was computed for a chord-based Reynolds number of 200,000 and an angle of attack of 9 degrees as shown in Figure 4. This case was chosen to confirm capability of the flow solver to predict boundary layer transition, development of the turbulent boundary layer and possible interactions of tip vortices and the turbulent boundary layer. LES with approximately 40 million computational grids adequately resolves this flow.

The time-averaged distributions of the measured static pressure around the aerofoil indicate that at the mid span of the aerofoil transition to turbulence is completed by around 0.3 *C*, with *C* denoting the chord length, downstream of the leading edge while it is delayed to at around 0.8 *C* near the tip due to the reduction in the aerofoil loading (adverse pressure gradient) there. Although not shown in this paper, transition points were accurately predicted by the present LES. The bottom figure of Figure 4 compares the time-averaged distribution of the static pressure on the suction and pressure surfaces of the aerofoil near the tip (0.02 *C* apart from the tip). The negative peak at around 0.35 *C* downstream of the leading edge indicates the passage of the tip vortex, which is accurately predicted by the present LES. The power spectra of the fluctuating static pressures at the end surface are also well predicted by the present LES as shown in Figure 5.

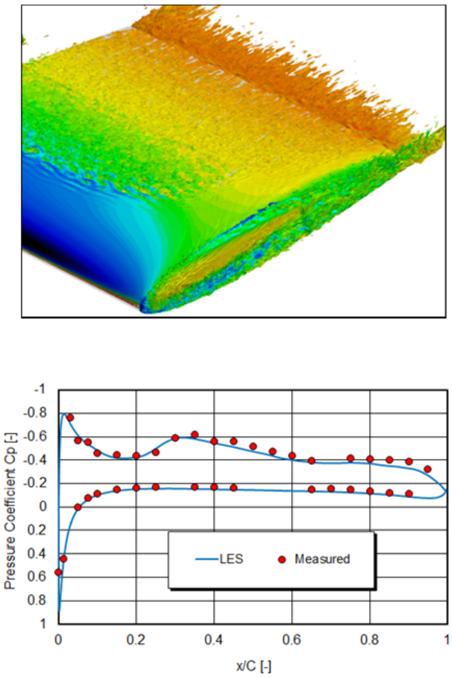
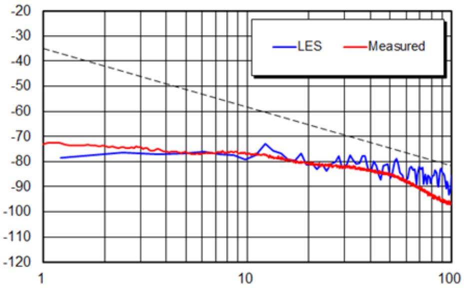
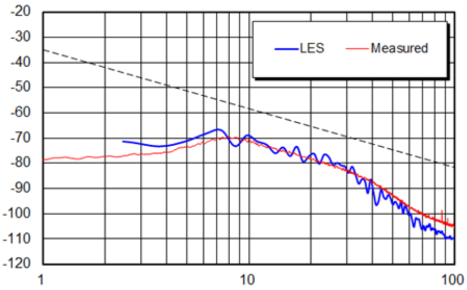


Figure 4. Instantaneous vortical strucrues and static pressure distribution computed for flow around a NACA0012 aerofoil with an open tip at $\boldsymbol{R_e = 2.0 \times 10^5}$ and $\boldsymbol{\alpha = 9°}$.



(a) x=0.1 C



(b) x=0.6 C

Figure 5. Comparisons of power spectra of fluctuating static pressure at end surace.

ENGINEERING APPLICATIONS

The LES and acoustical solvers have been applied to a number of engineering problems (see Kato et al., 2003, and Kato et al., 2007 for detail). Due to the limited space available, only one such example is presented in this paper, where flow and acoustical field were computed for a small cooling fan as shown in Figure 6.

The outer dimension of the test fan is 80 mm and a seven-bladed impeller rotates at 3,400 rpm, resulting in blade's chord based Reynolds number of 30,000. The boundary layers on the blades are presumably laminar both for the suction and pressure surfaces. LES with approximately 4 million grids adequately resolves all the important scales in this flow. In fact, grid sensitivity studies confirmed that LES with 40 million grids gives virtually the same results as the present grid does.

The blade passing frequency (BPF) is approximately 400 Hz for the test fan. A small cooling fan of this type radiates intense tonal sound at the BPF and its higher harmonics that are results of the blade-strut potential interactions. It is important to predict these intense tonal sounds as well as the broadband sounds up to around 4 k Hz.

The sound generated by potential interaction is not represented by Lighthill equation (5) and therefore its direct application is not appropriate. For this small fan, the extent where intense fluctuations in the surface pressure take place is limited to the impeller and the casing, which can be regarded as compact sound source. We therefore used surface pressure fluctuations computed by the incompressible LES as the input data for the subsequent acoustical computation. The acoustical solver can also be used for this purpose. Namely, we first compute the tailored Green's function that represents acoustical field around the fan and then multiply the source fluctuations computed by incompressible LES with the tailored Green's function to obtain the acoustical field (Takayama, 2011). Figure 7 compares sound pressure spectra predicted in this way for two different flow rate ratios. It is confirmed that for both flow rate ratios, the sound pressure spectra are fairly accurately predicted by the present method.

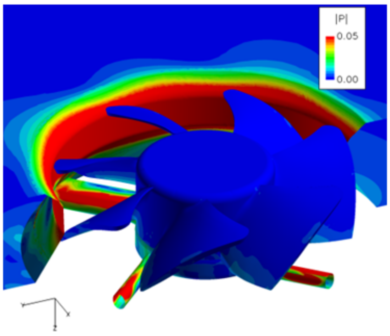


Figure 6. Surface pressure fluctuations predicted for a small cooling fan.

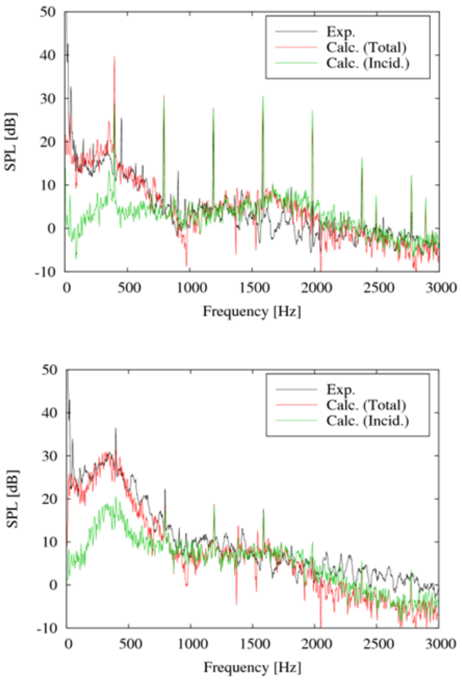


Figure 7. Comparisons of far-field sound for a small cooling fan.

CONCLUDING REMARKS

Flow and acoustical solvers for LES and acoustical predictions have been developed and validated for several classes of basic as well as industrial flows. Dynamic Smagorinky Model (DSM) is adopted for LES while Helmholtz equation is solved for the acoustical pressure by using Lighthill's acoustical tensor computed by the incompressible LES. Both solvers are designed such that they are to speed up to one million processing cores, and at this moment their parallel scalability has been confirmed up to 8,192 processing cores. Validation studies for basic flows show that the fully-resolved LES predicts fluid flow with an expected level of accuracy. In particular, it predicts frequency spectra of the fluctuating velocities and pressure that are almost identical to the measurements. Sound pressure spectra radiated from a small industrial fan was accurately predicted by a combined use of the flow and acoustical solvers.

ACKNOWLEDGMENTS

The flow solver (FrontFlow/blue) and acoustics solver (FrontFlow/blue-acoustics) have been developed and validated in "Innovative Simulation Software Project" sponsored by the Ministry of Education, Culture, Sports, Science and Technology (MEXT). Front/Flow/blue is primarily developed by Mr. Y. Yamade with a support of Dr. T. Takayama of Mizuho Information and Research Institute while FlowFlow/blue-acoustics is developed by Dr. Y. Guo of The University of Tokyo. The automatic grid refiner has been developed by Prof. S. Yoshimura of The University of Tokyo while the voxel-mesh generator is being developed by Dr. Ono of The University of Tokyo. The matrix solver based on IDR method was coded and tested by Drs. Y. Shizawa and M. Iizuka of Research Organization for Information Science and Technology. The square cylinder LES and wind tunnel measurements were done by Mrs. N. Masuda and T. Tomura of Nihon University and S. Mizutani of Kogakuin University. The measured data for the sound pressure spectra for the small cooling fan were provided by Canon Inc. in collaboration with The University of Tokyo. The wind tunnel measurements were done under supervision of Dr. Y. Suzuki of Nihon University. Without his contribution to this project, the validation studies could not have proceeded as successfully as presented in this paper.

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