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A ROLE OF HELICITY IN TURBULENT DIFFUSION

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ABSTRACT

A role of helicity in turbulent diffusions is investigated. Helicity is a typical quantity dealt in the sense of Eulerian description, but we are going to reveal a role of helicity through the lagrangian turbulent flows. In this study, we investigate temporal distributions of local helicity fields. Its ststistics will be compared to the statistics of lagrangian velocity and acceleration of the fluids. It would be said certain relations through the comparisions. As an application, study for the temporal helicity is applied for turbulent tranport or diffusion problems such as heavy particle dispersion problem.

INTRODUCTION

Lagrangin helicity is defined as

$$\int_0^T \mathbf{u}(\mathbf{x},t)\cdot\boldsymbol{\omega}(\mathbf{x},t)dt, \qquad (1)$$

where time $t \in [0,T]$, $\mathbf{x}$ is the position of fluid particles, $\mathbf{u}(\mathbf{x},t)$ is the velocity of fluid particles, and $\boldsymbol{\omega}(\mathbf{x},t)$ is the vorticity of the fluid particle at an instance. Since we are interested in the helicity density, we define the temporal helicity as $\mathbf{u}(t)\cdot\boldsymbol{\omega}(t)$. This is the instantaneous interaction between the velocity and vorticity through the time correlated. We would like to correspond to the local helicity of Eulerian fluid mechanics, $\mathbf{u}(\mathbf{x})\cdot\boldsymbol{\omega}(\mathbf{x})$.

Intermittency of helicity in Eulerian turbulence is a well-known unsolved problem(Moffat and Tsinober, 1992). Intermittency of helicity usually comes from highly fluctuating part and intermittent distributon of helicity is promisingly appeared in the highly rotating structures. This is guessed from Fig. 1 in (Choi and Lee, 2008). Besides intermittency of helicity, the core which has highly strong ratation rather than other part of coherently vortical structures, has present the most prominent alignment by velocity and vorticity. In the Eulerian turbulence, the role of helicity is intimately related to the rotating structures, roughly, turbulent flow structures.

In this study, we are interested in behaviors of Lagrangian fluid particles in turbulent transport or diffusion problems. Especially, temporal helicity defined in the above has a similar function in the Lagrangian turbulence problems as local helicity in the Euler turbulence problems. We introduce the basic statistics of Lagrnagian helicity(H_L) such as probability distribution functions(PDF) $P(H_L)$, autocorrelation $\rho_H(t) = \langle H_L(t_0)H_L(t_0+t)\rangle/\langle H_L^2(t_0)\rangle$, and comparsion between statistics of H_L and acceleration $\mathbf{a}$.

Table 1: Forcing parameter

N	particle	Re_λ	ν	k_f	ϵ_f	T_L^f
64^3	10,000	46.9	0.03	$2\sqrt{2}$	0.055	0.4312

Finally, we used direct numerical simulation for observations in this paper. We consider background flows and the fluid particles in the background flows. In order to obtain the turbulent flows, the Navier-Stokes equations and continuity equation were solved by the spectral method for spatial discretization and the third-order Runge-Kutta scheme for time advancement in a $2\pi^3$ cubic domin. Most calculations were carried out with 64^3 and there is the Table 1. for forcing parameters. For the maintenance of stationarity, we used the forcing scheme proposed by Eswaran and Pope(Eswaran and Pope, 1988) that the artificially forced low-wavenumber velocity components generate statistically stationary turbulence fields. Four point Hermite interpolation(Choi et al., 2003) is also used to interpolate particles from the grid points. units.

HELICITY STATISTICS IN THE LAGRANGIAN TURBULENCE

Probability Distribution Functions

Statistics including PDF's for Lagrangian fluid flows are observed in the several papers(Vedulat et al., 1999), (Mordant et al., 2002), and so on. In this research, we observe the quantities of Lagrangian fluid particles such as velocity, vorticity, acceleration and helicity. Fig. 2 presents distributions for Vorticity, Acceleration, and Helicity for Lagrangian fluid particles. Defferent from the Eulerian statistics manifested in Fig. 4, Lagrangian statistics in Fig. 2 represents quite different observations about temporal helicity and acceleration.

Autocorrelations

One- and two-particle lagrangian acceleration correlation(Yeung, 1997) is investigated and heavy particle be-

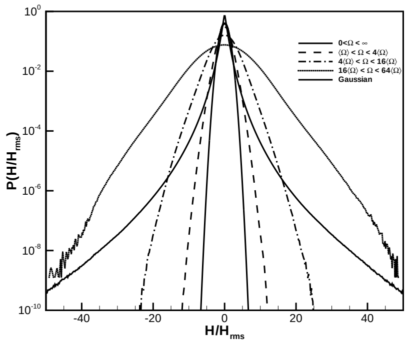

Figure 1: PDF of Helicity($P(H/H_{rms}|_{\langle\Omega\rangle})$) normalized by the rms value of the total helicity field, conditioned by four different enstrophy levels such as $\langle\Omega\rangle \leq \Omega \leq 4\langle\Omega\rangle$(dash-dot-dotted line), $4\langle\Omega\rangle \leq \Omega \leq 16\langle\Omega\rangle$(dash-dotted line), $0 \leq \Omega < \infty$(the total enstrophy field)(solid line), and $16\langle\Omega\rangle \leq \Omega \leq 64\langle\Omega\rangle$(dashed line).

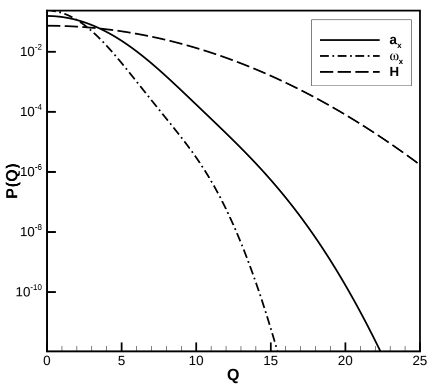

Figure 2: PDF of Vorticity, Acceleration, and Helicity for Lagrangian fluid particles

haviors are also studied in (Jung and Lee, 2008). In this section, we investigate autocorrelations for velocity, vorticity, acceleration, and H_L, and the definitions for those autocorrelations are as follows, respectively:

$$\begin{aligned} \rho_{\mathbf{u}(t)} &= \frac{\langle \mathbf{u}(t_0)\mathbf{u}(t_0+t)\rangle}{\langle \mathbf{u}^2(t_0)\rangle} \\ \rho_{\boldsymbol{\omega}(t)} &= \frac{\langle \boldsymbol{\omega}(t_0)\boldsymbol{\omega}(t_0+t)\rangle}{\langle \boldsymbol{\omega}^2(t_0)\rangle} \\ \rho_{\mathbf{a}(t)} &= \frac{\langle \mathbf{a}(t_0)\mathbf{a}(t_0+t)\rangle}{\langle \mathbf{a}^2(t_0)\rangle} \\ \rho_{H_L}(t) &= \frac{\langle H_L(t_0)H_L(t_0+t)\rangle}{\langle H_L^2(t_0)\rangle} \qquad (2) \end{aligned}$$

Fig. 3 presents the autocorrelations of Vorticity and Acceleration for Lagrangian fluid particles.

Statistics for Helicity and Acceleration of fluid particls

In (S. Lee et al, 2005) and(C. Lee et al., 2005), onset of intermittent distribution of acceleration is closely related to the movement of vortical structures, where enstrophy and dissipation as the measures of rotation and dissipation are strongly involved or not. In another numerical experiment, statistics of local helicity is compared to the statistics of acceleration(Fig. 4). Since the gredient of helicity is the axial

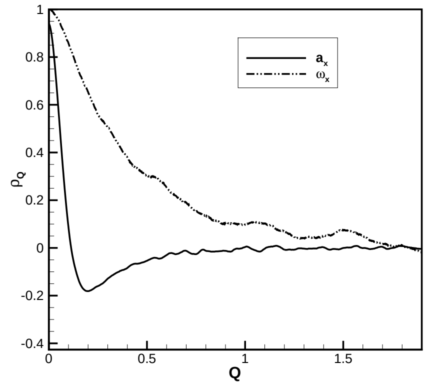

Figure 3: Autocorrelations of Vorticity and Acceleration for Lagrangian fluid particles

part of the gradient of the interations by velocity and vorticity, so called Vortex stretching, helicity is closely related to the stretching of vortex.

Meanwhile, Fig. 4 presents the similar statistics for helicity in Eulerian turbulence and acceleration. This comparison, however, can be justified by referring to the study(Biferale, 2004) in which the authors established the brigde between Eulerian and Lagrangian statistics in small enough scale regions. Namely, acceleration is defined by dividing velocity by sweeping time which is very short time and small scale ranges. Helicity is also dealt with the similar attitude. As a numerical result, local helicity of Eulerian fluid is compared to the acceleration in Fig. 4.

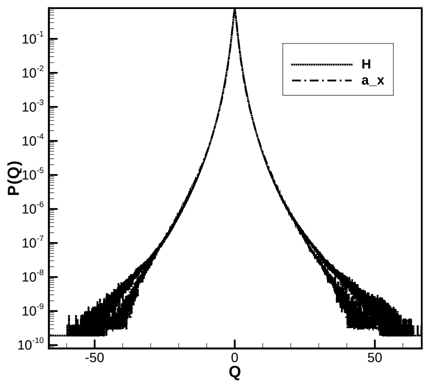

Figure 4: Distributions of Helicity and Acceleration in Eulerian statistics

CONCLUSION

Helicity is a key quantity to have three dimensionality in Eulerian fluid turbulence. In this paper, this helicity defined in Eulerian sense is extended to the largrangian fluid turbulence for the aim to apply study to the turbulent transpot or diffusion problems. First, Lagrangian helicity H_L is defined, and then basic statistics are presented such as PDF and autocorrelation of H_L. And helicity distribution was compared to intermittent distribution of acceleration. The clue of this comparison comes from the paper of Lee et al., 2004. Intermittency of helicity in lagrangian fluid flows is investigated. This make us imagine the existence of cascade structures and anomalous scalins for H_L. To obtain a concrete conclusion about the intermittency of helicity, it requires to study existence of spectrum of Lagrnagian helicity. Second, we can assume the comparison between Lagrangian helicity and acceleration that can give the relation between the three-dimensionality and turbulence transport and particle dispersion problems. In the later works, authors would like to study more explicit explanaton about the role of lagrangian helicity in turbulent transport: cascade and spectrum of helicity, helical behaviors of fluid particle and backgroud flow structures, calculation in the higher resolution circumstances.

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