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FRACTAL-GENERATED TURBULENT SCALING LAWS FROM A NEW SCALING GROUP OF THE MULTI-POINT CORRELATION EQUATION

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ABSTRACT

Investigating the multi-point correlation (MPC) equations for the velocity and pressure fluctuations in the limit of homogeneous turbulence a new scaling symmetry has been discovered. Interesting enought this property is not shared with the Euler or Navier-Stokes equations from which the MPC equations have orginally emerged. This was first observed for parallel wall-bounded shear flows in Khujadze and Oberlack (2004) though there this property only holds true for the two-point equation. Hence, in a strict sense there it is broken for higher order correlation equations. Presently using this extended set of symmetry groups a much wider class of invariant solutions or turbulent scaling laws is derived for the decay of homogeneous and homogeneous-isotropic turbulence which is in stark contrast to the classical power law decay arising from Birkhoff's or Loitsiansky's integrals. Beside two classical solutions two new scaling laws have been derived. In particular, we show that the experimentally observed specific scaling properties of fractal-generated turbulence (see Hurst and Vassilicos, 2007 and Seoud and Vassilicos, 2007) fall into this new class of solutions. Due to this specific grid a breaking of the classical scaling symmetries due to a wide range of scales acting on the flow is accomplished. This in particular leads to a constant integral and Taylor length scale downstream of the fractal grid and the exponential decay of the turbulent kinetic energy along the same axis. These particular properties can only be conceived from MPC equations using the new scaling symmetry. The latter new scaling law may have been the first clear indication towards the existence of the extended statistical scaling group.

MULTI-POINT EQUATION IN THE LIMIT OF HOMOGENEOUS TURBULENCE

We investigate the symmetry and invariance structure of the infinite set of multi-point correlation (MPC) equations for the velocity and pressure fluctuations $\mathbf{u}(\mathbf{x},t)$ and $p(\mathbf{x},t)$ respectively in the limit of homogeneous turbulence (for the full equation of inhomogenous turbulence see Oberlack, 2000)

$$
\begin{aligned}
&\frac{\partial R_{i_{\{n+1\}}}}{\partial t} + \sum_{l=1}^{n}\left(-\left.\frac{\partial P_{i_{\{n\}}[0]}}{\partial r_{m_{(l)}}}\right|_{[m_{(l)}\mapsto i]} + \frac{\partial P_{i_{\{n\}}[l]}}{\partial r_{i_{(l)}}}\right) \\
&-\nu\sum_{l=1}^{n}\sum_{m=1}^{n}\left(\frac{\partial^2 R_{i_{\{n+1\}}}}{\partial r_{k_{(m)}}\partial r_{k_{(l)}}} + \frac{\partial^2 R_{i_{\{n+1\}}}}{\partial r_{k_{(l)}}\partial r_{k_{(l)}}}\right) \\
&+\sum_{l=1}^{n}\left(-\frac{\partial R_{i_{\{n+2\}}[i_{(n+1)}\mapsto k_{(l)}]}[\mathbf{x}_{(n+1)}\mapsto\mathbf{x}]}{\partial r_{k_{(l)}}}\right. \\
&\left.+\frac{\partial R_{i_{\{n+2\}}[i_{(n+1)}\mapsto k_{(l)}]}[\mathbf{x}_{(n+1)}\mapsto\mathbf{r}_{(l)}]}{\partial r_{k_{(l)}}}\right) = 0, \qquad (1)
\end{aligned}
$$

where n varies from 1 to ∞. The system (1) is extended by its corresponding continuity equations of the form

$$
\begin{aligned}
&\sum_{j=1}^{n}\frac{\partial R_{i_{\{n+1\}}[i_{(0)}\mapsto k_{(j)}]}}{\partial r_{k_{(j)}}} = 0 \ , \\
&\frac{\partial R_{i_{\{n+1\}}[i_{(l)}\mapsto k_{(l)}]}}{\partial r_{k_{(l)}}} = 0 \quad \text{for} \quad l = 1,\ldots,n \ .
\end{aligned} \qquad (2)
$$

In the latter equations the MPC tensor is defined as

$$
R_{i_{\{n+1\}}} = R_{i_{(0)}i_{(1)}\ldots i_{(n)}} = \overline{u_{i_{(0)}}(\mathbf{x}_{(0)})\cdot\ldots\cdot u_{i_{(n)}}(\mathbf{x}_{(n)})} \ , \qquad (3)
$$

and the four variations of it appearing in (1) are given by

$$
\begin{aligned}
&R_{i_{\{n+1\}}[i_{(l)}\mapsto k_{(l)}]} = \\
&\quad\overline{u_{i_{(0)}}(\mathbf{x}_{(0)})\cdot\ldots\cdot u_{i_{(l-1)}}(\mathbf{x}_{(l-1)})u_{k_{(l)}}(\mathbf{x}_{(l)})\cdot} \\
&\quad\overline{\cdot u_{i_{(l+1)}}(\mathbf{x}_{(l+1)})\cdot\ldots\cdot u_{i_{(n)}}(\mathbf{x}_{(n)})}, \qquad (4)
\end{aligned}
$$

$$
\begin{aligned}
&R_{i_{\{n+2\}}[i_{(n+1)}\mapsto k_{(l)}]}[\mathbf{x}_{(n+1)}\mapsto\mathbf{x}_{(l)}] = \\
&\quad\overline{u_{i_{(0)}}(\mathbf{x}_{(0)})\cdot\ldots\cdot u_{i_{(n)}}(\mathbf{x}_{(n)})u_{k_{(l)}}(\mathbf{x}_{(l)})}, \qquad (5)
\end{aligned}
$$

$$
\begin{aligned}
&P_{i_{\{n\}}[l]} = \\
&\quad\overline{u_{i_{(0)}}(\mathbf{x}_{(0)})\cdot\ldots\cdot u_{i_{(l-1)}}(\mathbf{x}_{(l-1)})p(\mathbf{x}_{(l)})\cdot} \\
&\quad\overline{\cdot u_{i_{(l+1)}}(\mathbf{x}_{(l+1)})\cdot\ldots\cdot u_{i_{(n)}}(\mathbf{x}_{(n)})}, \qquad (6)
\end{aligned}
$$

t is time and the correlation distance is defined according to

$$
\mathbf{r}_{(l)} = \mathbf{x}_{(l)} - \mathbf{x}_{(0)} \quad \text{with} \quad l = 1,\ldots,n \ . \qquad (7)
$$

In the definitions (4) and (6) the overbar is meant to be continued over the entire right hand side.

Similar to Navier-Stokes or Euler equation a Poisson type of equation for the pressure-velocity correlations $P_{i_{\{n\}}}$ may be derived applying a divergence though in principle not need for the further considerations.

SYMMETRIES OF THE MULTI-POINT EQUATION

In the limit of $|\mathbf{r}| \gg \eta_K$, i.e. for length scales beyond the viscosity dominated Kolmogorov scale (Oberlack, 2000), we find a new scaling symmetry $\mathbf{G_{s3}}$ of the system (1). The system also admits the classical scaling groups $\mathbf{G_{s1}}$ and $\mathbf{G_{s2}}$ representing the independent scaling of space and time

$$\mathbf{G_{s1}}: \ \tilde{t} = t, \ \tilde{r}_{i_{(l)}} = r_{i_{(l)}} e^{a_1}, \quad \tilde{R}_{ij} = R_{ij} e^{2a_1}, \ \tilde{R}_{ijk} = R_{ijk} e^{3a_1}, \cdots, \tag{8}$$

$$\mathbf{G_{s2}}: \ \tilde{t} = e^{a_2} t, \ \tilde{r}_{i_{(l)}} = r_{i_{(l)}}, \quad \tilde{R}_{ij} = R_{ij} e^{-2a_2}, \ \tilde{R}_{ijk} = R_{ijk} e^{-3a_2}, \cdots, \tag{9}$$

$$\mathbf{G_{s3}}: \ \tilde{t} = t, \ \tilde{r}_{i_{(l)}} = r_{i_{(l)}}, \quad \tilde{R}_{ij} = R_{ij} e^{a_3}, \ \tilde{R}_{ijk} = R_{ijk} e^{a_3}, \cdots. \tag{10}$$

The latter scaling symmetries are Lie groups which define transformations that leave the differential equation under analysis invariant if written in the new variables and independent of the group parameters.

It is important to note that $\mathbf{G_{s3}}$ is clearly distinct i.e. linearly independent from the classical scaling groups in fluid mechanics. Interesting enough this property is not shared with the Euler or Navier-Stokes equations from which the MPC equations have originally emerged. Hence it is a purely statistical property of the equations (1) and subsequently referred to as statistical scaling group (SSG).

This was first observed for parallel wall-bounded shear flows (see Khujadze and Oberlack, 2004) though there this property only holds true for the two-point equation. Hence, in a strict sense there it is broken for higher order correlation equations. In fact, the new scaling group $\mathbf{G_{s3}}$ is due to the linearity of (1) which arise out of the assumption of homogeneity. At the same time linearity of (1) implies that there is the general superposition group admitted by (1) a property shared by all linear differential equations.

Beside the above symmetries the system (1) admits the classical symmetry translation in time

$$\mathbf{G_t}: \ \tilde{t} = t + a_4, \ \tilde{r}_{i_{(l)}} = r_{i_{(l)}}, \ \tilde{R}_{ij} = R_{ij}, \ \tilde{R}_{ijk} = R_{ijk}, \cdots. \tag{11}$$

and a translation in correlation space which is also a group not admitted by Euler or Navier-Stokes equations (and not to be mistaken for the classical translation in space)

$$\mathbf{G_t}: \ \tilde{t} = t, \ \tilde{r}_{i_{(l)}} = r_{i_{(l)}} + a_{i_{(l)}}, \ \tilde{R}_{ij} = R_{ij}, \ \tilde{R}_{ijk} = R_{ijk}, \cdots. \tag{12}$$

Note that the latter is broken due to the Schwarz inequality in correlation space, e.g. for the two-point tensor for homogeneous turbulence $R_{\alpha\beta}(\mathbf{r})^2 \leq R_{\alpha\alpha}(0) R_{\beta\beta}(0)$.

INVARIANT SOLUTIONS AND TURBULENT DECAY SCALING LAWS

Classical theories on decaying turbulence such as Birkhoff's and Loitsyansky's integral entirely rely on the groups $\mathbf{G_{s1}}$ and $\mathbf{G_{s2}}$. There these two groups give rise to a one-parameter family of similarity solution where e.g. the turbulent kinetic energy decays and the integral length scale increases according to a power law. The exponent is usually settled by one of the above proposed conserved integrals.

Presently using the above extended set of symmetry groups (8)-(11) a much wider class of invariant solutions or turbulent scaling laws is derived for homogeneous turbulence. For this we need to define an invariant solution, usually called similarity solution, employing the following three concepts.

(i) Any Lie symmetry group, and the groups (8)-(11) with the group parameters a_1-a_4 are among those, have an equivalent infinitesimal representation defined by

$$\tilde{\mathbf{x}} = \mathbf{x} + \boldsymbol{\xi}(\mathbf{x},\mathbf{y})\varepsilon + O(\varepsilon^2) \ \text{ and } \ \tilde{\mathbf{y}} = \mathbf{y} + \boldsymbol{\eta}(\mathbf{x},\mathbf{y})\varepsilon + O(\varepsilon^2) \ . \tag{13}$$

with $\mathbf{x}$ and $\mathbf{y}$ the vector of independent and dependent variables respectively and $\tilde{\mathbf{x}}$ and $\tilde{\mathbf{y}}$ refer to the corresponding variables in the transformed space, ε is the group parameter and the infinitesimals of the Lie symmetry are defined as $\boldsymbol{\xi}(x,y) = \left.\frac{\partial \tilde{\mathbf{x}}}{\partial \varepsilon}\right|_{\varepsilon=0}$ and $\boldsymbol{\eta}(x,y) = \left.\frac{\partial \tilde{\mathbf{y}}}{\partial \varepsilon}\right|_{\varepsilon=0}$. Lie's first theorem states that the infinitesimals are sufficient for the recovery of the full symmetry transformation (see e.g. Bluman and Kumai, 1989).

(ii) The condition of invariance, i.e. implementing any symmetry transformation into its associated equation leaves the equation unchanged written in the new variables, has an infinitesimal correspondent. This is defined by

$$\left[X \mathcal{F}(\mathbf{x}, \mathbf{y}, \mathbf{y}^{(1)}, \ldots, \mathbf{y}^{(m)}) \right]\Big|_{\mathcal{F}=0} = 0 \tag{14}$$

with $\mathcal{F} = 0$ the set of equations under investigation, here (1), and X is given by

$$X = \xi_i \frac{\partial}{\partial x_i} + \eta_j \frac{\partial}{\partial y_j} \ . \tag{15}$$

(iii) Suppose the Lie symmetries of an equation are given, as is the case here, the invariant solution $\boldsymbol{\theta}(x)$ is defined according to the condition

$$X\left(\mathbf{y} - \boldsymbol{\theta}(\mathbf{x})\right) = 0 \ \text{ on } \ \mathbf{y} = \boldsymbol{\theta}(\mathbf{x}) \ . \tag{16}$$

The latter condition constitutes a hyperbolic partial differential equation which may be solved using method of characteristic. The corresponding ordinary differential equation

$$\frac{\mathrm{d}x_1}{\xi_1} = \frac{\mathrm{d}x_2}{\xi_2} = \cdots = \frac{\mathrm{d}y_1}{\eta_1} = \frac{\mathrm{d}y_2}{\eta_2} = \cdots \tag{17}$$

is referred to as invariant surface condition of Lie group theory which leads to self-similar or, more general, invariant solutions.

For the present case the infinite set of equations (1), which in the turbulence community called scaling laws, we obtain the invariant surface condition

$$\frac{\mathrm{d}t}{a_2 t + a_4} = \frac{\mathrm{d}r_{(i)}}{a_1 r_{(i)}} = \frac{\mathrm{d}R_{(ij)}}{[2(a_1 - a_2) + a_3] R_{(ij)}} = \cdots \tag{18}$$

with the group parameters a_1-a_4 descending from the groups (8)-(11) and here written in infinitesimal form and the indices in brackets denote no summation but instead each component is to be taken separately.

It is important to note that any solution for an arbitrary set of group parameters of the latter system allows for an invariant solution of (1). Three different cases may be distinguished.

Firstly we consider the case without the new SSG (10) i.e. $a_3 = 0$. Further assuming $a_1 \neq a_2$ and $a_4 \neq 0$ we find the following invariants of the system (18)

$$\hat{\mathbf{r}}_{(1)} = \frac{\mathbf{r}_{(1)}}{(t+t_0)^n} \ , \quad R_{ij}(\mathbf{r}_{(1)}, t) = (t+t_0)^{-m} \hat{\mathcal{R}}_{ij}\left(\hat{\mathbf{r}}_{(1)}\right) \ , \quad \ldots \tag{19}$$

with $n = a_1/a_2$, $t_0 = a_4/a_2$ and $m = 2(1 - a_1/a_2)$. The variables $\hat{\mathbf{r}}_{(1)}$ and $\hat{\mathcal{R}}_{ij}$ are the constants of integration of the invariant surface condition (18) or in other words the invariants of the system. They are to be taken as new independent and dependent variables of the system (1) leading to a similarity reduction depicting the classical power law behavior. Therein $m = 6/5, n = 2/5$ and $m = 10/7, n = 2/7$ respectively correspond to Birkhoff's and Loitsiansky's integrals.

At this point a know sub-class of the invariant solution (19) may immediately be derived by considering that the scaling of space is broken $a_1 = 0$ or in other words a symmetry breaking constant length is acting on the decay process. Practically this may conceived e.g. in a DNS with periodic boundary conditions and letting the decay process proceed in a small box of fixed size. Therewith the usual growth of length scales, in particular the integral length scale, is inhibited. Hence preventing of the scalability of length immediately leads to $n = 0$ and $m = 2$.

For the present purpose of primarily understanding the scaling behaviour of fractal generated turbulence we need to consider both the breaking of the two classical scaling groups due to external symmetry breaking quantities to be detailed from a physical point of view below. Hence, we set $a_1 = a_2 = 0$. Further, for the present case a non-zero a_3 related to the new scaling group is need in order to allow for the construction of an invariant solution at all. Hence, employing the latter two informations into equation (18) we observe two important conclusions.

First, due to $a_2 = 0$ and combining the first and the last term in (18) we have an exponential scaling of the two- and MPC with time. Second, because $a_1 = 0$ we have no scaling of space and hence any $\mathbf{r}_{(i)}$ itself is an invariant. Following the methodology above this leads to a similarity solution for the infinite set of MPC tensors (1) where the first term in the row, i.e. the two-point tensor, has the following form

$$\hat{\mathbf{r}}_{(1)} = \mathbf{r}_{(1)} \quad , \quad R_{ij}(\mathbf{r}_{(1)}, t) = e^{-t/t_0} \mathcal{R}_{ij}\left(\hat{\mathbf{r}}_{(1)}\right) \quad , \quad \ldots \tag{20}$$

where $\mathcal{R}_{ij}$ is the similarity variable of the reduced set of MPC equations independent of time and $t_0 = -a_4/a_3$.

In order to compare (19) and (20) to experimentally observable one-point quantities we introduce the Reynolds stress tensor $\overline{u_i u_j}$ and the integral length scale L_i as functionals of R_{ij}

$$\overline{u_i u_j} = R_{ij}(\mathbf{r} = 0, t) \tag{21}$$

and

$$L_i = \frac{1}{2K} \int R_{kk}(\mathbf{r})\, \mathrm{d}r_i \ . \tag{22}$$

Employing these definitions and implementing (19) and (20) into the latter we respectively obtain the rather different turbulent scaling laws

$$\overline{u_i u_j} \sim (t + t_0)^{-m} \quad , \quad L_i \sim (t + t_0)^n \tag{23}$$

and

$$\overline{u_i u_j} \sim e^{-t/t_0} \quad , \quad L_i \sim const. \tag{24}$$

where the first one is the classical algebraic decay law while the second one corresponds to a new exponential decay law. Note that (23) also covers the sub-class of a constant integral length scale decay law with $n = 0$ and $m = 2$ as given above.

In Hurst and Vassilicos, (2007) it was first reported that fractal-generated turbulence in a wind tunnel experiment may lead to an exponential decay law for the turbulent kinetic energy according to (24) and it was more fully consolidated in Seoud and Vassilicos, (2007). For certain cases they

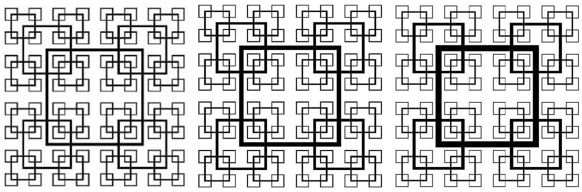

Figure 1: Set of fractal square grids used in the wind tunnel experiment of Vassilicos etal. 2007. t_r is the scaling factor between the largest to the smallest bar thicknesses. For the figures above t_r is given by 2.5, 5 and 8.5 - from left to right.

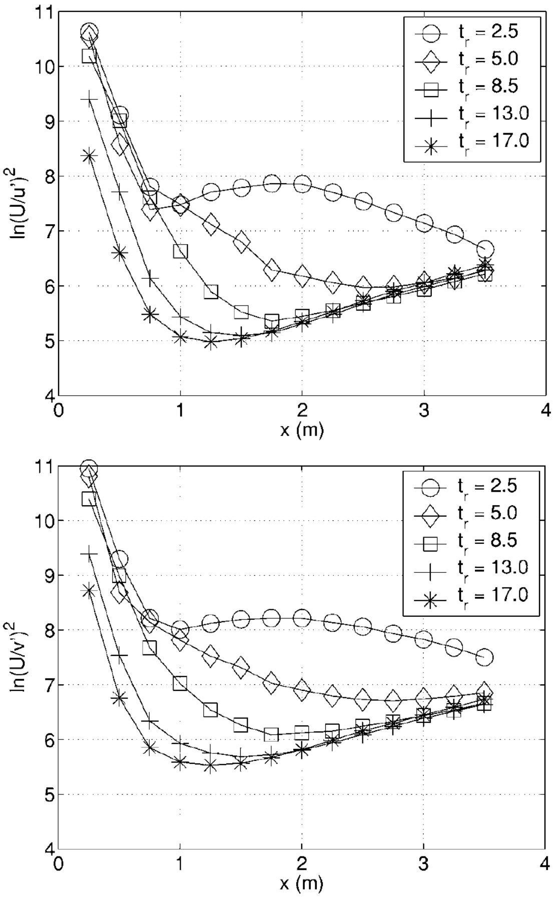

Figure 2: $\ln[(U/u)^2]$ and $\ln[(U/v)^2]$ as functions of x (in meters) for all five space-filling fractal square grids revealing the straight line in agreement with equation (24) on which the turbulence decay curves generated by all these grids eventually asymptote to. t_r defines the scaling factor between the largest to smallest bar thicknesses. All results are taken from Hurst and Vassilicos, (2007).

also find a constant integral length scale (22) (and also Taylor length scale) downstream of the fractal grid. A variety of different fractal grids were employed for the experiment such as cross-grids, square-grids and I-grids. A set of three different square grids are shown in figure 1. Therein t_r is the thickness ratio defined as the scaling factor between the largest to smallest bar thicknesses.

The data for the turbulent kinetic energy and the integral length scale showing both the behaviour according to (24) are given in figure 2 and 3. Three key results may be taken from 2 and 3. First, we observe that only the higher thickness ratios allow for the establishment of an exponential decay law i.e. beginning with $t_r = 8.5$ and higher the new scaling laws is clearly visible. Second, the development of

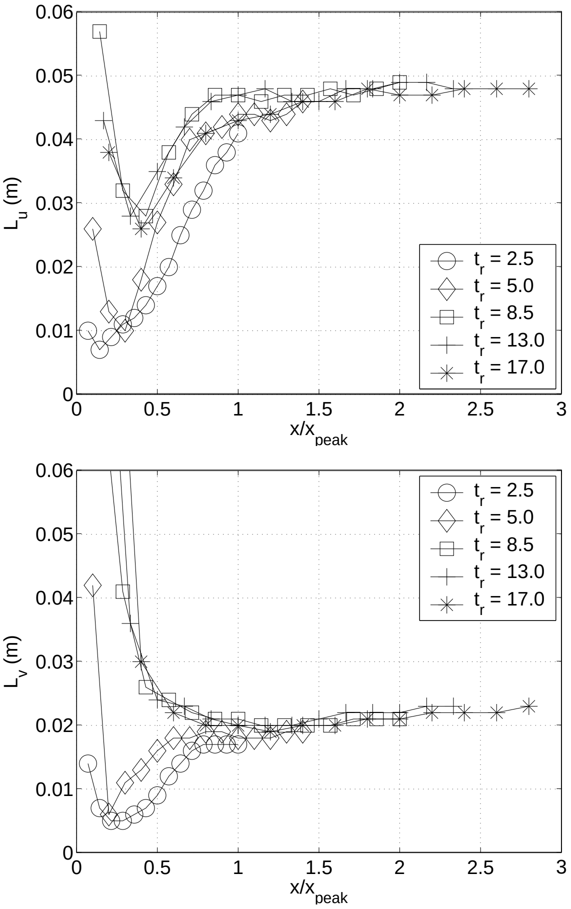

Figure 3: Longitudinal L_u and lateral L_v integral scales as functions of x/x_{peak} approaching a constant downstream of the grid. Space-filling fractal square grids at $U = 10m/s$ in the $T = 0.46m$ wind tunnel have been used. t_r defines the scaling factor between the largest to smallest bar thicknesses. All results are taken from Hurst and Vassilicos, (2007).

the exponential decay downstream of the grid becomes faster for increasing thickness ratios and is the largest for $t_r = 17$. Third, the constant integral length scale downstream of the grid appears to be less sensitive to the thickness ratio.

The physical interpretation of the latter results and in particular the fact that for large t_r the new scaling laws are established faster are due to the fact that a broad bandwidth of external scales have been imposed on the flow which are symmetry breaking. In the present case we have in fact only one scaling group of space. In the wind tunnel experiment however the scaling of time, here denoted by a_2, may be re-interpreted as another scaling group of space due to the relation $\tau = a/U$ where a is any fractal grid length scale and U is the constant mean velocity in the wind tunnel.

Hence, no matter how interpreted the fractal grids impose multiple scales, either time or length scales, on the flow and this is in particular true for large t_r. As a result we observe a symmetry breaking of a_1 and a_2 i.e. $a_1 = a_2 = 0$ which directly leads to the given multi-point scaling law (20) or the related one-point scaling law (24).

Beside the classical algebraic scaling laws including the sub-class of a decay at a constant length scale and the latter new exponential scaling law for decaying turbulence, we report another new turbulent scaling law which, to the best or the authors knowledge, has never been observed experimentally or reported from theoretical considerations.

For this we consider that there is a constant time scale, say τ_0, acting on the flow i.e. the scaling of time is broken and hence $a_2 = 0$. Employing the latter into the invariant surface condition (18) and integrate it leads to the following invariant solution for the correlation tensors

$$\hat{\mathbf{r}}_{(1)} = \mathbf{r}_{(1)} e^{-t/\tau_0}, \;\; R_{ij}(\mathbf{r}_{(1)}, t) = e^{-(2+\gamma)t/\tau_0} \mathcal{R}_{ij}\left(\hat{\mathbf{r}}_{(1)}\right), \ldots . \tag{25}$$

Translated into the language of one-point quantities we obtain

$$\overline{u_i u_j} \sim e^{-(2+\gamma)t/\tau_0} \quad , \quad L_i \sim e^{t/\tau_0} \tag{26}$$

where in the latter two equations γ emerged from the new scaling group $\gamma = a_3/a_4$. Imposing the assumption $\gamma = 0$ we observe that there is only the factor 2 between the the turbulent decay and the growth of the integral length scale.

CONCLUSIONS AND OUTLOOK

A variety of significant conclusions may be drawn from the present results. First, four different scaling laws for decaying turbulence have been identified two of which are classical and two new ones. The latter two both exhibit exponential behavior.

Second, in the context of one of the new scaling laws a new extended statistical scaling symmetry which goes beyond the Euler and Navier-Stokes, has been indirectly observed in the fractal grid turbulence experiments for the first time. The latter experimental result appears to correspond to one of the new scaling laws.

Third, the second new scaling law has been derived, even valid in a somewhat reduced form without the new scaling group. This scaling behavior corresponds to a constant time scale acting as a symmetry breaking ingredient onto the flow. At this point it is unclear how this scaling law may be experimentally generated.

Finally, we may note that none of the existing turbulence models in particular RANS models i.e. two equation and Reynolds stress transport model, admit the three scaling groups. Hence they are all incapable to mimic the above presented behavior. In the infinite Reynolds number limit essentially all RANS models admit two scaling groups.

A RANS type of model with multiple length scales is under development which is supposed to be admitting three scaling groups and at the same time able to mimic both the classical algebraic decay as well as the exponential decay depending on the length scale initial conditions.

The authors are deeply indebted to Ch. Vassilicos for very helpful discussion on fractal generated turbulence and leaving the figures from the experiments to their disposition.

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