Sixth International Symposium on Turbulence and Shear Flow Phenomena
Seoul, Korea, 22-24 June 2009

MODELLING THE RAPID PRESSURE-STRAIN CORRELATION IN THE RAPID DISTORTION LIMIT

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ABSTRACT

The rapid part of pressure-strain correlation is one of the key elements in DRSM that has received significant research attention. However, the existing model proposals still exhibit some apparent deficiencies when subjected to flows with rapid distortion. For instance, the initially isotropic turbulence subjected to rapid distortion, Sjögren & Johansson (2000) showed that all the existing rapid pressure-strain models would deliver the identical path in the anisotropy-invariant map (AIM) for homogeneous plane strain and shear flows. The RDT result shows two distinct curves reflecting different flow physics (POPE, 2000). Lee *et al.* (1990) shows RDT may give an accurate description of the turbulence at realistic strain rates in a study, using RDT and DNS, of turbulence subjected to a homogeneous shear flow. We believe that a good turbulence model should match RDT for rapid deformations. Present work presents a possible candidate to overcome these deficiencies through the inclusion of the strain and rotation rate tensors S and Ω in the expansion of the fourth rank M-tensor. Numerical results show the test model proposed here is effective in reflecting the rapid distortion effects on turbulence.

INTRODUCTION

In Differential Reynolds-Stress turbulence Models (DRSM), eddy-viscosity hypothesis for the constitutive relation between the Reynolds stress and strain rate is no longer required. As is based on the closure of the Reynolds-stress transport equations originated from the Navier-Stokes equations, the DRSM is much better in recovering turbulence physics than the eddy-viscosity model (EVM). With the rapid development of computer technology and numerical solution algorithm, there is an increasing demand for the application of the DRSM to solve industrial aerodynamic problems in the last decade.

However, the pace of DRSM application to practical engineering problems is often hampered by the deficiencies existed in the basic assumptions embodied in the current modeling framework. It is known that one of the main difficulties in closing the transport equations for the Reynolds stress lies in the modeling of the unknown pressure-strain correlation. From the Poisson equation for fluctuating pressure, the pressure-strain correlation have been conveniently separated into two parts (Launder et al. 1975): the *slow term* (Φ_{ij}^{s}) representing turbulence/ turbulence interactions and the *rapid term* (Φ_{ij}^{r}) the mean-flow-gradient/turbulence interaction. The pressure-strain correlation is known to play a redistribution role among turbulence energy components. The rapid term represents the fast response of the Reynolds stresses to the variation in the mean flows in the redistribution process. It is sensitive to the nature of the mean-flow velocity gradients, e.g. in the form of shear or strain rates. In the rapid distortion limit $\left(S\,k/\varepsilon \to \infty\right)$, the rapid pressure-strain correlation becomes a dominant term over the slow part which is negligibly small. In the traditional rapid pressure-strain models, the Green's function solution to the Poisson equation for fluctuating pressure can be used to express the pressure-strain correlation in terms of two-point velocity correlations [3]. With this approach, a fourth-order ***M***-tensor (see later for Eq.(4)) arises from the rapid pressure-strain correlation. Modeling of the rapid term then turns into the determination of the ***M***-tensor form. The existing models (such as LRR (Launder et al., 1975), SSG (Speziale et al., 1991,), FLT (Fu et al., 1987)) all assume that the ***M***-tensor of the rapid term is a function of the Reynolds-stress anisotropy tensor, $b_{ij} = \overline{u_i u_j}\big/\left(2k\right) - 1/3\delta_{ij}$, alone. The logic of this approach is clear that the ***M***-tensor can be approximated with a series of b_{ij} expansion. Thus, different models reflect the truncation of this expansion at various degrees. This approach works reasonably well for flows close to local equilibrium or flows with less mean-flow distortion. For flows with large velocity gradients or mean-flow distortion, the current modeling strategy requires significant further extension to account for the rapid response in the pressure-strain correlation.

When turbulence is subjected to rapid distortion, the description of turbulence evolution simplifies significantly, leading to the so-called rapid-distortion theory (RDT) equations (Taylor & Batchelor, 1949). In this limit, the physics of rapid pressure-strain correlation can be studied in relative isolation as the complicating effects of slow pressure-strain correlation and dissipation are absent. Larsson's DNS data (Larsson, 1996) show, gradually increasing the mean strain rate, DNS should give results converging towards the RDT solution. In DNS data of Lee *et al.* (1990), it is observed that the Reynolds-stress anisotropy tensor is compatible with that in sublayer of a turbulent channel flow at a comparable shear rate made dimensionless by turbulent kinetic energy and its dissipation rate; and this shear rate produces structures in homogeneous turbulent shear flow similar to the 'streaks' that are present in the sublayer of wall-bounded turbulent shear flows. It is also shown that this shear rate is so high that the anisotropic behavior of structural quantities predicted by the rapid distortion theory is remarkably compatible with that predicted by DNS. They pointed out that RDT contains the essential mechanism responsible for the development of turbulence structures in the presence of high shear rate, typical of the near-wall region in a turbulent shear flow.

This means that the RDT is not only a limiting state, it exists in real flows. Thus, RDT results are important references to the rapid pressure-strain model proposals.

In recent years, some new pressure-strain models (Johansson & Hallbäck,1994; Sjögren & Johansson, 2000; Girimaji et al., 2003) have been presented to capture the anisotropy evolution in the rapid limit, but problems still exist. It will be shown later that it is almost an impossible task for the class of rapid pressure-strain models developed with the present modeling strategy to adequately capture the anisotropy invariants evolution in the rapid limit. The same defect in these models, for instance, leads to undamped oscillations of the anisotropy in the case of initially anisotropic turbulence subjected to rapid rotation. The correct response should be a damped oscillation of anisotropy tensor toward the initial value of the structure anisotropy tensor (Mansour et al. 1991). When the evolution of initially isotropic turbulence was subjected to rapid distortion, Sjögren & Johansson (2000) showed that all existing rapid pressure-strain models are unable to differentiate the very different flow physics related to the homogeneous shear and plane strain flows on the anisotropy-invariant map (AIM). It is highlighted in the results of RDT analysis that the plane strain and plane shear give rise to two distinct evolution paths (Pope, 2000).

In the present paper, the cause of deficiencies in the rapid pressure-strain models is investigated. It is realized that the ***M***-tensor requires a new modeling strategy to cure the model defects. The present work finally leads to the proposal of a new rapid pressure-strain model capable to reflecting turbulence behaviors at the rapid distortion limit.

ANALYSES OF DEFICIENCIES IN CLASSICAL RAPID PRESSURE-STRAIN MODELS

In the rapid distortion limit, the *rapid* pressure dominates the fluctuating pressure field, the *slow* pressure can thus be neglected. Hence, the second-moment equation can be reduced to the RDT form

$$\underbrace{\frac{D\overline{u_i u_j}}{Dt}}_{c_{ij}} = \underbrace{\frac{\partial}{\partial x_k}\left(-\overline{p^r u_j}\delta_{ik} - \overline{p^r u_i}\delta_{kj}\right)}_{d_{ij}^r} - \underbrace{\left(\overline{u_i u_k}\frac{\partial U_j}{\partial x_k} + \overline{u_j u_k}\frac{\partial U_i}{\partial x_k}\right)}_{P_{ij}} + \underbrace{\overline{p^r\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)}}_{\Phi_{ij}^r} \qquad (1)$$

Here, d_{ij}^r is the rapid diffusion term (it will disappear in homogeneous turbulence) and Φ_{ij}^r the rapid pressure-strain correlation term which is the only term needs modelling.

If we scrutinize the above transport equation of the homogeneous turbulence a little further in terms of the Reynolds-stress anisotropy tensor b_{ij}, then, in the RDT limit, where the effects of slow pressure-strain correlation and dissipation are absent, we have:

$$db_{ij}/dt = P_{ij}^b + \Phi_{ij}^{br} = T_{ij} \qquad (2)$$

Here, $b_{ij} = \overline{u_i u_j}/2k - (1/3)\delta_{ij}$. In the modelling of the rapid term Φ_{ij}^r, it is first written as

$$\Phi_{ij}^r = 2k\frac{\partial U_l}{\partial x_m}\left(M_{mlij} + M_{mlji}\right) \qquad (3)$$

Here, the fourth-rank ***M***-tensor is given by the integral of the two-point velocity correlation.

Within the current "one point" modeling strategy for the pressure-strain correlation, the ***M***-tensor is considered expandable in a polynomial of the Reynolds-stress anisotropy tensor b_{ij}, i.e.,

$$M_{mlij} = f\left(b_{ij}\right) \qquad (4)$$

Johansson & Hallbäck (1994) derived the most general expression for the rapid pressure-strain models in the context of the current approach for the Reynolds stress models.

$$\left.\begin{aligned} T_{ij} &= \left[q_1 \mathrm{I}_{bS} + q_9 \mathrm{I}_{bbS}\right]G_{ij}^{(1)} + q_2 G_{ij}^{(2)} + q_3 G_{ij}^{(3)} + q_4 G_{ij}^{(4)} \\ &\quad + \left(q_5 \mathrm{I}_{bS} + q_{10}\mathrm{I}_{bbS}\right)G_{ij}^{(5)} + q_6 G_{ij}^{(6)} + q_7 G_{ij}^{(7)} + q_8 G_{ij}^{(8)} \end{aligned}\right\} \qquad (5)$$

Here the $G_{ij}^{(\lambda)}$ represent the following tensor bases formed by the Reynolds stress anisotropy, the strain rate and the the rotation tensors, b_{ij} , $S_{ij} = 0.5\left(\partial U_i/\partial x_j + \partial U_j/\partial x_i\right)$ and $\Omega_{ij} = 0.5\left(\partial U_i/\partial x_j - \partial U_j/\partial x_i\right)$, respectively. Mathematically, they are

$$\left.\begin{aligned} &G_{ij}^{(1)} = b_{ij} && G_{ij}^{(3)} = b_{ik}S_{kj} + S_{ik}b_{kj} - \frac{2}{3}\mathrm{I}_{aS}\delta_{ij} \\ &G_{ij}^{(5)} = b_{ik}b_{kj} - \frac{1}{3}\mathrm{II}_b\delta_{ij} && G_{ij}^{(7)} = b_{ik}b_{kl}\Omega_{lj} - \Omega_{ik}b_{kl}b_{lj} \\ &G_{ij}^{(2)} = S_{ij} && G_{ij}^{(4)} = b_{ik}\Omega_{kj} - \Omega_{ik}b_{kj} \\ &G_{ij}^{(6)} = b_{ik}S_{kl}b_{lj} - \frac{1}{3}\mathrm{I}_{bbS}\delta_{ij} && G_{ij}^{(8)} = b_{ik}b_{kl}\Omega_{lm}b_{mj} - b_{im}\Omega_{mk}b_{kl}b_{lj} \end{aligned}\right\}$$

where $\mathrm{I}_{bS} = b_{kl}S_{lk}$, $\mathrm{I}_{bbS} = b_{kl}b_{lm}S_{mk}$, the q_i , may depend on the second and third invariants of b_{ij} , namely $\mathrm{II}_b = b_{kl}b_{lk}$ and $\mathrm{III}_b = b_{kl}b_{lm}b_{mk}$. Any dependence on the Reynolds number is not relevant in the rapid distortion limit.

The transport equations for the invariants II_b and III_b in RDT limit can be derived from Equation (2), through some elaborate tensor algebra, that finally lead to

$$\left.\begin{aligned} d\mathrm{II}_b/dt &= \left[2q_2 + \left(2q_1 + q_6\right)\mathrm{II}_b + 2q_5\mathrm{III}_b\right]\mathrm{I}_{bS} \\ &\quad + \left[4q_3 + 2q_9\mathrm{II}_b + 2q_{10}\mathrm{III}_b\right]\mathrm{I}_{bbS} \\ d\mathrm{III}_b/dt &= \left[q_3\mathrm{II}_b + q_5\mathrm{II}_b^2/2 + \left(q_6 + 3q_1\right)\mathrm{III}_b\right]\mathrm{I}_{bS} \\ &\quad + \left[3q_2 + \left(q_{10}\mathrm{II}_b^2 + q_6\mathrm{II}_b\right)/2 + 3q_9\mathrm{III}_b\right]\mathrm{I}_{bbS} \end{aligned}\right\} \qquad (6)$$

In pure rotation ($S = 0$), we easy get

$$d\,\mathrm{II}_b/dt = 0\text{,}\ d\mathrm{III}_b/dt = 0\,\cdot \qquad (7)$$

So, current model predictions fail to give damped oscillations in the turbulence anisotropy. The cause can be attributed directly to the absence of the rotation term Ω in Eq. (6), in general, In general, the missing of the strain term S_{ij} there may also be undesirable.

To further elucidate the implication of Equations (6), it is appropriate to consider the behaviour of these two equations in the case of homogeneous two-dimensional mean flows. For initially isotropic turbulence subjected to two-dimensional mean flows in RDT limit, Reynolds stresses will have one principal axis that is always perpendicular to the plane of the flows. We set the eigenvalues of the principal axis to be b_{33} (b_{33} is thus an invariant also, and $b_{23}=0$, $b_{31}=0$). In this case, we are easy to get $\mathrm{I}_{bbS}/\mathrm{I}_{bS}=-b_{33}$. Then, the time evolution for this component is given as

$$db_{33}/dt=\left[q_1\mathrm{I}_{bs}+q_9\mathrm{I}_{bbS}\right]a_{33}+q_3\left(-2\mathrm{I}_{bS}/3\right)+ \left[q_5\mathrm{I}_{bS}+q_{10}\mathrm{I}_{bbS}\right]\left(b_{33}^2-\mathrm{II}_b/3\right)+q_6\left(-\mathrm{I}_{bbS}/3\right) \tag{8}$$

Now we can write Equations (3), and (5) in more simple form

$$\left.\begin{aligned} d\mathrm{II}_b/\left(\mathrm{I}_{bS}dt\right)&=d\mathrm{II}_b/d\tau=\left(f_1\left(\mathrm{II}_b,\mathrm{III}_b\right)-f_2\left(\mathrm{II}_b,\mathrm{III}_b\right)b_{33}\right)\\ d\mathrm{III}_b/\mathrm{I}_{bS}dt&=d\mathrm{III}_b/d\tau=\left(g_1\left(\mathrm{II}_b,\mathrm{III}_b\right)-g_2\left(\mathrm{II}_b,\mathrm{III}_b\right)b_{33}\right)\\ db_{33}/\mathrm{I}_{bS}dt&=db_{33}/d\tau=\begin{bmatrix} h_1\left(\mathrm{II}_b,\mathrm{III}_b\right)-h_2\left(\mathrm{II}_b,\mathrm{III}_b\right)b_{33}-\\ h_3\left(\mathrm{II}_b,\mathrm{III}_b\right)b_{33}^2-h_4\left(\mathrm{II}_b,\mathrm{III}_b\right)b_{33}^3\end{bmatrix}\end{aligned}\right\} \tag{9}$$

It is important to note that the Equations (9), representing a dynamic system of order three, are self-closed in terms of function of II_b , III_b and b_{33} in two-dimensional mean flows.

Thus, all two-dimensional flows will have the same path on the AIM. The above analysis shows two important deficiencies in the current rapid pressure-strain models, no matter how high and how complex the expansion in the Reynolds-stress anisotropy tensor is. The cause of the problem may be connect with the absence of the strain rate and rotation rate tensors, S and Ω , in Eq (6) which are insensitive to flows.

A NEW PROPOSAL FOR THE RAPID PRESSURE-STRAIN MODEL

As discussed above, we think the current expansion of M-tensor is insufficient and should include the effect of strain and rotation rate tensors. Although the mean-flow quantities do not appear directly in the M-tensor expression, the two-point nature of the correlations of the fluctuating velocity gradients suggests that they are inherently related to the mean velocity gradients. While M-tensor involves information that is not contained in Reynolds-stress anisotropy tensor [4], mean-flow quantities can affect M-tensor by change the lost information. This is likely true in many cases, a nonlinear expression in the mean velocity gradient should be considered more general. In the present work, the nonlinearity in the mean velocity gradients is considered in the M-tensor. Here we assume,

$$M_{likj}=M_{likj}^{b}\left(b\right)+M_{likj}^{\Omega b}\left(b,\Omega^*\right) \tag{10}$$

with $\Omega^*=\left(0.5\cdot(U_{i,j}-U_{j,i})\right)/\sqrt{U_{l,m}U_{l,m}}$ · $M_{likj}^{b}\left(b\right)$ is expansion of tensor b alone. This make new model can be easy to combine with current turbulence theory. In this paper, we select the form of FLT model (Fu, et al., 1987) to displace $M_{likj}^{b}\left(b\right)$ and $M_{likj}^{\Omega b}\left(b,\Omega^*\right)$ is the expansion of tensor b and Ω^*. $M_{likj}^{\Omega b}\left(b,\Omega^*\right)$ can be seen as a correction term of $M_{likj}^{b}\left(b\right)$. Here, we select a linear expansion form of $M_{likj}^{\Omega b}\left(b,\Omega^*\right)$,

$$\begin{aligned} M_{\mathrm{likj}}^{\Omega b}=&\ \alpha_1\left(\Omega^*{}_{lj}\delta_{ki}+\Omega^*{}_{ij}\delta_{kl}+\Omega^*{}_{lk}\delta_{ji}+\Omega^*{}_{ik}\delta_{jl}\right)+\alpha_2\left(\Omega^*b-b\Omega^*\right)_{li}\delta_{kj}\\ &+\alpha_3\left(\Omega^*b-b\Omega^*\right)_{kj}\delta_{li}+\alpha_4\left(\left(\Omega^*b\right)_{lj}\delta_{ki}+\left(\Omega^*b\right)_{ij}\delta_{kl}\right.\\ &\left.+\left(\Omega^*b\right)_{lk}\delta_{ji}+\left(\Omega^*b\right)_{ik}\delta_{jl}\right)+\alpha_5\begin{pmatrix}\left(b\Omega^*\right)_{lj}\delta_{ki}+\left(b\Omega^*\right)_{ij}\delta_{kl}+\\ \left(b\Omega^*\right)_{lk}\delta_{ij}+\left(b\Omega^*\right)_{ik}\delta_{lj}\end{pmatrix}\\ &+\alpha_6\left(\Omega^*{}_{lj}b_{ki}+\Omega^*{}_{ij}b_{kl}+\Omega^*{}_{lk}b_{ji}+\Omega^*{}_{ik}b_{jl}\right)\end{aligned} \tag{11}$$

In determining the coefficients, two kinematical constraints (continuity, normalization) lead to

$$\left.\begin{aligned} &\alpha_1=0 \quad \alpha_2=\mathrm{e}_1+\frac{7}{3}\mathrm{e}_2 \quad \alpha_3=-\frac{7}{2}\mathrm{e}_1+\frac{1}{6}\mathrm{e}_2\\ &\alpha_4=\mathrm{e}_1 \quad \alpha_5=\mathrm{e}_2 \quad \alpha_6=-2.5\left(\mathrm{e}_1+\mathrm{e}_2\right)\end{aligned}\right\} \tag{12}$$

As can be seen from the above, we reduce 6 free coefficients to 2 free coefficients. Considering 2 free coefficients in FLT model, the new model has 4 free coefficients. In the end, the 4 free coefficients are set as,

$$\left.\begin{aligned} &t=\left(0.5+0.4\mathrm{II}_W\right)F \quad r=-2F \quad \mathrm{e}_1=-0.2F\\ &\mathrm{e}_2=0 \quad \mathrm{II}_W=\Omega^*_{mn}\Omega^*_{nm} \quad F=1+27\mathrm{III}_b-9\mathrm{II}_b\end{aligned}\right\} \tag{13}$$

Thus, the model is closed.

COMPARISONS WITH RDT RESULTS

Because of the deficiencies mentioned above, the current rapid pressure-strain models work very poorly in RDT limit. In the present model validation, a number of homogeneous flows have been considered including plane strain, plane shear, turbulence subjected to pure rotation, axisymmetric contraction and axisymmetric expansion flows. In the following figures, "present model" refers to the rapid pressure-strain correlation model developed in the present work, "RDT" is for the data of RDT, "GL" the Gibson-Launder model and "SSG" the Sarkar, Speziale & Gatski model.

Figure 1 illustrates the performances of the GL, SSG and the present model against RDT data in a plane strain case. The existing models can only give reasonable results at very small time, at large evolution time the results seem to diverge with RDT data in the streamwise energy component. The reason is clear that there is no dissipative mechanism in the Reynolds stress governing equations due to the assumption of rapid distortion, then, all energy components will grow exponentially after a certain time.

The homogeneous plane shear case is illustrated in Figure 2. To save the space, only two components variations are given. The new model seems to match the results well. But the very important behaviour is from the phase plot of the Anisotropy Invariant Map (AIM) which is showing in Figure 3. There, it is clearly seen that the present model exhibits two distinct curves, one for the plane strain and the other for the plane shear flows. They are also close to the RDT data. The results obtained with the existing pressure-strain models, e.g., from GL and SSG model, give only one curve on the AIM. The reason had been discussed before that the existing modelling strategy fails to capture the "large scale" effect.

Figure 3 shows the development of the Reynolds stress anisotropy tensor subjected to a frame rotation. As indicated in Eq.(7) the absence of shear leads to no growth or decay in the second or third invariants of the Reynolds stress anisotropy tensor. This is equivalent to see the model results of Figure3(b) where d b_{33} /dt =0 for the existing models, GL and SSG. The behaviour of the other Reynolds stress anisotropy tensor components are basically of damped oscillation in time evolution as shown in Figure 3(a). The present model is able to reflect the decay of the b_{33} component, damping also occurred in b_{11}, but as can be seen that the modelled frequency is somehow not matching. The reason may be that the present model included only the linear expansion in the modelling of the M-tensor, high frequency damping may be related to higher order expansion in vorticity.

Figures 5 and 6 present the results for axisymmetric contraction and axusymmetric expansion. In the case of contraction, since the flow is basically accelerating, the streamwise turbulent kinetic energy component decreases in the time evolution. RDT results suggest that it should remain so. The conventional model results however lead to exponential growth after the initial decay, in contradictory to the RDT. The continued decay is obviously a ‘large scale’ effect, not a consequence of energy dissipation. The new model is able to reflect this subtle behaviour.

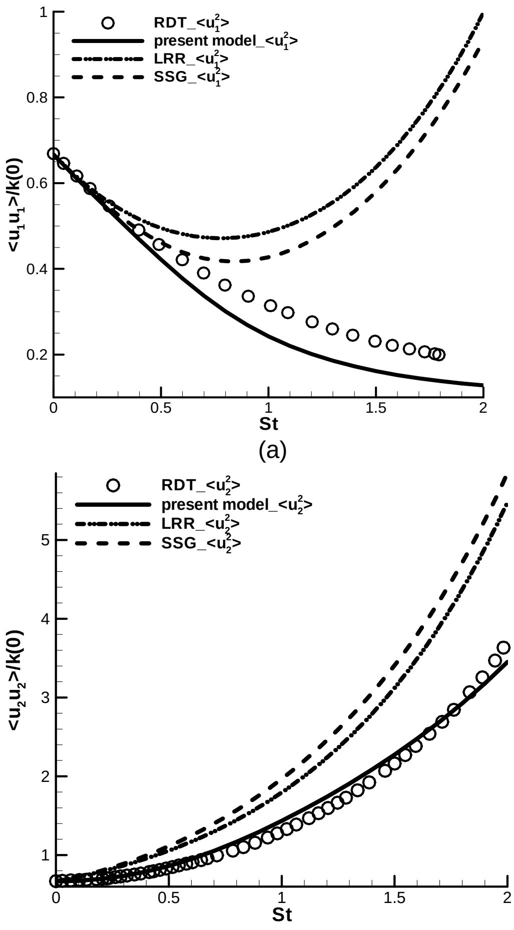


Figure 1. Comparison of Reynolds stress development in plane strain distortion from three turbulence models with RDT data. (a) the streamwise component, (b) the normal component.

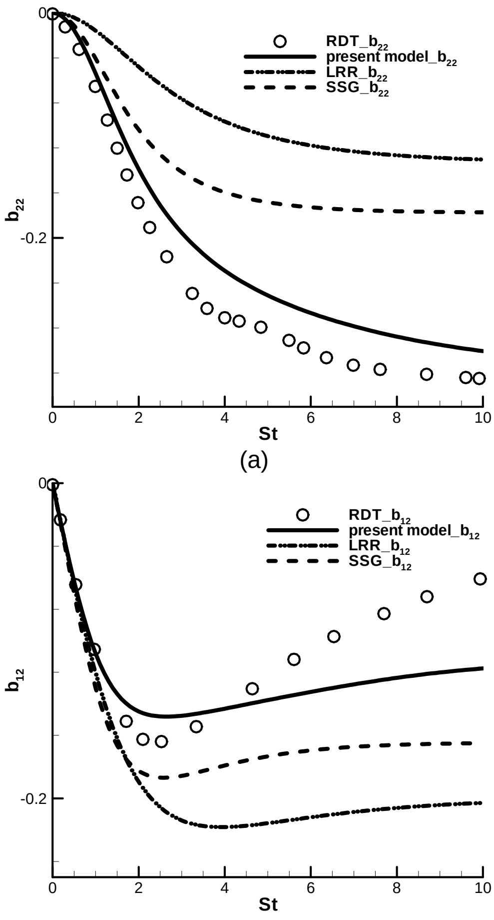


Figure 2. Reynolds-stress anisotropies for homogeneous shear flow. (a) the cross-stream compoenent, (b) the shear stress component.

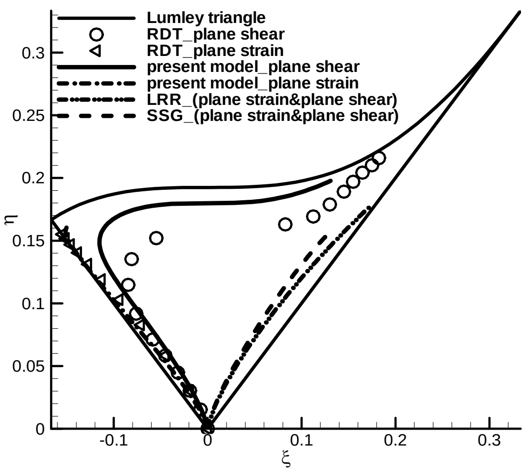
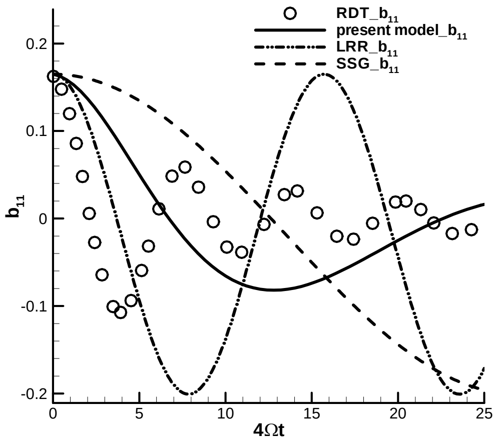
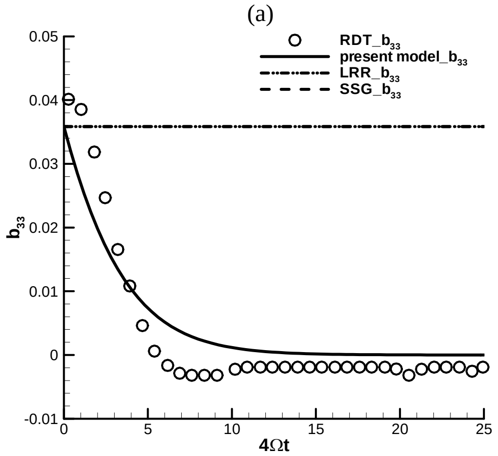


Figure 3. AIM paths in plane strain and homogeneous shear



(a)



(b)

Figure 4. Reynolds-stress anisotropies development subiect to rotation

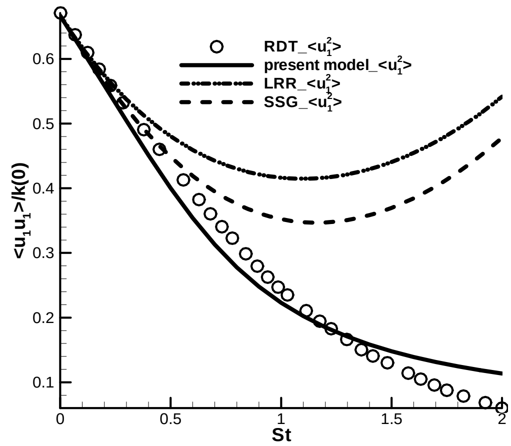
In the case of axisymmetric expansion flow, turbulence is experiencing a decelerating mean flow field, this would cause the turbulence energy increase from the beginning of the expansion. This is also shown here in Figure 6.

CONCLUSION

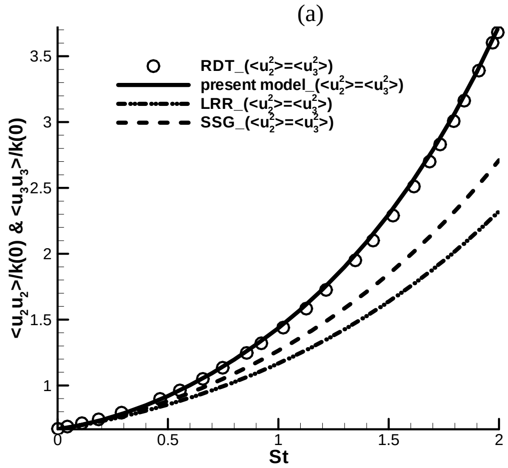
It is shown that the anisotropy evolution can be analyzed through an approach where a set of invariants are computed instead of the individual anisotropy components. The analysis is given for two dimensional mean flows and pure rotation flow. It is shown that all classical pressure-strain models give the same trace in the anisotropy-invariant map for plane strain and homogeneous shear flow, and undamped oscillations of the anisotropy measures for a pure rapid rotation in rapid limit.

A new approach to improving the prediction of the anisotropy evolution in rapid limit, where the $\boldsymbol{M}$ -tensor of rapid pressure-strain correlation is expandable in the Reynolds-stress anisotropy tensor (b_{ij}) and mean rotation rate tensor (Ω^*_{ij}). This extension allows two different traces in the anisotropy-invariant map for plane strain and homogeneous shear flow, and a damped oscillatory solution for the anisotropy components in the situation of pure rotation, which are not even qualitative captured by classical rapid pressure-strain models.

We believe that a good turbulence model should match RDT for rapid deformations and classical modeling for weak deformations. Present work presents a possible way to extend classical model to rapid deformations field.

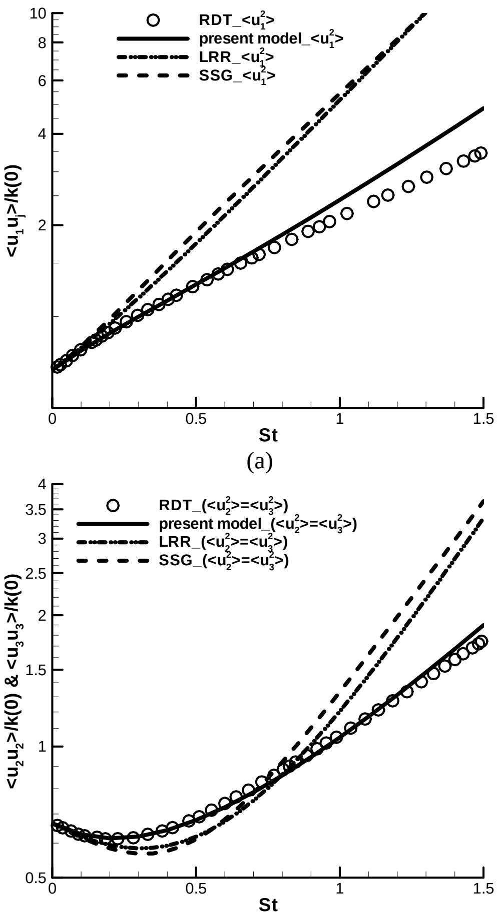


(a)



(b)

Figure 5. Reynolds-stress components for axisymmetric contraction



(a)

(b)

Figure 6. Reynolds-stress components for axisymmetric expansion

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