DIRECT NUMERICAL SIMULATION OF TURBULENT FLOW IN AN AXISYMMETRIC SUPERSONIC DIFFUSER

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ABSTRACT
Effects of deceleration and mean dilatation on the turbulence structure of supersonic flow in a diffuser with an incoming supersonic fully-developed turbulent pipe flow are studied by means of DNS. Strong enhancement of the turbulence intensities is observed when the flow undergoes deceleration. Turbulence production and pressure-strain terms in the Reynolds stress budgets are found to increase dramatically leading to increased Reynolds stresses. The central role of pressure-strain correlations in modifying the turbulence structure in these flow conditions is demonstrated.

INTRODUCTION
Decelerated compressible wall-bounded turbulent shear flows are still of great practical and theoretical interest and provide a challenge for improved turbulence modeling. Incompressible adverse pressure gradient (APG) shear flows have been studied experimentally among others by Nagano et al. (1997) and numerically by Coleman et al. (2003) and Lee & Sung (2008). When the flow is compressible and undergoes an APG, the turbulence structure is affected not only by mean strain and shear, but also by mean compression. In his pioneering work, Bradshaw (1974) studied effects of mean dilatation on the turbulence structure in wall-bounded flows in the context of engineering calculation methods. He found that mean dilatation effects have a greater impact on the turbulence structure than would be expected from the extra production terms in the Reynolds stress transport equations. He also mentioned indirect effects caused by the pressure-strain correlation tensor in such flows which can add ‘overwhelmingly’ to those of the extra production terms. In their experimental investigation of APG supersonic turbulent boundary layers, Fernando & Smits (1990) studied the behaviour of the Reynolds stresses and of the large-scale structures. Recently, Ghosh et al. (2008) performed DNS/LES of supersonic axisymmetric nozzle flow and also LES of supersonic diffuser flow using a fully-developed pipe flow at the inlet in order to study, among other features, Bradshaw’s indirect effect of the pressure-strain correlations on the turbulence structure. They found that the Reynolds stresses and pressure-strain correlations are dramatically increased in the diffuser even though the production by mean strain and mean compression is relatively small compared to that by mean shear. Now, supersonic internal flows subjected to APGs are more complicated than corresponding flows subjected to favourable pressure gradients (nozzle flow) so that their analysis needs greater care. When the Mach number of the incoming flow is at a rather low overall supersonic level and the flow contraction is moderate, substantial transonic regions can occur. It is the aim of this paper to gain insight into the complex dynamics of a supersonic diffuser flow which develops from an incoming fully-developed pipe flow. The results will provide a data-base for validation and improvement of Reynolds stress models. Apriori-tests of two existing pressure-strain models are carried out using DNS data.

NUMERICAL DETAILS
The governing Navier-Stokes equations are solved in a characteristic-type pressure-velocity-entropy form on non-orthogonal curvilinear coordinates. Fifth order compact upwind schemes with low dissipation (Adams & Shariff, 1996) have been used for the convection terms and sixth order compact central schemes (Lele, 1992) for the molecular transport terms. The flow field is advanced in time using a 3rd order low-storage Runge-Kutta scheme (Williamson, 1980). Fully-developed supersonic turbulent flow in a pipe serves as inflow condition for the diffuser flow. The walls are kept at the same constant temperature in both the flows. The centerline Mach number \( M_c \) and friction Reynolds number \( Re_\tau \) of the incoming flow are 1.75 and 300. \( Re_\tau \) is defined using the friction velocity \( u_\tau = \sqrt{T_\tau/\rho} \), the pipe radius \( R \) and the kinematic viscosity at the wall, \( \nu_w(T_w) \). The Mach number \( M_e \) is the ratio of the centerline velocity and local speed of sound. The domain length of each configuration (pipe or diffuser) is \( L = 10R \). The adverse axial pressure gradient averaged over the first half of the diffuser and normalized with the local displacement thickness and the local wall shear stress (Clauser parameter) is 5. The ratio of diffuser radius to pipe radius at the end of the computational domain is 0.93. The number of grid points used to discretize the pipe domain is \( 256 \times 256 \times 140 \) in streamwise, circumferential and radial directions while that for the diffuser is \( 384 \times 256 \times 140 \). The higher resolution in the diffuser is required to capture the increased turbulence activity occurring due to deceleration of the flow. The pipe and diffuser flow simulations are coupled using MPI routines. The concept of characteristics is applied to set inviscid inflow conditions for the diffuser flow. The incoming characteristics are computed from the periodic pipe flow simulations and are received at every time-step in the diffuser computation through MPI. Partially non-reflecting outflow conditions are used in the subsonic region of the outflow plane. No sponge layer has been used (Ghosh et al., 2008).

RESULTS
Instantaneous and mean flow features
A snapshot of instantaneous axial velocity fluctuations, normalized with the local friction velocity, and presented in an \((x,r)\)-plane that contains the axis, is shown in Figure 1. An increase in near-wall ‘sweep-ejection’ activity as the flow is decelerated, can be observed in the first half of this carpet.
Effects of deceleration on the turbulence structure

Deceleration of the supersonic flow in the diffuser leads to an increase in turbulence intensities. As a result, both solenoidal and dilatational dissipation rates, \( \rho \epsilon = \mu u'w', \) \( \rho \epsilon = \frac{1}{3} \mu (u'_x u'_y + u'_y u'_z + u'_z u'_x) \) are enhanced (figs. 8, 9). An increase in pressure-dilatation correlation \( p'u'_{i,j} \) is also observed. However, compressible dissipation rate as well as pressure-dilatation correlation continue to have negligible contributions to the TKE budget as in the incoming supersonic pipe flow. The increase in solenoidal dissipation rate can be explained intuitively from its transport equation (Kreuzinger et al., 2006) in which a production term appears that contains mean dilatation. The Reynolds stresses increase monotonously through the compression region, both in the near-wall region as well as in the core. Here we show the axial Reynolds stress and the Reynolds shear stress scaled with the local wall shear stress (figs. 10, 11). The rms axial and radial intensities scaled with constant friction velocity, \( u_{r,o} \) of the incoming flow (figs. 12, 13) reveal the same trend, but in this scaling the magnitude of the enhancement is reduced. It should be noted that while in incompressible boundary layers under the influence of APG, the turbulence intensities are decreased in the near-wall region and increased in the outer layer (Lee & Sung, 2008), in the compressible diffuser flow, we see an increase in turbulence intensities both near the wall as well as in the core region.

The axial Reynolds stress production is increased and the production term is now decomposed into contributions from mean shear, extra strain rate and mean compression as in eq.(1). Extra strain rate and mean compression lead to small increases in the production of the axial stress as seen in fig. 14. But, the major increase in production is due to production by mean shear. The mean shear itself changes only marginally in the peak production region (as shown in fig. 11 at \( x/L = 0.2 \) and 0.35) which means that the increase in the Reynolds shear stress is the main reason for the increase in production by mean shear. This is contrary to findings in incompressible decelerated channel flows (Coleman et al., 2003) where decreased mean shear leads to decreased production (and hence decreased turbulence intensities) in the near-wall region. The production term in the shear stress equation is similarly decomposed (eq. 2) and again production due to mean shear \( \text{shear1} \) is the main reason for an increase in Reynolds shear stress (fig. 15). The term \( \text{shear2} \) has a small negative contribution, while production by mean dilatation and extra rate of strain remain equally small. The remarkable increase in production by mean shear is caused

![Image](https://via.placeholder.com/150)

Fig. 1: Axial velocity fluctuations, normalized with local \( \sqrt{\tau_{w}/\mu w} \), in a \((x, r)\)- plane of the diffuser. Amplitudes range from -15 to +15. Flow is from left to right.
by increased radial stress, which in turn increases due to the

dramatic increase in the redistributive pressure-strain cor-

relations (figs.16, 17). Thus the pressure-strain correlations
play a significant role in controlling turbulence production
in this flow. Both pressure fluctuations and strain rate fluc-

tuations are found to increase.

$$P_{xx} = -\mu u_x \frac{\partial u_x}{\partial r} - \frac{1}{3} \mu \frac{\partial u_r}{\partial x} \frac{\partial u_r}{\partial r}$$

Apriori-test of pressure strain models

The pressure-strain correlation can be written as the sum of a 'slow' part proportional to the local turbulence

anisotropy and a 'rapid' part proportional to mean strain

rates. Lai & So (1990) applied a near-wall extension of the

Fig. 2: Bulk quantities.

Fig. 3: Centerline quantities.

Fig. 4: Inner layer quantities.

Fig. 5: Local mean Mach number. $x/L = 0 \ldots$, 0.2(—), 0.35(--), 0.6(---), 0.95(----)

Fig. 6: Mean density, temperature $x/L = 0(\ldots), 0.2(\ldots), 0.35(\ldots)$

Fig. 7: Van Driest transformed velocity $x/L = 0(\ldots), 0.15(--), 0.35(--), 0.6(--), 0.95(----)$

Straight line: $u^+ = 2.5h y^+ + 5.5$
LRR model (Launder et al., 1975) to incompressible pipe flows and found reasonable agreement with DNS data. So et al. (1998) derived a near-wall, variable density extension of the SSG model (Speziale et al., 1991) using Morkovin’s hypothesis and applied this to supersonic boundary layers. In this paper, we use near-wall variable density extensions of both these models to predict the axial pressure-strain correlations using DNS data. The objective is not only to see how these models perform in this complex flow, but also to look at the behaviour of the rapid and slow parts of the pressure-strain correlations.

Following So et al. (1998), the pressure-strain correlations are expressed as

\[(\rho' u'_i j + \rho' u'_j i) = \rho (\Phi_{ij} + \Phi_{j}^w) \tag{3}\]

For ease of notation, we use Cartesian coordinates. The near-wall LRR model for \(\Phi_{ij}, \Phi_{j}^w\) as in Lai & So (1990) are:

\[\Phi_{ij} = -2C_1 \epsilon_s b_{ij} - \alpha (P_{ij} - \frac{2}{3} \delta_{ij} \bar{P}) - \beta (D_{ij} - \frac{2}{3} \delta_{ij} \bar{P}) - 2\gamma k S_{ij} \tag{4}\]

\[\Phi_{j}^w = f_w [2C_1 \epsilon_s b_{ij} - \frac{\epsilon_s}{k} (u_i \bar{u}_j n_1 n_1) + \alpha^* (P_{ij} - \frac{2}{3} \delta_{ij} \bar{P})] \tag{5}\]

The near-wall SSG model for \(\Phi_{ij}, \Phi_{j}^w\) as in So et al.
are observed. The model and a marginal under-prediction by the LRR model are closer to the DNS data. A slight over-prediction by the SSG model and a marginal under-prediction by the LRR model are observed.

The constants used are the same as given in Lai & So (1998). The wall function $f_w$ is given by

$$ f_w = \exp(-\frac{ARe_\theta}{60}) $$

as explained in So et al. (1994).

The axial pressure-strain correlations ($\Pi_{xx}$) given by the LRR and SSG models (eqs. 3 -7) are evaluated using DNS data at two axial locations along the diffuser and compared with $\Pi_{xx}$ obtained from the DNS in figs. 16 and 17. Substantial divergence from the DNS data in the near-wall region is observed although beyond a certain distance from the wall, 1 - $r/R = 0.3$, there is some sort of collapse. At $x/L = 0.2$, both models underpredict the peak in $\Pi_{xx}$ at 1 - $r/R = 0.2$ and show non-physical peaks closer to the wall. At $x/L = 0.35$, the predictions of both the models are closer to the DNS data. A slight over-prediction by the SSG model and a marginal under-prediction by the LRR model are observed.

**CONCLUSIONS**

Effects of a weak adverse pressure gradient on the turbulence structure of supersonic flow in a diffuser with incoming supersonic fully-developed pipe flow is investigated by means of DNS. Large increases in turbulence intensities occur when the flow undergoes deceleration. Turbulence production and pressure-strain correlations are increased. Analysis of the production terms shows that although the extra strain rate and mean compression increase turbulence production, their role is small compared to that of the redistributive pressure-strain correlations whose increase leads to a major increase in turbulence production by mean shear. Both pressure and strain rate fluctuations increase and further analysis is currently being performed to explain this.

Apriori tests of near-wall, variable-density extensions of LRR and SSG models for the axial pressure-strain correlations using DNS data reveal large modifications in both the rapid and slow parts of the models. The prediction of the axial pressure-strain correlations by both the models is found to be reasonable in the core region of the diffuser but needs to be improved in the near-wall region.
**Figures**

18, 19: \( \Pi_{xx} \) in the diffuser at \( x/L = 0.2 \) (left) and \( x/L = 0.35 \) (right).

---, DNS, – – – LRR, – – – SSG.

20, 21: Rapid (left) and slow (right) parts of \( \Pi_{xx} \) in the diffuser. – – – LRR, – – – SSG.

\( x/L = 0.1, 0.2, 0.35 \) from top to bottom of fig.

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**References**


