Sixth International Symposium on Turbulence and Shear Flow Phenomena
Seoul, Korea, 22-24 June 2009

ACCELERATING STATIONARITY IN LINEARLY FORCED ISOTROPIC TURBULENCE

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ABSTRACT

The physics of the linear forcing of isotropic turbulence allows for some useful estimates of the characteristic length scales of the turbulence produced during the statistically stationary phase. With such estimates we could practically define uniquely the stationary statistics by means of the box-size of the simulation, the linear forcing parameter and the viscosity of each case. We try to explain the estimates and we use them in order to pre-describe the resulting statistically stationary energy spectra, retrieving well documented analytical spectral relations. The produced spectra are in very good agreement with the numerically calculated average spectra during the statistically stationary phase, for a variety of values of the dimensionless linear forcing parameter. The analytical forms are used as a revised initial condition for the production of isotropic turbulence which accelerates remarkably the achievement of stationarity, minimizing any transient phase and could be generalized for the initialization of several different cases.

INTRODUCTION

Numerical simulations of isotropic turbulence play a key role in studying basic features of turbulent flows. The two most frequently studied types of isotropic turbulence are freely decaying, and forced statistically stationary turbulence. For studies in which one wishes stationarity for statistical sampling, forced turbulence is preferable over decaying turbulence. In 2003, a very interesting paper by Lundgren proposed an alternative to the band-limited methods of forcing turbulence, using a linear forcing factor. Apart from its simplicity, the profound advantage of linear forcing is the possibility of applying this method in both physical and Fourier space. Problems that do not admit fully periodic boundary conditions, for instance simulating interactions of turbulence with combustion in which conditions upstream and downstream of the flame are inherently different, are often simulated using numerical codes formulated in physical space, such as finite differences. The application of band-limited forcing schemes requires knowledge of the wave numbers and Fourier-transformed velocities, quantities that are not readily available in codes formulated in physical space. Rosales and Meneveau (2005) have shown that the application of linear forcing in both physical and spectral space renders practically equivalent results, reflecting the profound equivalence of the method in both spaces. Thus, the linear-forcing method opens wide opportunities for application in both physical and spectral space. Furthermore, the resemblance of the forcing parameter to an applied shear promises the achievement of stationary spectra, where the structure of the large scale is more realistic. In this direction, Ludgren (2003) showed that linear forcing produces statistics at scales between the integral scale and the inertial range (e.g., structure function curving) that resemble the curving observed from experimental data.

In this study, we continue the work of Rosales and Meneveau (2005), investigating the statistical stationarity that is produced by applying the linear forcing method in spectral space, and we try to quantify the characteristic scales of the statistically stationary turbulence produced. More specifically, we are focusing on the finding of Rosales and Meneveau that the energy-containing length scale, which characterizes the large eddies of the turbulence, approaches a stationary value, proportional to the dimensions of the problem. The constancy of this well-defined stationary length scale L, determines uniquely the statistics during the stationary phase and it could be

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supported by the dimensional analysis of the linear forcing method. In this direction, we extend the past analysis explaining the constancy of that length scale by giving some theoretical reasoning for the value of the constant of the proportionality between L and the box-size of the simulation. This reasoning is based on the limitations of the linear-forcing application, in terms of the possible separation of the scales of turbulence. Furthermore, we investigate the stationary spectra of the turbulence produced by the linearly forcing method, and we show that they can be fitted accurately by the analytical Pope's (2000) formula using the linear forcing parameter, the computational size and the viscosity that characterize each case. The analytical forms are used for initializing linearly forced DNS of isotropic turbulence, minimizing sharp transients and accelerating remarkably the computational time needed for stationarity.

STATIONARY ISOTROPIC TURBULENCE BY THE LINEAR FORCING METHOD

Linear forcing is applied in the Navier-Stokes equations by including the linear term $\mathbf{g} = A\mathbf{u}$, proportional to the velocity. The time evolution of the energy spectrum becomes

$$\partial_t E(k,t) = -\partial_k T_k(k,t) - 2\left(\nu k^2 - A\right) E(k,t) \qquad (1)$$

where $T_k(k,t)$ is the function of the spectral transfer of energy. Integrating (1) and taking the energy balance for the statistically stationary state we see that the dissipation rate, ε, is linked with the turbulent kinetic energy, K, through

$$\varepsilon = 2AK = 3Au_{rms}^2 \qquad (2)$$

where u_{rms} is the RMS of the fluctuating velocity. Rosales and Meneveau showed that, independently of the initial conditions, the application of the linear forcing drives the turbulence in a statistical steady state where the turbulent kinetic energy oscillates around an average value, satisfying equation (2). Furthermore, as expected in a statistical sense, this steady state was invariant between runs with a physical space based finite-difference code and runs using standard pseudospectral methods.

The above relation (2) between the dissipation and the turbulent kinetic energy is an immediate physical consequence of the energy balance, where the energy injection rate equals the dissipation rate for stationarity. It also defines the characteristic eddy turnover time scale of the turbulence, during the statistically stationary phase,

$$\tau = u_{rms}^2 / \varepsilon = \left(3A\right)^{-1} \qquad (3)$$

which could be visualized as the characteristic time lag between energy injection and its eventual dissipation. In order to clarify this, we present, in figure 1, the evolutions of the energy injection rate and the dissipation rate for 30 turnover times (dimensional time 150 for this case). The results are from new linearly forced DNS of isotropic turbulence with a linear forcing parameter A = 0.0666 and viscosity $\nu = 1.041 \cdot 10^{-3}$ in a $(2\pi)^3$ computational domain (more details on the DNS will be given later). The two different curves almost coincide when the evolution of the dissipation is shifted forward by the time increment given from (3), $t = (3A)^{-1}$.

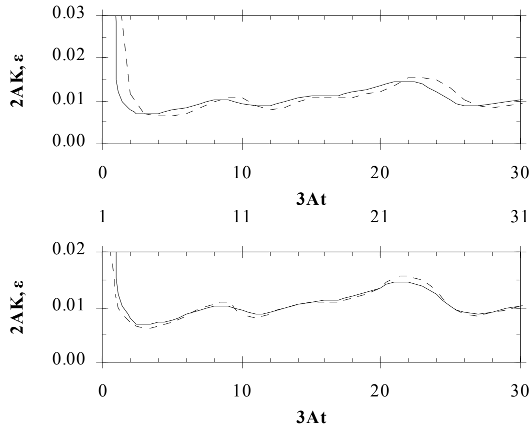

Figure 1: Evolution of the energy injection, $2AK$ (solid line), and dissipation, ε, (dashed line) rates (upper graph). When the evolution of the dissipation evolution is shifted by $\tau = (3A)^{-1}$, the two curves almost coincide (lower graph).

The same picture can be drawn from the more strict investigation of the correlation between the two evolutions versus their time lag, shown in figure 2. It turns out that the correlation coefficient maximizes when the time difference equals one turnover time-scale.

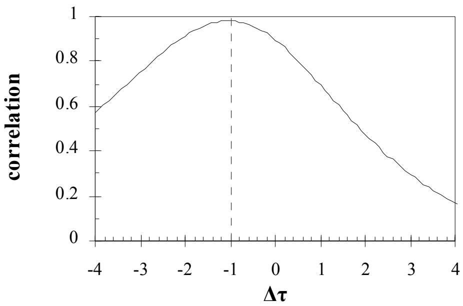

Figure 2: Correlation between the energy injection rate and dissipation rates versus their time difference, for the case presented in fig. 1.

What it is not immediately evident is a second, very interesting, relation that links the values of ε and K or u_{rms}, averaged at the stationary phase, with the dimensions of the problem. More specifically, Rosales and Meneveau showed that the energy containing length scale, $L = u^3/\varepsilon$, characterizing the large eddies, approaches a stationary value, proportional to the dimensions of the problem, l,

$$L = u_{rms}^3 / \varepsilon = c\, l \qquad (4)$$

Rosales and Meneveau used numerical results from several DNS runs, at different Reynolds numbers ($30<Re_\lambda<200$), and estimated the constant of the proportionality to be c = 0.19. The constancy of this well-defined stationary length scale L, could be supported by the dimensional analysis of the linear forcing method. In the following, we extend the past analysis in a more systematic way that gives a more physical justification for the calculated value of the constant c. Apart from the physical interpretation of the linear forcing method, the accurate calculation of c is important because, in combination with equation (2), it defines uniquely the statistics of the application of the linear forcing during the stationary phase. For instance, the RMS averaged velocity approaches $u_{rms}=3cAl$, and the averaged dissipation rate, $\varepsilon = 27c^2A^3l^2$. Also, the values of the Reynolds numbers characterizing the produced stationary isotropic turbulence are uniquely defined using equations (2) and (4).

CHARACTERISTIC SCALES AND SIMILARITY OF LINEARLY FORCED TURBULENCE

The application of the linear forcing in isotropic turbulence imposes a profound time scale which is the reciprocal of the linear forcing parameter A (time^{-1}). The other scales characterizing the application of the linear forcing is the dimension of the box, l (length), and the viscosity, ν (length^2 time^{-1}). The only dimensionless parameter that can be formed combining these scales is then

$$\mathrm{Re}_A = 3Al^2/\nu \qquad (5)$$

which corresponds to a kind of a Reynolds number of the linear forcing and defines the similarity of the problem. Therefore, cases referring to the same value of this parameter should show a similar behavior, approaching the same values of the dimensionless Reynolds number $\mathrm{Re}_L=uL/\nu$, and consequently of $\mathrm{Re}_\lambda=u\lambda/\nu$ as well. Using the definition of L and the stationarity implication (2), we express

$$\mathrm{Re}_L = 3AL^2/\nu \qquad (6)$$

and thus, taking the ratio $\mathrm{Re}_L/\mathrm{Re}_A$, we obtain

$$\mathrm{Re}_L/\mathrm{Re}_A = (L/l)^2 \qquad (7)$$

which implies that L should depend only on the box-size, through equation (4). At the same time, using the definition of the Taylor length scale $\lambda = (10\nu K/\varepsilon)^{1/2}$, and taking the statistical balance of dissipation and energy injection from equation (2), we see that the Taylor length scale is independent of the box-size, l, depending only on the ratio ν/A, through

$$\lambda = (5\nu/A)^{1/2} \qquad (8)$$

As expected from equations (5) – (7), for a given box size (which uniquely defines the length scale $L=cl$, through equation 4), the value of the turbulence Reynolds number is proportional to the ratio of the linear forcing divided by the viscosity, A/ν, and hence this ratio defines the dynamical similarity of all linearly forced simulations. The more often used Taylor-scale Reynolds number, $\mathrm{Re}_\lambda = (15\mathrm{Re}_L)^{1/2}$ or, equivalently, $\mathrm{Re}_\lambda = c\ (15\mathrm{Re}_A)^{1/2}$, has a slower, $\sim (A/\nu)^{1/2}$, dependence, as a consequence of the simultaneous change in λ values through equation (8).

The above discussion shows that the calculation of the statistically stationary values of turbulence reduces to choosing a value of L given the box size l. We try to approach this problem by comparing the L and λ scales for a given box-size. The physical interpretation of the Taylor λ scale has the sense of an intermediate scale between the clearly defined and well-separated $\eta = \nu^{3/4}/\varepsilon^{1/4}$ and L scales (see also Pope, 2000). In contrast to the stationary value of L, which depends only on the dimensions of the box (equation 4), the stationary value of λ increases inversely to $(A/\nu)^{1/2}$ through equation (8). Setting $2\nu k_c^2E(k_c) = 2AE(k_c)$ in equation (1), we determine the specific wave number k_c, where the stationary time-averaged energy production rate equals the stationary time average of the dissipation rate and relates to λ through

$$k_c = (A/\nu)^{1/2} = 5^{1/2}/\lambda \qquad (9)$$

For this specific value of the wave number, the time-averaged transfer rate $\langle\partial T_k(k_c)/\partial k\rangle$ becomes zero, and the averaged energy transfer function $\langle T_k(k_c)\rangle$ maximizes. The minimum possible wave number k_c , representing a fully three-dimensional state is $k_{min} = 3^{1/2}2\pi/l$, therefore, we should seek the limits of the applicability of the linear forcing (in terms of the production of statistically stationary isotropic turbulence) within this wave number and the immediate following one $k = 4\pi/l$, in order to keep the least three-dimensional spectrum components with a non-zero net energy injection rate active. Through equation (9), this demand bounds the maximum meaningful value of λ, which permits the production of statistically stationary isotropic turbulence to

$$l\sqrt{5}/2 < 2\pi\ \lambda_{\max} < l\sqrt{5}/\sqrt{3} \qquad (10)$$

Furthermore, the application of the linear forcing drives turbulence to a stable equilibrium state where the energy dissipation and the energy transfer rate minimizes. This is equivalent with the minimization of the energy injection rate which equals $2AK = 3Au^2$. Independently of the value of A, this implies the minimization of the L scale, in accordance with the minimization of the scale difference between the large and the small structures. The lowest value of L which permits separation of the eddy scales equals the maximum possible, for three-dimensional linearly forced isotropic turbulence, Taylor length-scale, λ. This limiting equality $L_{\min}= \lambda_{\max}$, defines

$$(5/4)^{1/2}/2\pi < L_{\min}/l < (5/3)^{1/2}/2\pi \qquad (11)$$

which bounds the constant of proportionality in equation (3) between $0.178 < c < 0.205$, in excellent agreement with the numerical estimation of the coefficient $c = 0.19$ by Rosales and Meneveau (2005).

In figure 3, we present results of the averaged produced Re_λ as a function of the characteristic Re_A from a series linearly forced DNS of isotropic turbulence that we have recently carried out. We have used an MPI version of a pseudospectral code, which has been implemented in the vectoral language and tested for accuracy, grid independence, and scalability (Kassinos et al., 2007). The runs presented here have a resolution of 128^3 - 256^3 Fourier modes in a $(2\pi)^3$ computational domain and time advance is accomplished by a third-order Runge-Kutta method. The initial conditions for the velocity were common to all cases, starting with a pulse of energy at low wave numbers in Fourier space and a random distribution of phases for the Fourier modes. The linear forcing parameter was the same, $A = 0.0666$, for all the runs, and the viscosity, ν, varied from $2.22 \cdot 10^{-2}$ to $4.63 \cdot 10^{-4}$. The total simulation time was 120 turnover time scales ($t = 600$), but the statistics were collected during the last 90. The statistically stationary average values obtained from the simulations are shown as symbols in figure 3. The error bars indicate the calculated deviations. The upper line in figure 3 corresponds to $\mathrm{Re}_\lambda = 0.205(15\mathrm{Re}_A)^{1/2}$ and the lower to $\mathrm{Re}_\lambda = 0.178(15\mathrm{Re}_A)^{1/2}$, that is, the lines indicate the bounding expected values based on the analysis presented in the previous section. Despite the deviation (10–15%), the statistically averaged results lie well within the previously calculated range.

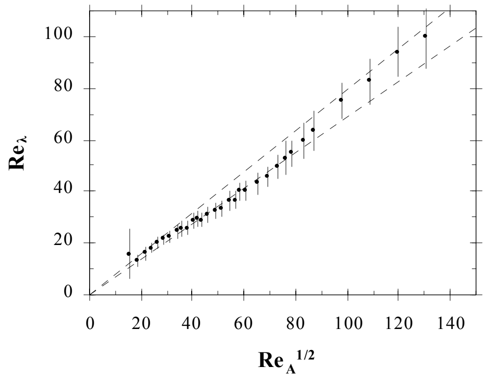

Figure 3: Statistical stationary averages and deviation of Re_λ versus the characteristic Re_A of each case (symbols), from new linearly forced DNS. The two lines correspond to the solutions $\mathrm{Re}_\lambda = c\,(15\mathrm{Re}_A)^{1/2}$, with $c = 0.178$ (lower line) and $c = 0.205$ (upper line).

LINEARLY FORCED DNS WITH A PREDEFINED INITIALLY STATIONARY SPECTRUM

In this section, we apply the above calculated statistically stationary values to the analytical spectrum given in Pope (2000) and we show the agreement of the produced forms with the statistically stationary spectra of linearly forced isotropic turbulence from numerical simulations. The calculated spectra are then used as a refined initial condition for new linearly forced DNS of isotropic turbulence, in order to accelerate the time needed for stationarity.

New initial spectrum

The Pope's (2000) spectrum is analytically defined as

$$E(k) = C\varepsilon^{2/3}k^{-5/3} f_L(k,L) f_\eta(k,\eta) \tag{12}$$

where C is a constant, and $f_L(k,L)$, $f_\eta(k,\eta)$ are non-dimensional functions which are specified bellow. The $f_L(k,L)$ function determines the shape of the energy-containing range at the low wave numbers and is given by

$$f_L(k,L) = \left(\frac{kL}{\sqrt{k^2L^2 + c_L}} \right)^{5/3+p_0} \tag{13}$$

where $L = u^3/\varepsilon$ (see equation 4) and the function $f_\eta(k,\eta)$ is given by (see also Kraichnan, 1976)

$$f_\eta(k,\eta) = \exp\left\{ -\beta \left[\left(k^4\eta^4 + c_\eta^4 \right)^{1/4} - c_\eta \right] \right\} \tag{14}$$

In the above, we use $\beta = 5.2$ as suggested in Pope (2000) (see also the experimental evidences by Saddoughi and Veeravalli, 1994). For large Reynolds numbers, and setting $p_0 = 2$ and $C = 1.5$, equal to the Kolmogorov constant (Sreenivasan, 1995), the values of the parameters c_L and c_η were estimated in Pope, for large Reynolds numbers ($\mathrm{Re}_\lambda \approx 1000$), as $c_L \approx 2.00$ and $c_\eta \approx 0.40$. However, for relatively low Reynolds numbers, the above values of the tuning parameters should be modified as will be shown. By introducing the stationary estimates for ε and $\eta^4 = \nu^3/\varepsilon$, in equations (12) – (14), we practically define a stationary spectrum, which depends solely on the values of A and ν

$$E(k) = \frac{13.5 L^5 A^2 k^2}{\left(k^2L^2 + c_L\right)^{11/6}} \exp\left\{ 5.2 \left[c_\eta - \left(\frac{\nu^3 k^4}{27 A^3 L^2} + c_\eta^4 \right)^{1/4} \right] \right\} \tag{15}$$

given that the value of L at the stationary phase is fixed ($L \approx 1.2$ for a cube with side 2π), through equation (11). The values of the parameters c_L and c_η are calculated by the requirements that the integrals (or the sums in discrete space) of the terms $E(k)$ and $2\nu k^2 E(k)$ equal the kinetic energy and the dissipation respectively. Using the non-dimensional spectrum form $E(x) = E(kL)/KL$, we see that it depends only on the Re_L number

$$E(x) = \frac{x^2}{\left(x^2 + c_L\right)^{11/6}} \exp\left\{ -5.2 \left[\left(\frac{x^4}{\mathrm{Re}_L^3} + c_\eta^4 \right)^{1/4} - c_\eta \right] \right\} \tag{16}$$

or, similarly, on the $\mathrm{Re}_\lambda = (15\mathrm{Re}_L)^{1/2}$ number of the turbulence. The connection with the parameters of the linear forcing method comes through equations (5) – (7), taking the constant $c \approx 0.19$. The dimensionless spectrum (16) should integrate to unity

$$\int_0^\infty E(x)\,dx = 1 \tag{17}$$

The same holds true for the dimensionless dissipation spectrum,

$$\int_0^\infty \frac{3x^2 E(x)}{\mathrm{Re}_L} dx = 1 \tag{18}$$

Through the relations (17) and (18), the values of c_L and c_η for the continuous spectral forms are determined uniquely. Indeed, a minor dependence on the resolution used should appear, since the values of the parameters c_L and c_η could slightly change depending on the range of the resolved wave numbers. However given that the criteria for fine resolution are fulfilled, the above determination of c_L and c_η is rather accurate.

In figure 4 we give the dependence of the two parameters on the Reynolds number. It is profound that for relatively large values of the Reynolds number the two parameters approach the values specified in Pope (2000), c_L = 2.00, and c_η = 0.40. For small Reynolds numbers however, they drastically deviate from the above calculations.

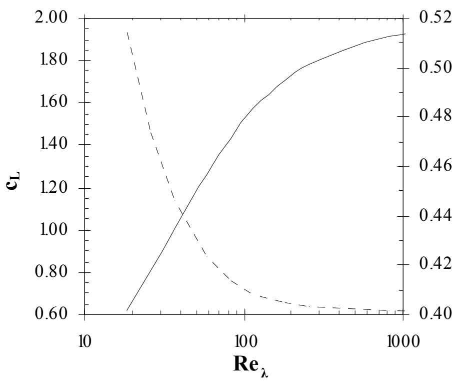

Figure 4. The dependence of the parameters c_L (continuous line) and c_η (dashed line) in Pope's (2000) spectrum on Re_λ

In a first attempt to validate the above calculated spectra (equations 15 - 18), we compare them with numerical results from linearly forced DNS of isotropic turbulence. More specifically, in figure 5 we illustrate the energy spectra which have been averaged during the statistically stationary period, (taken from the figure 2, presented in Rosales and Meneveau, 2005), and the corresponding results for the same cases, from equation (15). The presented cases refer to three different values of the linear forcing parameter A and they have a Re_λ which varies from 30 - 54. From the comparison it turns out that the numerically calculated stationary spectra are in very close agreement to, the analytical forms which, as it has been mentioned, are determined exclusively by the values of the viscosity v, and the linear forcing parameter A.

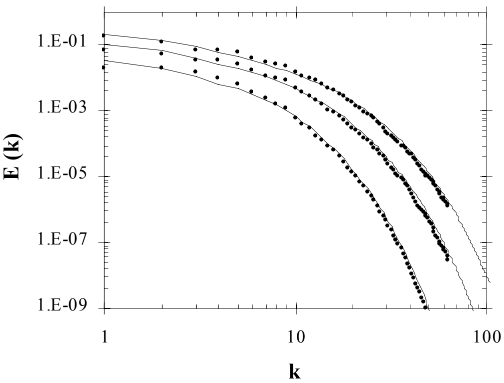

Figure 5: Energy spectra presented in Fig. 2 in Rosales and Meneveau (2005), averaged over the statistically stationary period (symbols) and the respective results (lines) from equation (15).

In Table 1, we give the values of the parameters which have been used for producing the spectra of the three different cases presented. One may note that as the Reynolds number increases, the tuning parameters c_L and c_η, gradually approach the values given in Pope (2000).

Table 1: The values of the parameters c_L and c_η for the cases presented in Fig. 5

A	v	Re_λ	c_L	c_η
0.0667	$4.491\cdot10^{-3}$	29	0.89	0.45
0.1333	$4.491\cdot10^{-3}$	43	1.07	0.43
0.2000	$4.491\cdot10^{-3}$	54	1.18	0.42

Simulations using the new initialization

Rosales and Meneveau (2005) have shown that, independent on the initial conditions, the linearly forced turbulence (for the same A and v values) approaches the same statistically stationary steady state. However, depending on the initialization used, the transient phases preceding stationarity could be remarkably long, thus increasing the number of the time-steps needed for the calculation of representative statistics. This can prove to be extremely time-consuming in runs at large Reynolds numbers demanding high resolution. On the other hand, starting the simulations from an initial spectrum close to the stationary shape could minimize transients, resulting in shorter runs and less demands in computational time. In order to investigate this possibility, we have performed linearly forced DNS of isotropic turbulence using two different initializations. Firstly, we have applied the same initial spectral form as in Rosales and Meneveau (2005). That is a solenoidal isotropic velocity field with random phases and the energy spectrum

$$E(k) = \frac{16}{\sqrt{\pi/2}} u_0^2 \frac{k^4}{k_0^5} \exp\left(-2\frac{k^2}{k_0^2}\right) \tag{19}$$

where u_0^2 is the initial rms velocity, and k_0 is the wavenumber at which the maximum of $E(k)$ maximizes. The choice of $k_0 = 2$ was proved as the most rapid and thus we kept this value in our simulations. The second initialization uses the stationary spectrum defined by equation (15). The comparison which is presented here corresponds to a linearly forced case with A = 0.0666 and v = $4.1\cdot10^{-4}$, which correspond to a Re_λ around 100. The runs have been done using the code which was described in the previous section, with a fine resolution of 256^3. In figure 6 and 7, we compare the evolutions of the turbulent kinetic energy and the Re_λ for the two different initializations.

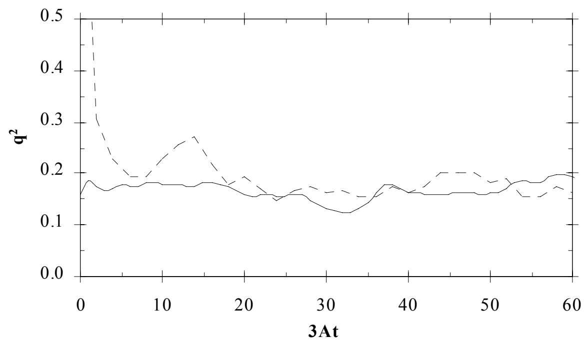

Figure 6: Evolution of twice the turbulent kinetic energy q^2=2K, for linearly forced DNS of isotropic turbulence with A = 0.0666 and v = $4.1\cdot10^{-4}$. The two lines correspond to different initializations: (a) initial spectrum as in Rosales and Meneveau (2005) (dashed line) and (b) initial spectrum given by equation (15) (solid line).

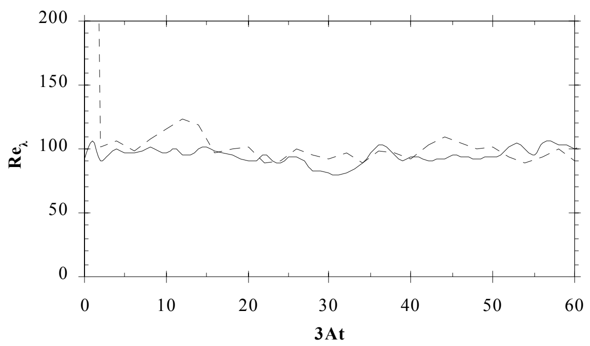

Figure 7: Evolution of Re_λ. Lines are as in figure 6.

In the case of the simulation using the initialization given by (19) the transient period is long because of the choice of an initial energy spectrum very different from the form to which it finally converges (fig. 8). The profound transient needs around 17 turnover times (approximately 15000 time steps) in order to converge to a statistically stationary state with small oscillations around the statistical mean. In the same figure, the simulation which was initialized using the spectrum given by (15) is already in a stationary state without any profound transient.

The picture becomes clearer in figure 8, where the evolution of the energy spectra during the two simulations is shown. The first simulation needs about 20 turnover times to reach the shape of the average spectrum of the second case, which is very close to its initial form given by equation (15).

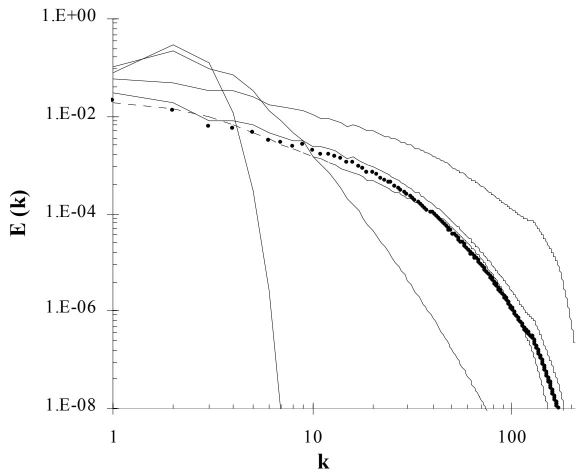

Figure 8: Energy spectra from the simulations presented here. (a) The simulation with an initial spectrum as in Rosales and Meneveau (2005) (continuous lines) for $t/\tau = 0$, 1, 5, 20 and (b) the simulation with an initial spectrum given by equation (15) (dashed line) and the average spectrum during all the simulation (points).

CONCLUSIONS

Previous studies of linearly forced isotropic turbulence have clearly shown that the energy containing length scale, L, characterizing the large eddies of the turbulence, approaches a stationary value, proportional to the dimensions of the size of the simulation. We tried to explain the constancy of that length scale giving, also, some theoretical reasoning for the value of the constant of proportionality with the box-size of the simulation, based on the separation of the scales of isotropic turbulence. The outcomes of the theoretical analysis are in excellent agreement with the calculated statistics from linearly forced DNS. Apart from the physical interpretation of the linear forcing method, the accurate calculation of the L scale is important, because in combination with the stationarity equation (2), it defines uniquely the statistics of the linearly forced turbulence during the stationary phase. In other words, we can practically define uniquely the stationary statistics by means of the box-size of the simulation, the forcing parameter and the viscosity. In this direction, we made use of these estimates in order to pre-describe analytically the resulting statistically stationary energy spectrum, retrieving well documented analytical spectral relations (Pope, 2000). The produced spectra are in very good agreement with the numerically calculated averaged spectra during the statistically stationary phase, for a variety of values of the dimensionless linear forcing parameter. The analytical forms can be used as a revised initial condition for the production of linearly forced isotropic turbulence, accelerating remarkably the achievement of stationarity, minimizing any transient phase and could be generalized for the initialization of several different cases.

Aknowledgment This work was supported by a Center of Excellence grant from the Norwegian Research Council to Center for Biomedical Computing. The authors would like to thank Prof. Antonis D. Koussis for fruitful discussions.

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