Sixth International Symposium on Turbulence and Shear Flow Phenomena
Seoul, Korea, 22-24 June 2009

STATISTICAL PROPERTIES OF A COHERENT STRUCTURE FUNCTION FOR HOMOGENEOUS ISOTROPIC TURBULENCE AND TURBULENT CHANNEL FLOWS

Hiromichi Kobayashi
Department of Physics,
Keio University,
4-1-1, Hiyoshi, Kohoku-ku, Yokohama, 223-8521, Japan
hkobayas@phys-h.keio.ac.jp

Yasuhiro Tominaga, Taisuke Kubota, Mamoru Tanahashi, and Toshio Miyauchi
Department of Mechanical and Aerospace Engineering,
Tokyo Institute of Technology,
2-12-1, Ookayama, Meguro, Tokyo, 152-8550, Japan
mtanahas@mes.titech.ac.jp, tmiyauch@mes.titech.ac.jp

ABSTRACT

Statistical properties of a coherent structure function F_{CS} are investigated using DNS data for homogeneous isotropic turbulence and turbulent channel flows. The function F_{CS} is defined as the second invariant Q of a velocity gradient tensor normalized by the magnitude E of the velocity gradient tensor. In homogeneous isotropic turbulence, the probability density function (pdf) of F_{CS} shows good agreement at different Reynolds number. The volume fraction of positive Q and the average of F_{CS} converge to a certain value at high Reynolds number. For the turbulent channel flows, the pdf of F_{CS} at different Reynolds number coincides very well at the distance from the wall in wall units. The profiles of the conditional average of positive and negative Q, the volume fraction, and the near-wall F_{CS}^2 are in good agreement at different Reynolds number and those profiles are given as a function of the distance from the wall in wall units.

INTRODUCTION

Understanding the intermittency of turbulence has been one of the central issues on the statistical theory of turbulence. According to the local similarity theory by Kolmogorov (1941), the fluctuations of all scales result in the normal probability distribution. The probability density function (pdf) of velocity u is indeed given by a normal probability distribution (see for example Anselmet *et al.*, 1984; Vincent & Meneguzzi, 1991). As is well-known, however, the longitudinal velocity difference normalized by the rms velocity $\Delta u/\left\langle u^2\right\rangle^{1/2}$ distributes like a skirt with exponential tails and deviates from the normal probability distribution (see for example Anselmet *et al.*, 1984; Vincent & Meneguzzi, 1991; Tsinober, 2001; Ishihara *et al.*, 2009). This is due to the spatial fluctuation of turbulent energy dissipation ε. The influence of the fluctuation of ε on the pdf of $\Delta u/\left\langle u^2\right\rangle^{1/2}$ appears in small scales. This is the so-called intermittency of turbulence.

Kolmogorov and Obukhov improved the Kolmogorov's hypothesis in 1941 taking account of the spatial fluctuation of ε regarded as a lognormal distribution (Kolmogorov, 1962). Anselmet *et al.* (1984) showed that the lognormal distribution agrees with the experimental data for the lower-order moments of the structure function, while it deviates for the higher-order ($n > 10$) moments. Many theories have been proposed to predict the higher-order moments of the structure function; e.g. the β model (Frisch *et al.*, 1978), the multifractal model (Meneveau & Sreenivasan, 1991), the log-Poisson theory (She & Leveque, 1994), and the trinominal model (Hosokawa *et al.*, 1996).

The pdfs of $\partial u_x/\partial x$ and $\partial u_x/\partial y$ normalized by rms velocity $\left\langle u^2\right\rangle^{1/2}$ also indicate the skirt-like distributions (Vincent & Meneguzzi, 1991; Ishihara *et al.*, 2009). Biferale (1993) derived the pdf of the velocity gradient from a multifractal model and showed that the increase in the exponential tails of the pdf is accompanied by the increase in Reynolds number Re. However, it is difficult to compare the pdf with different Re, and the pdf independent of Re is expected for the statistical theory of turbulence.

In turbulence, coherent structures – eddies – exist and it is revealed that the diameter and the maximum azimuthal velocity of the coherent fine-scale eddies have the universal scaling by the Kolmogorov microscale (η) and Kolmogorov velocity (u_k) in homogeneous isotropic turbulence, turbulent mixing layers, and turbulent channel flows using direct numerical simulation (Miyauchi & Tanahashi, 2001; Tanahashi *et al.*, 2004; Das *et al.*, 2006). The most expected diameter and maximum azimuthal velocity are $8-10\eta$ and $1.2-2.0u_k$, respectively. The eddies and tube-like structures are often extracted using the second invariant Q of the velocity gradient tensor proposed by Hunt *et al.* (1988). Since increasing in the maximum and minimum of Q is accompanied by the increase in Re, it is necessary to think out a new scaling to compare the statistical properties of the pdf, the average, and the volume fraction.

Kobayashi (2005) proposed a coherent structure function F_{CS} – the second invariant normalized by the magnitude of the velocity gradient tensor. The function F_{CS} indicates the coherent structure scaled by the strength of the shear stress and is useful to compare the statistical properties because F_{CS} has maximum and minimum as $-1 \leq F_{CS} \leq 1$. F_{CS} was used for a subgrid scale model in large-eddy simulation – the coherent structure model – and the model was demonstrated in a series of canonical turbulent flows (Kobayashi,

Re_τ	Re_{mc}	Domain size	$Nx \times Ny \times Nz$	Re_λ	Re_{lE}	N^3
180	3276	$4\pi\delta \times 2\delta \times 2\pi\delta$	$192 \times 193 \times 160$	60.1	201	128^3
400	8200	$3\pi\delta \times 2\delta \times \pi\delta$	$384 \times 385 \times 192$	97.1	516	256^3
800	17760	$2\pi\delta \times 2\delta \times \pi\delta$	$512 \times 769 \times 384$	119.5	692	324^3
1270	30320	$2\pi\delta \times 2\delta \times \pi\delta$	$864 \times 1239 \times 648$	175.4	1519	512^3

Table 1: Numerical conditions of DNS for turbulent channel flows (left) and homogeneous isotropic turbulence (right).

2005), MHD duct flows (Kobayashi, 2008), and complex flows – a backstep flow, a diffuser flow, and staggered jets (Kobayashi *et al.*, 2008). Wang *et al.* (2006) discussed the influence of near-wall anisotropy on the turbulence topologies predicted using large-eddy simulation in terms of the second and the third invariants in the invariant phase plane.

In this study, the statistical properties of F_{CS} are investigated for homogeneous isotropic turbulence and turbulent channel flows using the data of direct numerical simulation (DNS) at several values of Reynolds number.

COHERENT STRUCTURE FUNCTION

The coherent structure function F_{CS} is the function of the second invariant Q normalized by the magnitude of the velocity gradient tensor E, and is defined as follows.

$$F_{CS} = \frac{Q}{E} \tag{1}$$

$$Q = \frac{1}{2}\left(W_{ij}W_{ij} - S_{ij}S_{ij}\right), \quad E = \frac{1}{2}\left(W_{ij}W_{ij} + S_{ij}S_{ij}\right) \tag{2}$$

$$S_{ij} = \frac{1}{2}\left(\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j}\right), \quad W_{ij} = \frac{1}{2}\left(\frac{\partial u_j}{\partial x_i} - \frac{\partial u_i}{\partial x_j}\right) \tag{3}$$

where S_{ij} is the strain-rate tensor and W_{ij} is the vorticity tensor. F_{CS} has the maximum and minimum as follows.

$$-1 \le F_{CS} = \frac{W_{ij}W_{ij} - S_{ij}S_{ij}}{W_{ij}W_{ij} + S_{ij}S_{ij}} \le 1 \tag{4}$$

There is the following relation at the maximum and minimum:

$$F_{CS} \to -1 \quad (W_{ij}W_{ij} \to 0), \quad F_{CS} \to 1 \quad (S_{ij}S_{ij} \to 0) \tag{5}$$

NUMERICAL CONDITIONS AND METHODS

Table 1 shows the numerical conditions of DNS for turbulent channel flows and homogeneous isotropic turbulence. Re_τ is the friction Reynolds number based on the half-width of channel height δ, the friction velocity u_τ, and the kinematic viscosity ν. The Reynolds number Re_{mc} is based on the mean centerline velocity, δ, and ν. Nx, Ny, and Nz denote the grid points in streamwise (x), normal (y), and spanwise (z) directions. In table 1, Re_λ is the Reynolds number based on the rms of velocity, the Taylor's microscale, and ν, and Re_{lE} is based on the rms of velocity, the integral length based on energy spectrum, and ν. N indicates the grid point in a direction.

For turbulent channel flows, spectral methods are used in the streamwise (x) and spanwise (z) directions, and a fourth-order central finite difference scheme is used in the normal (y) direction. Computations were carried out until the turbulent flow field attains a statistical steady state. For homogeneous isotropic turbulence, spectral methods are used in homogeneous directions. Figure 1 shows the mean streamwise velocity profiles of turbulent channel flows at various Re_τ. These statistics were confirmed to be consistent with those obtained by Moser *et al.* (1999).

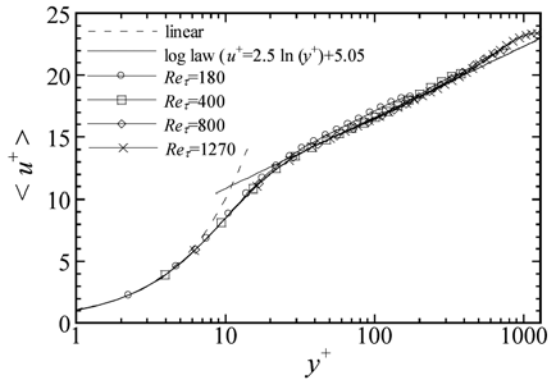

Figure 1: Mean streamwise velocity profiles of turbulent channel flows at various Re_τ.

Hereafter all results are obtained from instantaneous DNS data, and $\langle\rangle$ denotes the average in homogeneous directions.

RESULTS AND DISCUSSION

Figure 2 shows the total and local pdfs of Q for turbulent channel flows at y^+= 5, 20, 150, and the center at Re_τ = 180, 400, 800, and 1270, where the superscript $^+$ denotes the properties in the wall units. Each position of y^+=5, 20, and 150 is located at viscous sublayer, buffer layer, and log-law layer, respectively, as shown in the figure 1.

As shown in figure 2, the maximum and minimum values of Q increase as the Reynolds number increase. The large, positive value of Q indicates intermittent fine-scale coherent eddies (Tanahashi *et al.*, 2004). The pdf at $y^+ = 20$ has the widest skirts because strong fine-scale coherent eddies exist in the buffer layer. It is difficult to normalize the pdf of Q independent of the Reynolds number.

Figure 3 shows the pdf of Q^* for homogeneous isotropic turbulence at various Re_λ. where Q^* is normalized Q by the Kolmogorov length and the rms velocity. This scaling improves the dependence of the pdf of Q on Reynolds number. As a result, the fine-scale coherent eddies are scaled by the diameter and the maximum azimuthal velocity of themselves (Miyauchi & Tanahashi, 2001; Tanahashi *et al.*, 2004; Das *et al.*, 2006). However, each pdf of Q^* still deviates at different Re_λ.

The coherent structure function F_{CS}, however, reveals a different scene of the pdf. Figures 4 and 5 show the pdf of F_{CS} at y^+= 5, 20, 150, and the center of channel with the pdf for homogeneous isotropic turbulence (HIT) at various Reynolds number. The pdf of F_{CS} at different Re_τ shows quite good agreement at each location of y^+. Note that no pdf at Re_τ =180 is shown in the figure at y^+ = 150 because the profile of mean velocity deviates from the log-law. As

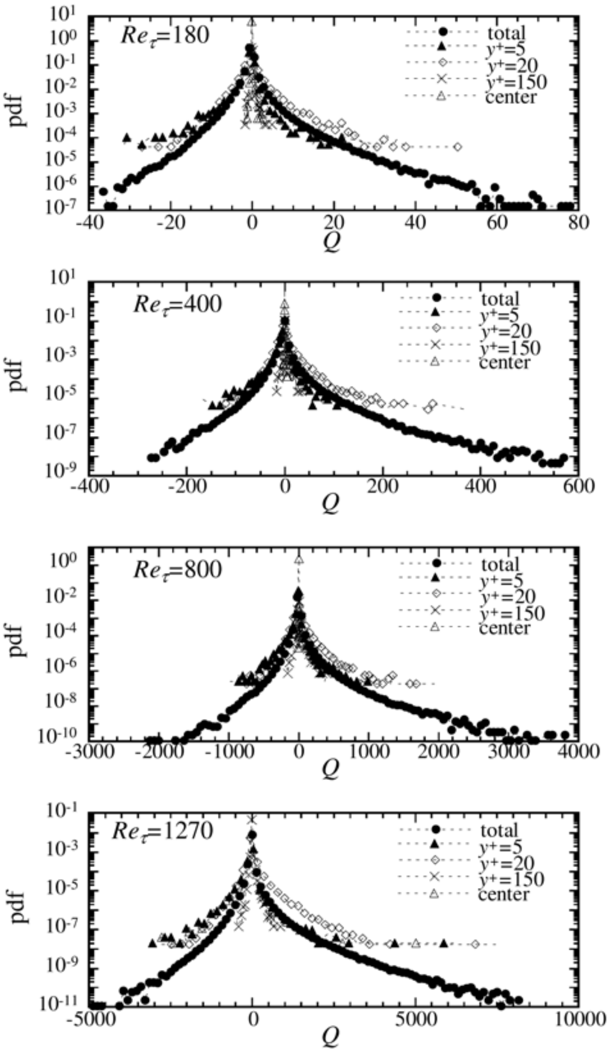

Figure 2: Total and local pdfs of Q for turbulent channel flows at y^+= 5, 20, 150, and the center at Re_τ = 180, 400, 800, and 1270; the superscript $^+$ denotes the properties in the wall units.

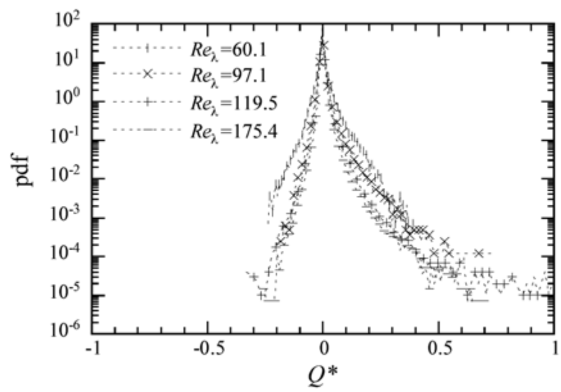

Figure 3: Pdf of Q^* for homogeneous isotropic turbulence at various Re_λ; Q^* is normalized Q by Kolmogorov length and rms velocity.

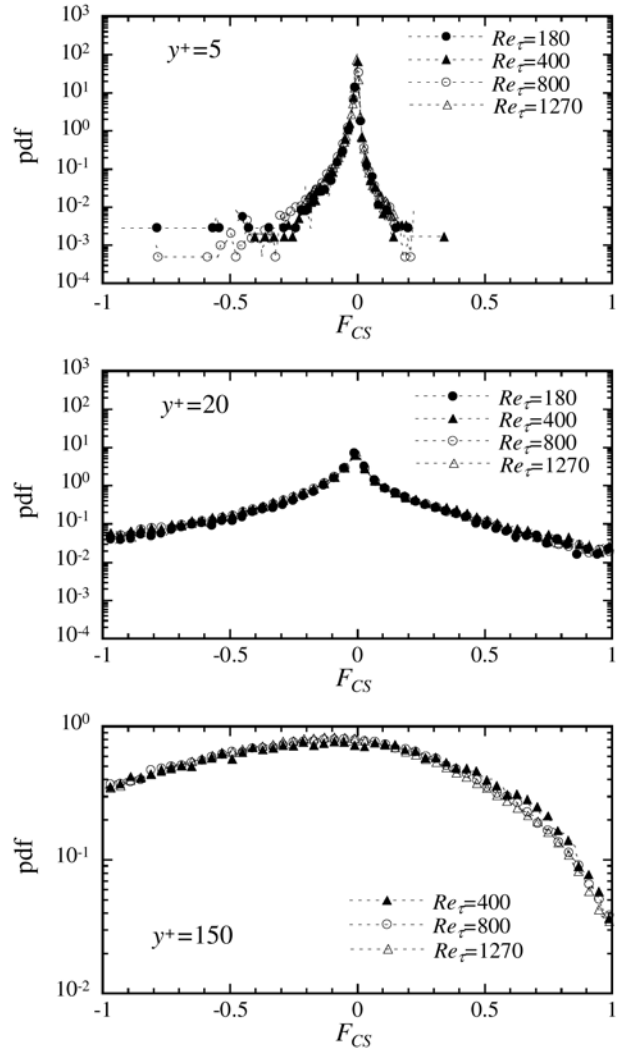

Figure 4: Pdf of F_{CS} at y^+= 5, 20, and 150 at various Reynolds number.

the location of y^+ moves to the center of the channel, the sharp profile of the pdf approaches the broad profile of the pdf of the homogeneous isotropic turbulence. This indicates that the pdf of F_{CS} depends on a distance from the wall.

For the homogeneous isotropic turbulence, the pdf of F_{CS} at different Re_λ coincides very well as shown in figure 5. Moreover, it is found that the pdf at the center of channel deviates from the pdf of homogeneous isotropic turbulence. The pdf at the center of channel shifts toward the negative F_{CS}. It is thought that the center of the channel is not the same as the homogeneous isotropic turbulence. This is because the shear stress from walls affects the coherent structures at the center of channel and the coherent structures align in the streamwise direction (Tanahashi *et al.*, 2004).

These pdfs independent of Reynolds number are expected to lead a new scaling model of the structure function.

Figure 6 shows the conditional average and the volume fraction of F_{CS} for positive or negative Q (or F_{CS}) in the y^+ direction. The profiles of the averaged F_{CS} and the volume fraction agree well regardless of the Reynolds number as a function of y^+.

In homogeneous isotropic turbulence, as increasing in Re_λ = 60.1, 97.1, 119.5, and 175.4, the volume fraction of positive Q decreases to 0.441, 0.432, 0.431, and 0.431

Contents Main

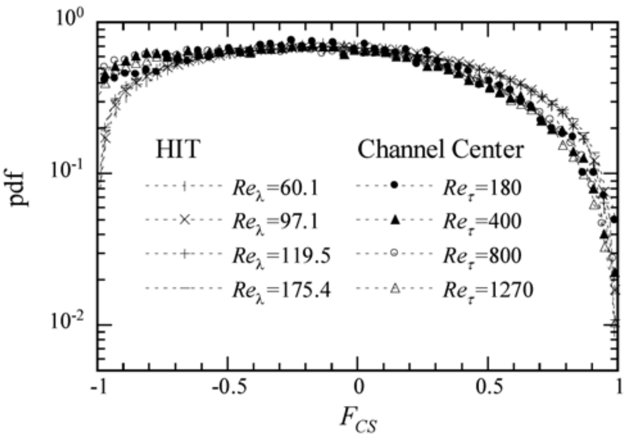

Figure 5: Pdf of F_{CS} at the center of channel with the pdf for homogeneous isotropic turbulence (HIT) at various Reynolds number.

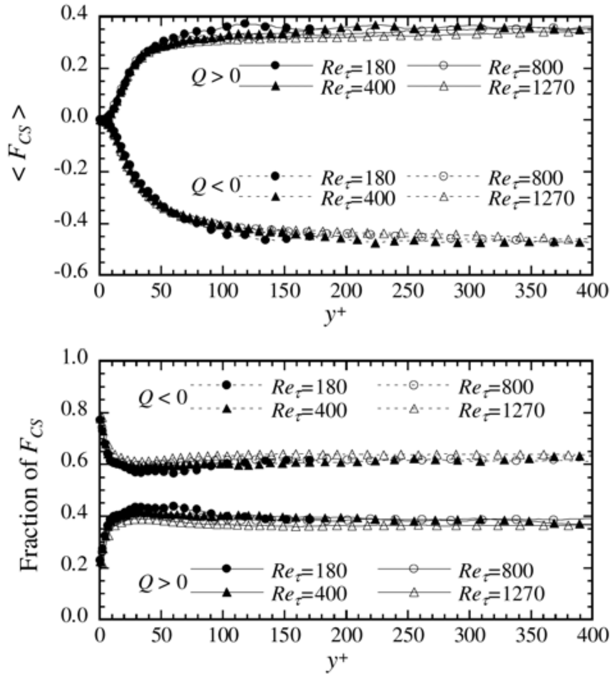

Figure 6: Conditional average, volume fraction of F_{CS} for positive or negative Q (or F_{CS}) in the y^+ direction.

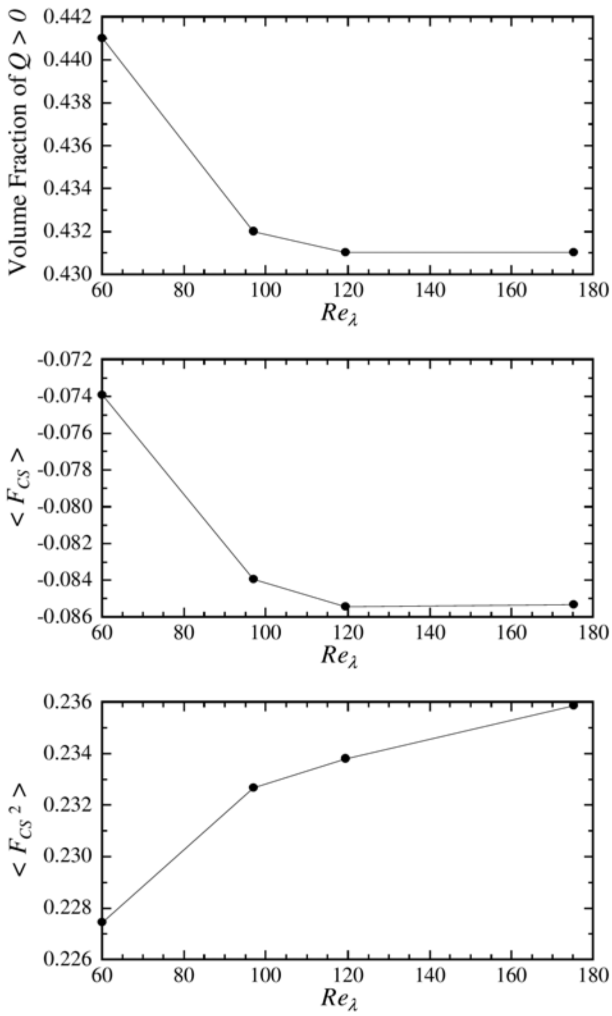

Figure 7: Volume fraction of positive Q (top), $\langle F_{CS} \rangle$ (middle), and $\left\langle F_{CS}^2 \right\rangle$ (bottom) as a function of Re_λ.

as shown in figure 7. It seems that the volume fraction of positive Q converges to a certain value of around 0.431 at high Reynolds number as well as $\langle F_{CS} \rangle$ approaches around $-$ 0.0854 as shown in figure 7. In the range of the present Re_λ, $\left\langle F_{CS}^2 \right\rangle$ does not converge to a certain value in figure 7. It is necessary to investigate it at higher Re_λ including whether $\left\langle F_{CS}^2 \right\rangle$ converges or not.

Figure 8 shows the F_{CS}^2 profiles at various Reynolds number in the y^+ direction. Under all conditions for all Q, $Q < 0$, and $Q > 0$, the near-wall distributions of F_{CS}^2 are in good agreement for high Re_τ, although the profiles at $Re_\tau = 180$ deviate owing to the effect of low Reynolds number. This would be used for a wall model in large-eddy simulation and has been already used for the subgrid scale model in large-eddy simulation (Kobayashi, 2005).

Figure 9 shows the pdf of Q (top), E (middle), and F_{CS} (bottom) at $Re_\lambda = 60.1$ in comparison with an initial velocity field. The initial velocity field is satisfied with the energy spectrum of $-5/3$ power, and random phase angles of the velocity are given, namely, the angle of the velocity vector is random. The initial velocity field gives the larger second invariants Q and magnitude of shear stress E relevant to the energy dissipation over the kinematic viscosity ε/ν. Those large values of Q and E are due to the high wave number unphysical fluctuations. After the sufficient time integration to develop turbulence, such high wave number fluctuations are dissipated, and then the velocity field at $Re_\lambda = 60.1$ is obtained. The time integration leads to the high pdf for $F_{CS} < -0.5$ at $Re_\lambda = 60.1$, whereas the pdf for $F_{CS} > 0.5$ of the initial field is close to that of $Re_\lambda = 60.1$. Note that the numerical simulations at $Re_\lambda = 60.1$ in Figs. 9 and 10 were implemented using the 4th order central finite difference method, so that the pdf deviates a little from that obtained by the spectral method in Fig. 5.

Since F_{CS} is Q divided by E, the contribution of the Q value to the pdf of F_{CS} is unclear. Let us look at the conditional pdf of F_{CS} by the Q values in Fig. 10. In the initial field, the positive and negative, large Q values do not mainly contribute to the pdf of F_{CS}, whereas at $Re_\lambda = 60.1$ the positive and negative, large Q values contribute to the pdf of the positive and negative, large F_{CS}. The small Q values mainly contribute to the pdf of the small F_{CS}. Therefore, as shown in Fig. 5, the difference of the pdf of

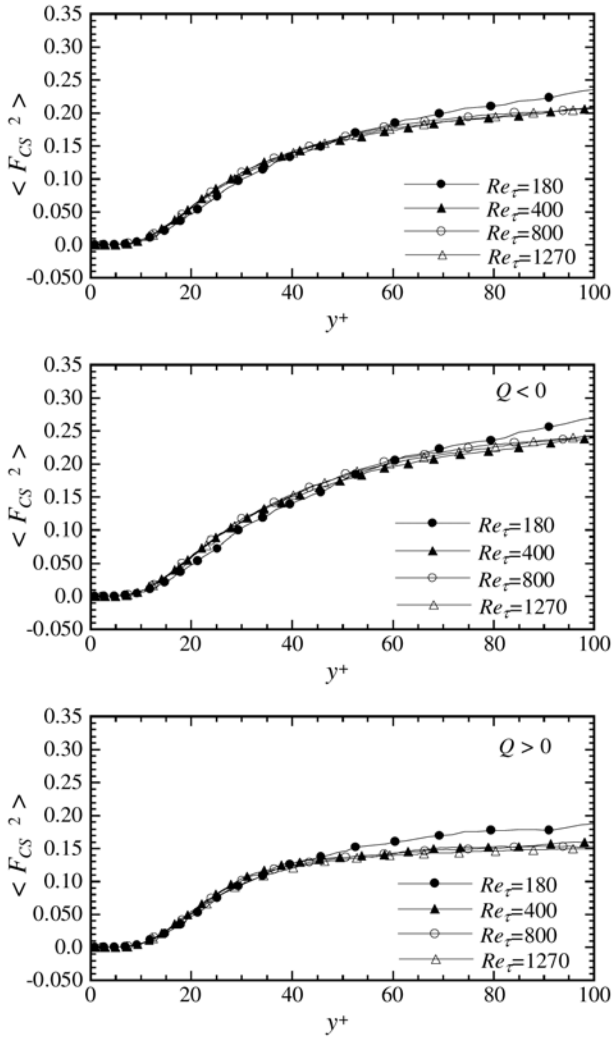

Figure 8: F_{CS}^2 profiles at various Reynolds number in the y^+ direction; (top) for all Q, (middle) for $Q < 0$, (bottom) for $Q > 0$.

F_{CS} between the HIT and channel center is due to the pdf of the positive and negative, large Q values. The higher pdf at the center of channel than that of the HIT at $F_{CS} = -1$ results from the negative, large Q values, whereas the lower pdf at the center of channel than that of the HIT around $F_{CS} = 0.8$ is due to smaller contribution of the positive, large Q values than the HIT. Another conditional pdf by the classification of vortex sheet and tube structures (Horiuti, 2001) would be more useful to look at the contribution of the coherent structure to the pdf of F_{CS}. It remains as the future study.

CONCLUDING REMARKS

The statistical properties of the coherent structure function F_{CS} was examined using DNS data at various Reynolds number for homogeneous isotropic turbulence and turbulent channel flows, and some concluding remarks are drawn below.

F_{CS} is defined as the second invariant of the velocity gradient normalized by the magnitude of the velocity gradient. Since F_{CS} has maximum value 1 and minimum value -1, the statistical properties using F_{CS} are useful to compare themselves at different Reynolds number. Despite the different Re_λ, the pdf of F_{CS} shows good agreement. For the channel flows, the pdf at different y^+ coincides very well for different Re_τ.

Moreover, the conditional average, the volume fraction of F_{CS} for positive and negative Q, and near-wall F_{CS}^2 profiles are given as a function of y^+, and yet those profiles are in good agreement at different Re_τ. In particular, it seems that the volume fraction of positive Q and the average of F_{CS} converge to a certain value at high Reynolds number for homogeneous isotropic turbulence.

It is expected that these statistical properties are used for a constrain to the existing models of the structure function, a new scaling model in statistical theory of turbulence, and a new wall model in large-eddy simulation.

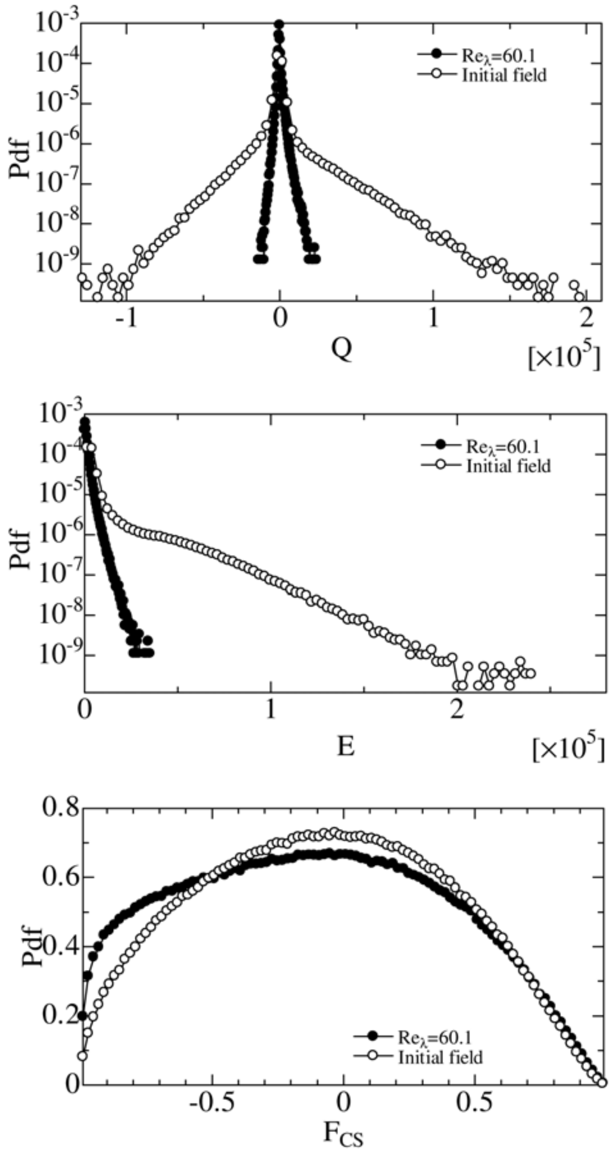

Figure 9: Pdf of Q (top), E (middle), and F_{CS} (bottom) at $Re_\lambda = 60.1$ in comparison with an initial velocity field.

ACKNOWLEDGMENTS

HK is deeply grateful to Professor F. Hamba for teaching how to produce the initial velocity field of the HIT. This work was supported by the Ministry of Education, Culture, Sports, Science and Technology of Japan, Grant-in-Aid for

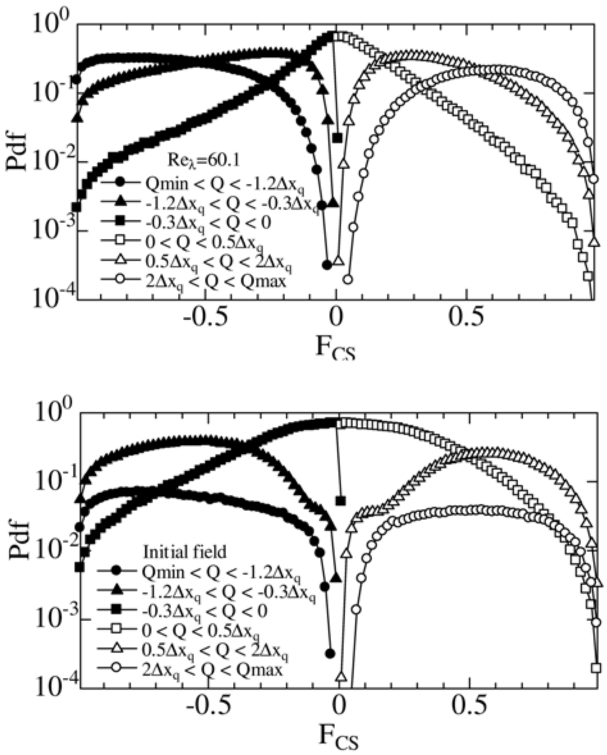

Figure 10: Conditional pdf of F_{CS} by the Q values; (top) at $Re_\lambda = 60.1$, $Qmin = -1.40 \times 10^4, Qmax = 2.42 \times 10^4, \Delta x_q = 3.82 \times 10^2$; (bottom) for an initial velocity field, $Qmin = -1.30 \times 10^5, Qmax = 2.04 \times 10^5, \Delta x_q = 3.34 \times 10^3$.

Young Scientists KAKENHI (B), Grant No. 20760119, and HK gratefully acknowledges its support.

REFERENCES

Anselmet, F., Gagne, Y., Hopfinger, E. J., & Antonia, R. A., 1984, "High-order velocity structure functions in turbulent shear flows", *J. Fluid Mech.*, Vol. 140, pp. 63–89.

Biferale, L., 1993, "Probability distribution functions in turbulent flows and shell models", *Phys. Fluids A*, Vol. 5, pp. 428–435.

Das, S. K., Tanahashi, M., Shoji, K., & Miyauchi, T., 2006, "Statistical properties of coherent fine eddies in wall-bounded turbulent flows by direct numerical simulation", *Theor. Comp. Fluid Dyn.*, Vol. 20, pp. 55–71.

Frisch, U., Sulem, P. L., & Nelkin, M., 1978, "A simple dynamical model of intermittent fully developed turbulence", *J. Fluid Mech.*, Vol. 87, pp. 719–737.

Horiuti, K., 2001, "A classification method for vortex sheet and tube structures in turbulent flows", *Phys. Fluids*, Vol. 13, pp. 3756–3774.

Hosokawa, I., Oide, S., & Yamamoto, K., 1996, "Isotropic turbulence: Important differences between true dissipation rate and its one-dimensional surrogate", *Phys. Rev. Lett.*, Vol. 77, pp. 4548–4551.

Hunt, J. C. R., Wray, A. A., & Moin, P., 1988, "Eddies, stream, and convergence zones in turbulent flows", In *Center for Turbulence Research Report*, CTR-S 88. Stanford, CA: Center for Turbulence Research.

Ishihara, T., Gotoh, T., & Kaneda, Y., 2009, "High-order velocity structure functions in turbulent shear flows", *Annu. Rev. Fluid Mech.*, Vol. 41, pp. 165–180.

Kobayashi, H., 2005, "The subgrid-scale models based on coherent structures for rotating homogeneous turbulence and turbulent channel flow", *Phys. Fluids*, Vol. 17, 045104.

Kobayashi, H., 2008, "Large eddy simulation of magnetohydrodynamic turbulent duct flows", *Phys. Fluids*, Vol. 20, 015102.

Kobayashi, H., Ham, F., & Wu, X., 2008, "Application of a local SGS model based on coherent structures to complex geometries", *Int. J. Heat Fluid Flow*, Vol. 29, pp. 640–653.

Kolmogorov, A. N., 1941, "The local structure of turbulence in incompressible viscous fluid for very large Reynolds numbers", *C. R. Dokl. Acad. Sci. USSR*, Vol. 30, pp. 301–305.

Kolmogorov, A. N., 1962, "A refinement of previous hypothesis concerning the local structure of turbulence in a viscous incompressible fluid at high Reynolds number", *J. Fluid Mech.*, Vol. 13, pp. 82–85.

Meneveau, C. & Sreenivasan, K. R., 1991, "The multifractal nature of turbulent energy dissipation", *J. Fluid Mech.*, Vol. 224, pp. 429–484.

Miyauchi, T. & Tanahashi, M., 2001, "Coherent fine scale structure in turbulence", In *IUTAM Symposium on Geometry and Statistics of Turbulence* (ed. T. Kambe et al.). pp. 67–76. Kluwer.

Moser, R. D., Kim, J., & Mansour, N. N. 1999 Direct numerical simulation of turbulent channel flow up to $Re_\tau = 590$. *Phys. Fluids* **11**, 943–945.

She, Z-S. & Leveque, E., 1996, "Universal scaling laws in fully developed turbulence", *Phys. Rev. Lett.*, Vol. 72, pp. 336–339.

Tanahashi, M., Kang, S. -J., Miyamoto, T., Shiokawa, S., & Miyauchi, T., 2004, "Scaling law of fine scale eddies in turbulent channel flows up to Re_τ=800", *Int. J. Heat Fluid Flow*, Vol. 25, pp. 331–340.

Tsinober, A., 2001, "An Informal Introduction to Turbulence", *Kluwer Academic Publishers*, The Netherlands.

Vincent, A. & Meneguzzi, M., 1991, "The spatial structure and statistical properties of homogeneous turbulence", *J. Fluid Mech.*, Vol. 225, pp. 1–25.

Wang, B.-C., Bergstorm, D. J., Yin, J., & Yee, E., 2006, "Turbulence topologies predicted using large eddy simulation", *J. Turbulence*, Vol. 7, pp. 1–28.

*