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SOME ASPECTS OF LINEAR AND NON LINEAR SCALE INTERACTIONS IN WALL TURBULENCE

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ABSTRACT

Simultaneous hotwire measurements across the vertical direction, obtained in the atmospheric surface layer in near-neutral conditions at $Re_\tau = \delta u_\tau/\nu \simeq 10^6$, are used to study interactions among the turbulent scales, from the very large scale motions (VLSM of order of $6 - 10\delta$) to the dissipative scales. In this contribution we want to focus on the linear and non linear character of such interactions. Two distinct ways are pursued, the first based on wavelet analysis, the second one based on the Mutual Information Content (MIC). The overall picture confirms that large and very large scale motions are actively participating in near wall turbulence activity. By wavelet analysis we can quantify the temporal evolution of the energetic contribution of different large and small (strictly time) scale motions, and thus study if VLSM locally influence smaller time scale processes. Such interaction can be investigated also probabilistically as the mutual influence between two different quantities, namely a large scale velocity time series and a (varying) velocity differences time series $\Delta u(t, z) = u(t + \tau) - u(t)$, representative (for small time lag τ) of the small scales of turbulence. MIC analysis allows a distinction to be made between the linear and non linear type of interaction, suggesting perhaps how much and where small scales are i) simply convected by or ii) "entangled" with large scales motions.

INTRODUCTION

In the last decade the role of very large scale motions in turbulent boundary layer has been significantly re-evaluated, not only in terms of kinetic energy and Reynolds stress, (see e.g. [1], [3], [8]) , but also as a source of strong scale interactions across the wall region (see e.g. the modulation effect by [4]). Besides turbulent boundary layer (TBL) measurements in laboratory large-scale flows (see e.g. [2], [10]), measurements in the near-neutral atmospheric surface layer (ASL) have also provided new insight onto the Reynolds number effects on high order flow statistics (see [7], [9] and reference therein). Despite limited control on the free stream velocity, surface roughness and boundary layer thickness, on one side, and stratification or buoyancy instability effects on the other side, selected measurements in the ASL in conditions of near thermal neutrality have been shown to provide a reliable picture of TBL at high Reynolds number, with limited drawback effects arising from positioning error and/or sensor finite size.

A recently discovered feature of TBL flows is the anomalous scaling of the near wall streamwise velocity fluctuation, a.k.a. near wall turbulent peak, with the Reynolds number [11], [9]. This has been interpreted as the result of the interaction between an outer scale that contributes to the definition of Reynolds number [11], and an inner scale that is known to control near wall turbulent processes, such as, e.g., wall streak separation, apparently independent of Reynolds number [5]. Experimental evidence of large (or very-large) scale oscillation in the log region modulating the amplitude of the near wall turbulent fluctuations was provided by [4]. Some more research is however needed to define unequivocally inner-outer interaction processes and, in a broader sense, any scale interaction process. The latter is indeed intrinsically complicated by the fact that scales must be also defined unambiguously.

We present here experimental observations from the SLTEST site in Utah [8] , at $Re_\tau = \delta u_\tau/\nu \simeq 10^6$, in near neutral conditions and in the transitionally rough regime ($k_s^+ = 50$). The dataset, consisting of 29 simultaneous single hotwire measurements of the streamwise velocity component u at high sampling frequency (5Khz) logarithmically spaced in the wall normal direction, is particularly suited to study scale interaction process in the temporal domain.

As we can see in the contour of the local mean velocity (averaging time of 1 s) depicted in figure 1, the dataset contains very large time scale oscillations of period of about 100s, corresponding to a time scale of roughly $10\delta/U$ at $z = 5m$, resembling, in the wall normal plane, the so called stripes observed by [4] in the plane parallel to the wall. Such oscillations, which are believed to represent the time signature of the very large scale motions, VLSM (see e.g. [3]), are observed to characterize the whole investigated region of the TBL down to the wall, and they are thus potentially capable of modulating the near wall turbulent intensity.

The main objective of this paper is thus to provide the statistical tools able to i) define different time scales, ii) relate each time scale to a variable that has its own temporal evolution - e.g. a low pass velocity signal - , iii) study if these variables, each describing the time history of a certain time scale process, are mutually influencing each other. Such mutual influence, in principle, can be simultaneous (or not), or linear (or not).

It is of particular interest to understand how much, and for which time scales, the linear and non linear characteristics of the interaction are respectively important. Indeed we can reformulate the same question in the context of turbulent structures. We can thus try to understand if small scales are simply advected by the VLSM or if small scales are also changing under the effect of VLSM (strong structural coupling occurs), eventually, if such distinction depends on the size or type of structure, or on the height from the wall.

The imprint of large scale motion on near wall turbulence has broad importance on the understanding of turbulent boundary layer flows and on the scaling of surface processes such as momentum, heat and mass fluxes, eventually controlling e.g. dust and sediment resuspension or water vapor mass balance.

RESULTS

In this section we provide two ways to define different time-scale processes based on tools including wavelets and mutual information content analysis. Furthermore we attempt to quantify the temporal interactions between a set of different time scales processes across the whole wall region. The main reason to use wavelets and MIC is to exploit the simultaneous information from the hotwire rakes limiting, as possible, a priori assumptions and embracing the complexity of the dataset. Since the outer length and velocity scales, i.e. the boundary layer height δ and the free stream velocity U_∞, were inferred from previous measurements (obtained at the same site, at sunset, in similar stability conditions), and not directly measured, we decided to present our results in dimensional form. Indeed, though the normalization of time with inner units can be provided with good accuracy, the physical meaning of inner units normalization in the context of VLSM can be misleading.

Wavelet: preliminary results

As opposed to Fourier mode decomposition, which assumes linear superimposition of infinitely long, non interacting sinusoids, a wavelet transform can provide a quantitative description of the energy distribution among different scales, as a function of time. Wavelets, as opposed to sine waves are required to be compact not only in the frequency domain but also in the time domain. Therefore, wavelet analysis is able to provide i) an unambiguous, energetically based, definition of the scale (localization in the frequency domain), and ii) the temporal evolution of the variable associated with such time scale (localization also in the time domain).

The continuous wavelet transform produces wavelet coefficients $W(a,t)$, which are a function of a (time) scale a and time t. The scale is a (temporal) compression or stretching of the Morlet wavelet function and has the same dimensions of time. Given a discrete time signal (with interval Δt), a is, strictly speaking, the number of points in which the single wavelet extending in time $a\cdot\Delta t$ is represented. a thus defines an unambiguous time scale, over which each compressed or stretched wavelet extends. $W(a_0,t_0)$ essentially measures how strongly a certain wavelet of scale a_0 *fits, or correlates with,* a finite portion of the signal of extension a_0 centered in t_0. We can now clarify that $W(a_0,t)$ is the time scale process related to the scale a_0. Eventually, the modulation of W in the a,t phase space thus indicates when, along the full time history, and how much, different time scales are (locally) energetically contributing.

$$W(a,t) = \int_{-\infty}^{\infty} f(\tau)\psi\left(\frac{\tau - t}{a}\right) d\tau \qquad (1)$$

where the *mother wavelet* ψ is the real Morlet wavelet, described by the following equation in the time domain:

$$\psi(t) = \exp(-t^2/2) * cos(5t) \qquad (2)$$

For the simple case of a continuous sinusoidal signal $\sin(\omega t)$, the dominant scale is denoted by a maximum $W(a = 1/\omega, t)$ varying in time as the sinusoid itself. For a linear superimposition of two sinusoids of different frequencies, the two scales, corresponding to the inverse of the two frequencies respectively, will be active and so on. For a turbulent signal, where different structures spanning over a large range of scales occur seldom or simultaneously, the wavelet maps are extremely complex and rich. In particular the distribution of $W(a,t)^2$ in the scale - time domain, allows for a quantitative local description of the flow in energetic terms.

It is worth noticing that the power spectrum derived by Fourier analysis can be replaced by the wavelet spectrum $S_w(a)$. Indeed we can write:

$$\Phi(k) = \int_{-\infty}^{\infty} |W(a,t)|^2 dt \qquad (3)$$

with the wavenumber $k = \frac{2\pi}{aU(z)}$ obtained as a function of the scale a and the mean velocity U at each height z. The wavelet derived 1D power spectra are shown, with different normalizations, in figure 2.

As we can see, not only the correct shape of the Fourier power spectrum is recovered, but also its smoothness is improved, in particular in the low wave number region (large scales) where the averaging procedure leading to the wavelet spectrum replaces the poor statistical representativeness of the Fourier coefficients associated with the nearby multiples of any chosen fundamental wavelength. The consistency of wavelet spectra confirms the interpretation of a as an energetically representative time scale, well localized in the frequency domain. Now that the scale is conceptually well defined, we can go back to the wavelet map and unveil its time history.

An example of a wavelet transform map is shown in figure 3 together with the velocity time series (at $z^+ \simeq 2\cdot 10^4$). The latter is superimposed with a low pass filtered velocity

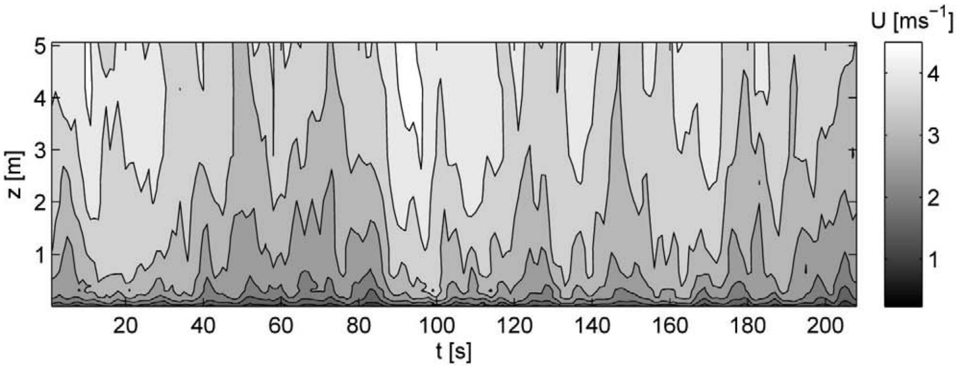

Figure 1: Contours of the local (averaging time of 1 s) mean velocity as a function of time and height from the wall.

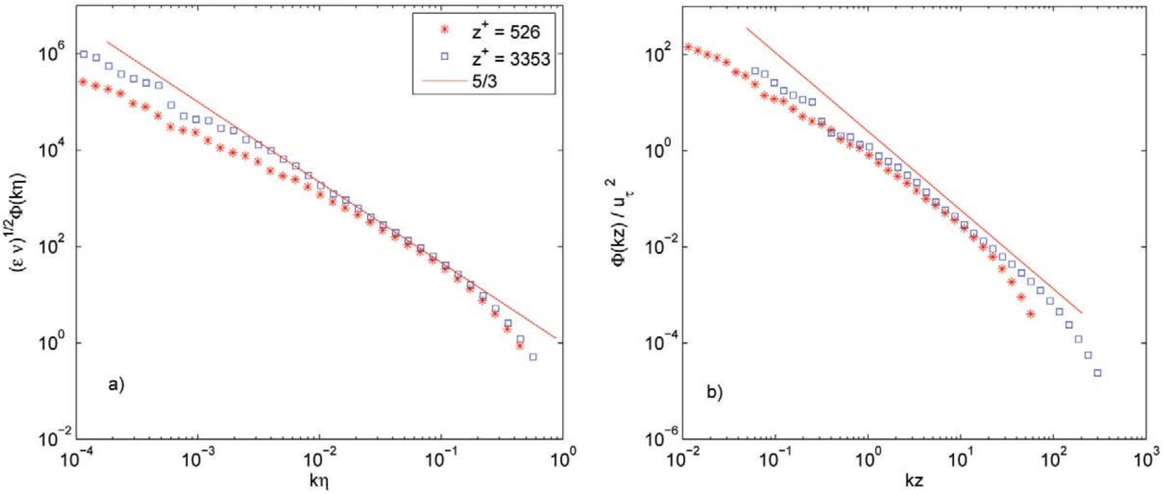

Figure 2: Dimensionless wavelet spectra a) Kolmogorov scaling, b) wall normal height scaling

($f\delta/U < 0.5$, cutoff frequency) chosen to outline the temporal signature of VLSM as analyzed in [4]. As we can note from the wavelet map (a subset $t = 60 - 110$s is shown) a cluster of relatively large values of wavelet coefficients indicates that a wide range of scales a are exceptionally energetic while the low passed velocity shows, near the wall region, a strong positive excursion (faster than the mean, between 90 and 110 s), see figure 1 and upper panel in 3.

It is noteworthy that, since the mother wavelet is symmetric in the time domain, when it is centered in the proximity of the beginning t_b or end t_e of the original $f(t)$ signal, its convolution is contaminated. Therefore in order to remove artifacts, we consider the wavelet map only in the central part defined as t_b + max(a)< t < t_e-max(a), where max(a) is the maximum scale considered in our analysis, i.e. the most stretched wavelet.

Following on the interpretation of the wavelet map in energetic terms, we can now make use of the simultaneous data acquisition across the vertical direction. For instance, we can ask ourselves how the energy associated with a certain scale a_0 is distributed in time across the wall layer. In the specific context of a turbulent flow, various type of coherent structures can possibly be related to common trends of a subset of neighboring scales a, and thus identified as coherent patterns in the wavelet map. So the distribution of any given a_0 in the z^+, t domain can shed some light on the mutual occurrence and intensity of different population of structures across the TBL.

The representation of $|W(a_0, t, z)|$ is improved by plotting at each height the envelope of $|W(a_0, t)|$, obtained by low pass filtering the full wavelet time history associated with the scale a_0. The reason is that the wavelet map retains the oscillating features of the Morlet mother wavelet, which is in fact a user defined input.

Low pass filtered $|W(a_i, t, z^+)|$ for different scales a_i are plotted in figure 4. The cutoff frequency was chosen as $f_c = 1/max(a) \simeq 0.06$ Hz, consistently for all scales. Scale interaction can be assessed by observing similar patterns over a wide range of scales, implying that the very large scale motion acting between 100 and 150 s leaves a strong and consistent signature on the whole range of smaller scales in the same periods. We can thus speculate that in such time frame, not only scale interaction is strong, but also a structural interaction occurs. We must however acknowledge that scale interactions are simply referred to as the simultaneous occurrence of a wide range of energetic wavelets when VLSM are also energetically strong. We indeed still miss if there is a cause - consequence relationship between the different scales and if the interaction is a one or two way coupling (e.g. VLSM are stimulating smaller scale turbulence, but not vice versa, or the other way around, such that VLSM are generated by scale growth processes). We could argue that quantifying the relative importance of linear and non-linear interactions can shed some light on those open question.

Mutual information content analysis

The interaction between two variables can be described following the mutual information content based on the standard Shannon entropy. An interesting application of such procedure is provided in [12], where the interaction between the large and the small scales of turbulence is investigated in the canopy sublayer. A quantity representative of the large scales was chosen as the square of the instantaneous deviation from the mean velocity $u_L(t,z) = (u(t,z) - U(z))^2$, (where $U(z)$ is the mean velocity at height z), while smaller scales were related to the square of the velocity differences time series $u_s(t,z) = (u(t+\tau) - u(t))^2$ for varying time separation τ. The entropies H of the distribution of each single variable, defined as a measure of its uncertainty, and of their mutual occurrence are given by :

$$
\begin{aligned}
H(u_L) &= -\sum p(u_L) \cdot ln(p(u_L)) && (4)\\
H(u_s) &= -\sum p(u_s) \cdot ln(p(u_s)) && (5)\\
H(u_L, u_s) &= -\sum p(u_L, u_s) \cdot ln(p(u_L, u_s)), && (6)
\end{aligned}
$$

where p indicates the (joint in case of two variables) probability density function and $\sum p(u_L) = \sum p(u_s) = \sum(p_L, u_s) = 1$. The energetic mutual information content, MIC, accounting for the non linear interaction between the two variables is quantified for different time lag τ and height z as:

$$MIC(u_L, u_s) = H(u_L) + H(u_s) - H(u_L, u_s) \quad (7)$$

The linear counterpart $L(u_L, u_s)$, assuming normal distribution of the two variables u_L ,u_s, and null covariance matrix C, is obtained as:

$$L(u_L, u_s) = \frac{1}{2}\left[\sum_{i=1}^{2} ln(c_{ii}) - \sum_{i=1}^{2} ln(\sigma_i)\right], \quad (8)$$

where c_{ii} and σ_i are the diagonal term and the eigenvalues of the covariance matrix C respectively. Eventually, the relative importance of non linearities in the interaction between u_L ,u_s can be quantified as:

$$\alpha(u_L, u_s) = \frac{MIC(u_L, u_s) - L(u_L, u_s)}{MIC(u_L, u_s)} \quad (9)$$

$\alpha(u_L, u_s)$ thus depends on the definitions of both large scale representative time histories and on the height z. While the time series associated with the small scale velocity depends on τ, the large scale velocity series is more complex to define.

Here below and in figure 5 we propose a possible alternative for the large scale velocity variable and ultimately we discuss the observed differences. Consistent with the low pass filtering proposed in the wavelet analysis, we can define different cutoff frequencies f_c and compute the large scale velocity time series as a low pass filtered velocity signal. We choose f_c=20, 50, 100, 200, 400 Hz, respectively associated with figure 5, a)-e). Then in panel 5 f), we adopt the same definition used in [12], where the large scale velocity is defined as a instantaneous deviation from the mean velocity $u_L(t,z) = (u(t,z) - U(z))^2$ and it is thus more related to energy containing eddies rather than to the very large scales of motion depicted e.g. in figure 1. In the latter case non linear interaction occurs mostly farther from the wall (large z^+) and with smaller scale turbulence, i.e. with decreasing small scale separation τ. However, when we consider a large scale definition approaching that of a very large scale of motion (smaller cutoff frequency f_c), the non linearities dominates a broader region in the τ, z^+ phase space, progressively including layers closer to the wall (small z^+) and an increasing range of small scales (small to large τ).

The overall trend can be summarized as it follows: the more the large scale relates to the meandering type of VLSM, the more it can be said that non linear interactions are important in the range of small scales. So while dissipative scales are non-linearly interacting with large scale energy-containing-eddies close to the wall, the same large scales are also non linearly interacting with the very large scale of motion throughout most of the wall layer.

CONCLUSION

In this contribution we present two different statistical procedures aimed at quantifying scale interaction processes in the atmospheric surface layer in near neutral condition and thus representative of high Reynolds number turbulent boundary layers. The first method is based on wavelet analysis, the second one based on the Mutual Information Content (MIC). Both results confirm that large and very large scale motions are actively participating in near wall turbulence activity. Non-linear types of interaction dominate with respect to linear interaction, suggesting that the amplitude modulation of near wall turbulence observed in the literature ([4], [6]) in high Reynolds number flow, i.e. larger fluctuations near the wall due to outer scale type of motions, is not simply due to an advection of small scale turbulent structures, but it is a result of a non trivial interaction process where small scales are progressively coupled with larger scale as we move farther from the wall. This is further confirmed by a wide range of wavelet scales that are all energetically active simultaneously.

More work is however needed to understand if the underlined scale interaction processes correspond to the mutual evolution (and mutual influence) of known structural types in wall turbulence (e.g. hairpins, packets, and VLSM).

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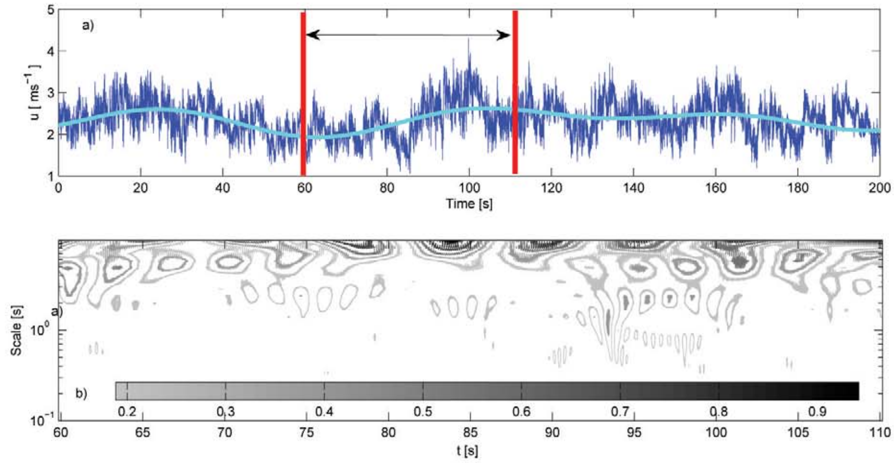

Figure 3: Streamwise velocity component at $z^+ \simeq 2 \cdot 10^4$ superimposed to the low pass Fourier filtered velocity ($f\delta/U < 0.5$) (left). Contour of the absolute value of the wavelet transform coefficients as a function of time, and of the time scale of each Morlet wavelet.

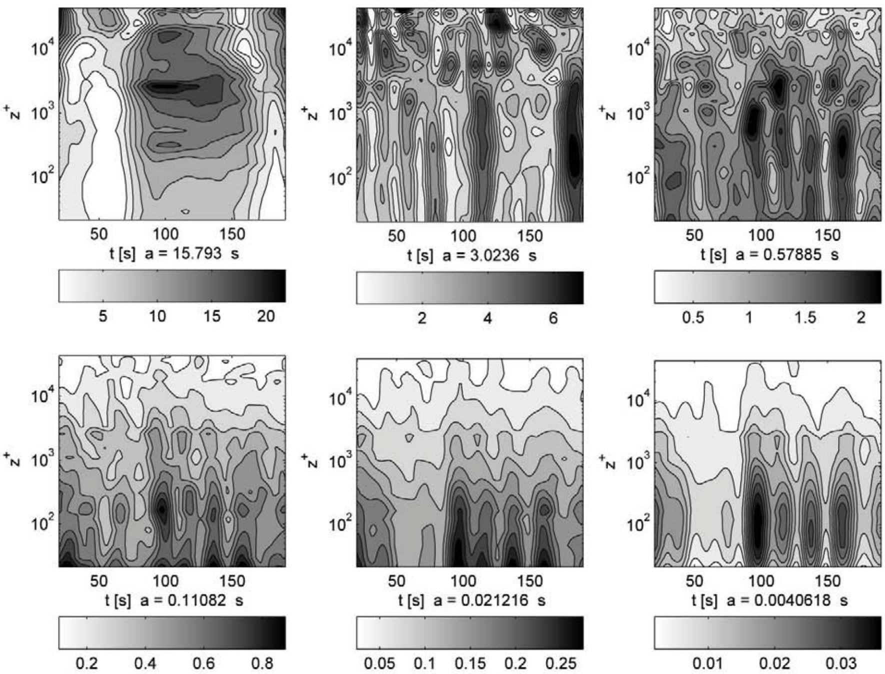

Figure 4: Contour of the absolute value of the wavelet transform coefficients $|W(a_i, t, z)|$ as a function of time, and vertical location for different chosen scales

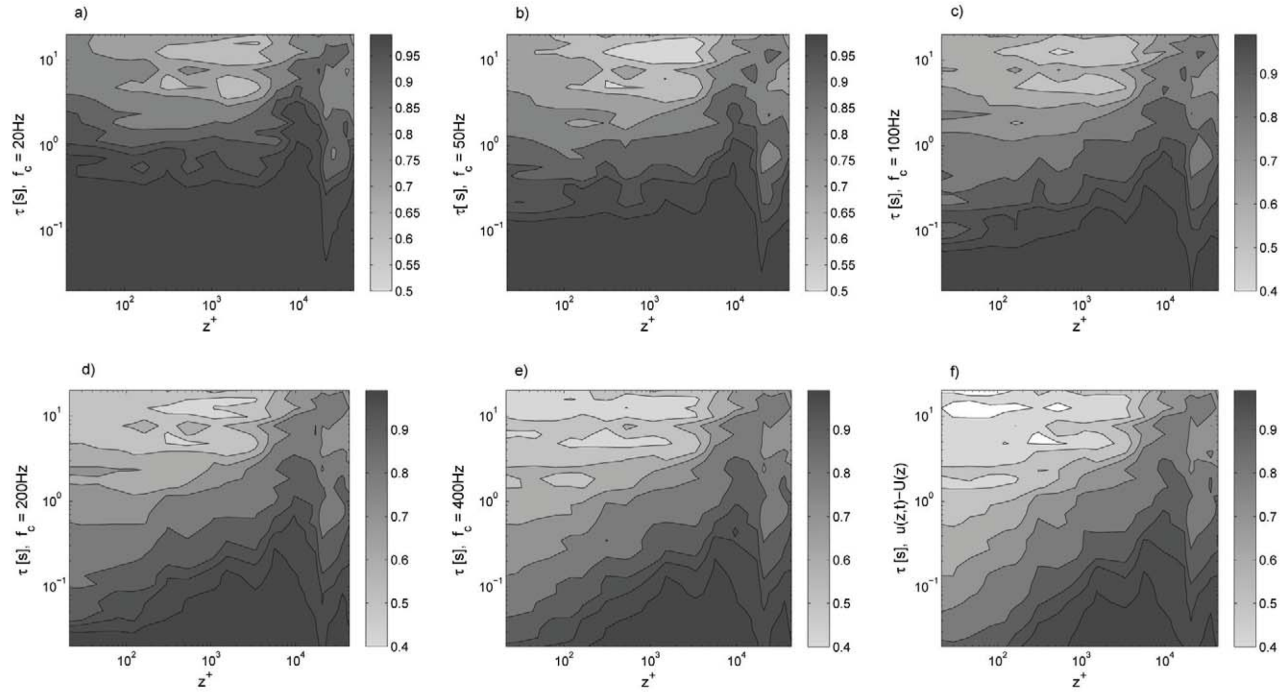

Figure 5: Contour plots of $\alpha(u_L, u_s)$ in the z^+, τ domain for different definition of large scale velocity signal. Low pass filtered velocity with increasing cutoff frequency is used in a)-e), while in f) we defined $u_L(t,z) = (u(t,z) - U(z))^2$

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