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INVESTIGATION OF FLUID PARTICLE DISPERSION IN STABLY STRATIFIED TURBULENCE

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ABSTRACT

Numerical simulations are used to study vertical dispersion of fluid particles in homogeneous turbulent flows with a stable stratification (Brethouwer and Lindborg, 2009). The results of direct numerical simulations are in good agreement with the relation for the long time fluid particle dispersion, $\langle \delta z^2 \rangle = 2\varepsilon_P t/N^2$, derived by Lindborg and Brethouwer (2008), though with a small dependence on the buoyancy Reynolds number. Here, $\langle \delta z^2 \rangle$ is the mean square vertical particle displacement, ε_P is the dissipation of potential energy, t is time and N is the Brunt-Väisälä frequency. Simulations with hyperviscosity are performed to verify the relation $\langle \delta z^2 \rangle = (1+\pi C_{PL})2\varepsilon_P t/N^2$ for $N^{-1} \ll t \ll T$, where N is the Brunt-Väisälä frequency and T is the turbulent eddy turnover time. The simulation results approach the relation for increasing stratification and we find that C_{PL} is about 3 in strongly stratified fluids. The onset of a plateau in $\langle \delta z^2 \rangle$ is observed in the simulations at $t \sim T$.

INTRODUCTION

Mixing and dispersion in stratified flows is a topic of utmost importance for environmental and climate processes. Several researchers have examined the vertical dispersion of fluid particles in stratified flows in order to obtain a better understanding of mixing in geophysical flows. Pearson et al. (1983) used a Langevin model to analyze the mean square of vertical fluid particle displacements $\langle \delta z^2 \rangle$ in stationary stratified flows. They predict that $\langle \delta z^2 \rangle$ reaches a plateau with $\langle \delta z^2 \rangle \sim \langle w^2 \rangle/N^2$ at $t \sim N^{-1}$, where w is the vertical velocity fluctuation and N is the Brunt-Väisälä frequency. Furthermore, they predict a linear growth, $\langle \delta z^2 \rangle \sim \langle w^2 \rangle t/N$, at long times when molecular diffusion alters the particle density. Kaneda and Ishida (2000) applied rapid distortion theory to study vertical dispersion in decaying stratified turbulence. They predict a plateau for $\langle \delta z^2 \rangle$ at long times which is consistent with direct numerical simulations (DNS). Nicolleau and Vassilicos (2000), Nicolleau and Yu (2007) and Nicolleau, Yu and Vassilicos (2008) observed $\langle \delta z^2 \rangle \sim E_K/N^2$ (E_K is mean kinetic energy) after long times in stationary stratified turbulence using kinematic simulations (KS). The influence of the changing particle density on the dispersion of the particles was neglected because molecular diffusion is not included in KS. Liechtenstein, Godeferd and Cambon (2005, 2006) used a linear model, KS and DNS to study dispersion in rotating and stratified turbulence. For decaying turbulence they observed that $\langle \delta z^2 \rangle \sim \langle w^2 \rangle/N^2$ after some time. A similar plateau was observed in DNS of decaying stratified turbulence by Kimura and Herring (1996). Venayagamoorthy and Stretch (2006) examined the role of the changing particle density on vertical dispersion. They observed that after about one eddy turnover time diabatic dispersion dominated in their DNS of decaying stratified turbulence, Van Aartrijk, Clercx and Winters (2008) were the first to study particle dispersion in DNS of stationary stratified turbulence. They observed a plateau with $\langle \delta z^2 \rangle \sim \langle w^2 \rangle/N^2$ at $t \sim N^{-1}$. However, some of the DNS showed a linear growth with $\langle \delta z^2 \rangle \sim t$ at long times caused by density changes of fluid particles by molecular diffusion.

RELATIONS FOR THE VERTICAL DISPERSION

In a recent paper we have analysed and derived relations for the vertical dispersion of fluid particles in stratified turbulence (Lindborg and Brethouwer, 2008). Assuming a statistically stationary and stratified homogeneous turbulent flow governed by the Boussinesq equations and integrating the governing equations along a fluid particle trajectory, we derived

$$\langle \delta z^2 \rangle = \frac{2}{N^2}\left[\varepsilon_P t \left(1 - O(\mathcal{R}^{-1/2})\right) + 2E_P\right] \qquad (1)$$

for $t \gg E_P/\varepsilon_P$. Here, ε_P is the dissipation of potential energy, E_P is the potential energy, $\mathcal{R} = \varepsilon_K/\nu N^2$ is the buoyancy Reynolds number, ε_K is the turbulent kinetic energy dissipation and ν is the viscosity. One term has be neglected using scaling arguments. Adiabtic displacements of fluid particles leads to the last term in (1) and gives a finite contribution to long time dispersion because it is constrained by the available energy. Changes of the density of fluid particles by molecular diffusion gives also a contribution to vertical dispersion. This diabatic dispersion contribution is represented by the first term on the right-hand-side of (1) and leads to $\langle \delta z^2 \rangle \sim t$ for $t \to \infty$. In geophysical flows generally $\mathcal{R} \gg 1$ and consequently the $O(\mathcal{R}^{-1/2})$-term in (1) can be neglected. However, in laboratory experiments or numerical simulations this term can give a significant contribution since $\mathcal{R}$ is then not always very large. Relation (1) is expected to be valid when $t \gg T$ where T is an eddy turnover time.

Strongly stratified turbulence with a high Reynolds number has an anisotropic inertial range at scales larger than the Ozmidov length scale (Brethouwer et al., 2007; Lindborg and Brethouwer, 2007). Assuming such an inertial range, we derived

$$\langle \delta z^2 \rangle = \frac{2}{N^2}\varepsilon_P t \left[1 + \pi C_{PL} - O(\mathcal{R}^{-1/2})\right], \qquad (2)$$

for $N^{-1} \ll t \ll E_P/\varepsilon_P$. Using documented observations, Lindborg and Brethouwer (2008) estimated that the constant $C_{PL} \approx 3$. The adiabatic dispersion, $\langle \delta z^2 \rangle = 2\pi C_{PL}\varepsilon_P t/N^2$, gives then the dominant contribution to dispersion in this period.

More background on the analysis and relations for vertical dispersion in decaying stratified turbulence can be found in Lindborg and Brethouwer (2008).

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NUMERICAL SIMULATIONS

The aim of this study is to test relations (1) and (2) for the vertical dispersion of fluid particles by numerical simulations. We have carried out a series of DNS of homogeneous stratified turbulence. In the DNS, a pseudospectral approach with triple periodic boundary conditions is applied to solve the Boussinesq equations. Horizontal vortical modes are forced at horizontal wave numbers $k_h \leqslant 3$. to obtain statistically stationary turbulence. Since the flow is highly anisotropic in strongly stratified flows, we follow the approach taken by Brethouwer et al. (2007) and use computational domains stretched in the horizontal directions. In all DNS $k_{max}\eta \approx 1$, where k_{max} is the largest resolved wave number and η is the Kolmogorov length scale. Simulations with hyperviscosity have also been carried out to test relation (2) in very strongly stratified turbulence. Numerical and physical parameters of the DNS and hyperviscosity simulations are presented in table 1 and 2 respectively.

Four sets of simulations are carried out where the buoyancy Reynolds number $\mathcal{R}$ is varied between the sets but is approximately equal for all simulations within each set, while the Froude number $F_h = \varepsilon_K/(NE_K)$, where E_K is the mean turbulent kinetic energy, is varied. The turbulent Reynolds number is defined as $Re = E_K^2/(\nu\varepsilon_K) = \mathcal{R}F_h^{-2}$. The Prandtl number, $Pr = \nu/\kappa = 0.7$ in all these simulations. The four sets are designated A, B, C and D. In the hyperviscosity simulations $\Delta z/l_O \simeq 7$ as in Lindborg and Brethouwer (2007), where $l_O = \varepsilon_K^{1/2}/N^{3/2}$ is the Ozmidov length scale and Δz the vertical grid spacing.

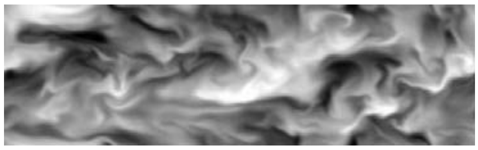
When the flow is statistically stationary, we track 12000 up to 96000 particles with a random initial distribution in the simulations. To obtain the particle velocity $\boldsymbol{u}_p$ at the particle position $\boldsymbol{x}_p$, we employ an interpolation scheme. More details on the numerical approach and forcing can be found in Brethouwer and Lindborg (2009) and Lindborg and Brethouwer (2007).

Table 1: Numerical and physical parameters of the DNS. L_h/L_v is the aspect ratio of the horizontal to vertical domain size and N_h, N_v are the number of nodes in the horizontal and vertical direction, respectively.

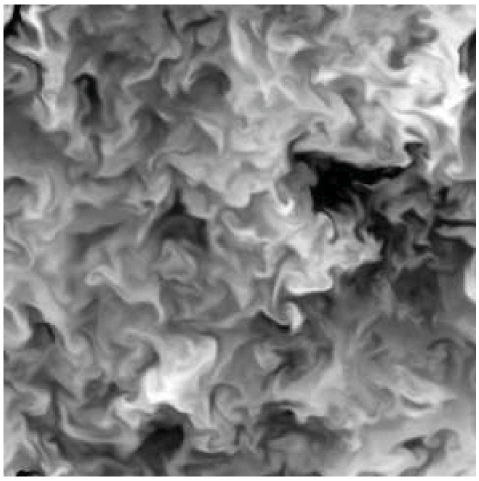
run	Re	F_h	$\mathcal{R}$	$\frac{L_h}{L_v}$	$N_h \times N_v$
A1	1100	0.03	0.9	2.0	128×80
A2	2100	0.02	0.9	3.3	256×96
A3	6300	0.01	0.9	5.0	512×128
B1	1000	0.1	9.3	2.0	128×80
B2	2500	0.06	9.3	3.3	256×96
B3	5500	0.04	9.5	5.0	512×128
B4	14000	0.03	9.9	6.0	1024×256
C	8300	0.07	38	1.0	512×512
D1	1300	2.2	6200	1.0	128×128
D2	2400	1.6	6200	1.0	256×256
D3	8900	0.8	5900	1.0	512×512

Table 2: Numerical and physical parameters of the hyperviscosity simulations.

run	F_h	$\frac{L_h}{L_v}$	$N_h \times N_v$
H1	0.0014	64	512×128
H2	0.0008	72	768×256
H3	0.0005	64	1024×512



(a)



(b)

Figure 1: Snapshots of the buoyancy field in a vertical plane in run B2 (a) and D2 (b).

RESULTS

Figures 1(a) and (b) show snapshots of the buoyancy field of runs B2 and D2, respectively. We can see the typical anisotropic structure in run B2 with strong stratification whereas run D2 with a weak stratification is much more isotropic.

If $\mathcal{R} \gg 1$ the relations (1) and (2) can be written as

$$\langle \delta z^2 \rangle^* = 1 + \frac{1}{2}t^*, \qquad t^* \gtrsim 1, \tag{3}$$

$$\langle \delta z^2 \rangle^* = \frac{1}{2}t^*\left(1 + \pi C_{PL}\right), \qquad F_h \ll t^* \ll 1. \tag{4}$$

Here, $\langle \delta z^2 \rangle^* = \langle \delta z^2 \rangle N^2/4E_P$ and $t^* = t/T$ are the nondimensional mean square of the vertical particle displacements and time respectively. The eddy turnover time is defined by $T = E_P/\varepsilon_P$.

Figure 2 shows the time development of $\langle \delta z^2 \rangle^*$ in the DNS together with relations (3) and (4). DNS with a similar value of $\mathcal{R}$ are grouped in the same plot. The initial period shows ballistic dispersion with $\langle \delta z^2 \rangle \sim t^2$. Thereafter, the growth of $\langle \delta z^2 \rangle$ slows down. The evolution of $\langle \delta z^2 \rangle^*$ should become independent of F_h when $R \gg 1$ and $F_h \ll 1$ according to our analysis. However, the curves in figure 2(a) still show a clear dependence on F_h for $t^* < 1$. The mean square displacement, $\langle \delta z^2 \rangle^*$, moves closer to the straight line representing (4) as F_h decreases, but no linear range is visible. We must conclude that we have to perform simulations with considerably lower F_h to test relation (4). We also see the onset of a plateau at $t^* \sim 1$ in figure 2(a), as expected. Such a plateau has also been observed by van Aartrijk et al. (2008), and indicates that the adiabatic mean square displacement has approached its upper bound $\langle \delta z^2 \rangle = 4E_P/N^2$, i.e. $\langle \delta z^2 \rangle^* = 1$. The adiabatic dispersion regime or the onset of a plateau cannot be seen in DNS results with $F_h \gtrsim 1$ displayed in figure 2(b).

After the slow down of vertical dispersion seen in figure 2(a), $\langle \delta z^2 \rangle^*$ grows faster again and approaches the asymp-

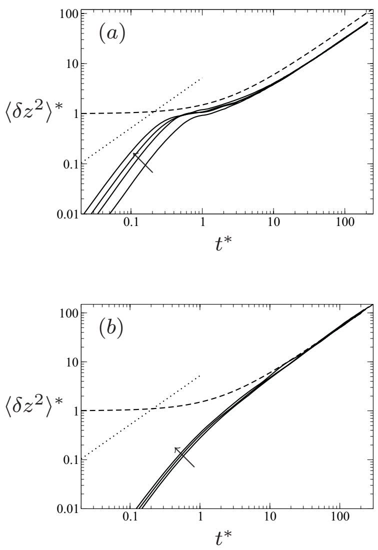


Figure 2: Time development of $\langle \delta z^2 \rangle^*$ vs. t^*. The dashed and dotted lines show relations (3) and (4) respectively, and the solid lines DNS results. The arrow indicates the direction of decreasing F_h or increasing Re. (a) B-runs ($\mathcal{R} \simeq 9$) and (b) D-runs ($\mathcal{R} \simeq 6000$).

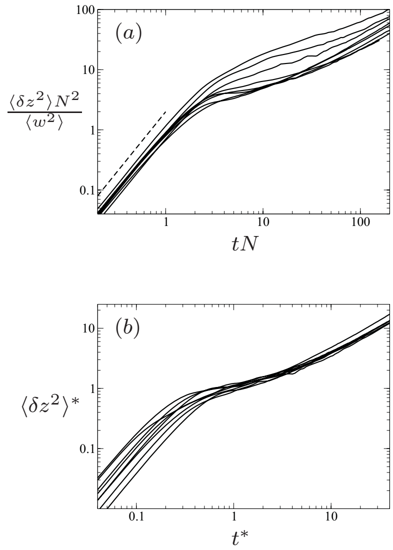


Figure 3: (a) Time development of $\langle \delta z^2 \rangle N^2 / \langle w^2 \rangle$ and (b) $\langle \delta z^2 \rangle^*$. The solid lines are the results of all A, B and C runs. The dashed line in (a) indicates $\langle \delta z^2 \rangle \sim t^2$.

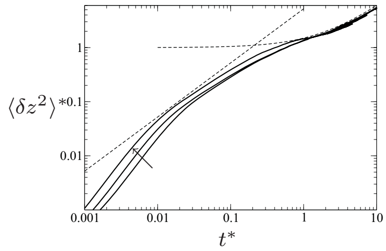


Figure 4: Time development of $\langle \delta z^2 \rangle^*$. The straight and bent thin dashed lines show relations (4) and (3) respectively. In both plots the solid lines show the result of the hyperviscosity simulations. The arrow indicates the direction of decreasing F_h.

totic diabatic dispersion limit (3) with $\langle \delta z^2 \rangle^* \sim t^*$. Noticeable is that the asymptotic diabatic dispersion limit is seen in DNS with strong as well as weak stratification. Furthermore, the plots show the collapse of $\langle \delta z^2 \rangle^*$ for $t^* > 1$ in DNS with approximately equal $\mathcal{R}$. The relation (1) predicts that $\langle \delta z^2 \rangle^* \to t^*/2$ for long times, as $\mathcal{R}$ is increased. We see that the simulation results are consistent with this prediction. Note that the linear growth at late times only can be observed in stationary flows. In decaying stratified turbulence, $\langle \delta z^2 \rangle$ goes to a constant as observed in many DNS (Kimura and Herring, 1996; Kaneda and Ishida, 2000; Venayagamoorthy and Stretch, 2006).

When Re is sufficiently high we can expect molecular diffusivity to have a small influence on the dispersion of particles (Venayagamoorthy and Stretch, 2006). We have carried three DNS with approximately the same F_h and $\mathcal{R}$ but Pr varying from 0.7 to 11.2, see Brethouwer and Lindborg (2009). The influence of Pr appeared to be relatively small if $\mathcal{R} \gg 1$, which illustrates the turbulent nature of the diabatic dispersion. In contrast, diabatic dispersion reduces strongly for increasing Pr in our DNS when $\mathcal{R} < 1$ and small-scale turbulent mixing is mostly absent. This was already shown by van Aartrijk et al. (2008).

In figure 3(a) most of the runs are collected. All runs initially display ballistic dispersion with $\langle \delta z^2 \rangle \approx \langle w^2 \rangle t^2$. Pearson et al. (1983) suggested that adiabatic dispersion should be bounded by $\langle \delta z^2 \rangle \simeq \langle w^2 \rangle / N^2$ and reach this limit at $t \simeq N^{-1}$. This behaviour was observed by van Aartrijk et al. (2008) in their DNS. In our DNS, scaling of $\langle \delta z^2 \rangle$ and t by $\langle w^2 \rangle / N^2$ and N^{-1} respectively does not lead to a collapse of the adiabatic dispersion plateau in the many DNS. The reason why we do not observe this scaling is that our DNS covers the regime $\mathcal{R} \gtrsim 1$ while van Aartrijk et al. (2008) considered the regime $\mathcal{R} \lesssim 1$. Figure 3(b) shows the time development of $\langle \delta z^2 \rangle^*$ for the same runs. In accordance with (3) the onset of the adiabatic dispersion plateau appears when $\langle \delta z^2 \rangle \simeq 4 E_P / N^2$. However, the range of F_h is too limited to firmly determine whether the onset appears when $t \sim E_P / \varepsilon_P$ ($t^* \sim 1$) or when $t \propto N^{-1}$. Nevertheless, it seems that the DNS data are in better agreement with Lindborg and Brethouwer's suggestion. Venayagamoorthy and Stretch (2006) also found that T is the relevant dispersion time scale.

Since the DNS do not reveal a clear inertial stratified turbulence range, we have carried out additional hyperviscosity simulations. Stratified turbulence features in hyperviscosity simulations, having a well defined inertial range, have extensively been examined by Lindborg (2006), Lindborg and Brethouwer (2007) and Brethouwer and Lindborg (2008). Figure 4 shows the time development of $\langle \delta z^2 \rangle^*$ in the hyperviscosity simulations together with relations (3) and (4). For increasing stratification, $\langle \delta z^2 \rangle$ moves closer to expression (4) for the adiabatic dispersion in the inertial range of stratified turbulence with $C_{PL} = 3$ for $N^{-1} \ll t \ll T$, but there is no extended range where it matches the relation. We can only speculate that it may require very extended inertial ranges as in geophysical flows to observe the behaviour expressed by (4).

CONCLUSIONS

We have used DNS and numerical simulations with hyperviscosity to examine vertical fluid particle dispersion in stationary stratified homogeneous turbulent flows (Brethouwer and Lindborg, 2009). The DNS are in good agreement with relation (1) for the mean square of the vertical fluid displacements $\langle \delta z^2 \rangle$ derived by Lindborg and Brethouwer (2008). For increasing stratification the adiabatic dispersion contribution moves closer to Lindborg and Brethouwer's suggestion $2\pi C_{PL} \varepsilon_P t/N^2$ with $C_{PL} \sim 3$ for $N^{-1} \ll t \ll T$, according to the simulations with hyperviscosity. However, the growth of $\langle \delta z^2 \rangle$ is somewhat slower than linear even in the simulation with the strongest stratification. At about $t \approx T$ we see the onset of a plateau since the adiabatic dispersion reaches its upper bound $4E_P/N^2$. In all DNS, spanning a quite extended range of Froude and Reynolds numbers, $\langle \delta z^2 \rangle$ approaches $2\varepsilon_P t/N^2$ in the long time limit. This linear growth of the vertical mean square displacement of fluid particles suggests that the vertical eddy diffusivity of stratified turbulence can be calculated as

$$K_\varepsilon = \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\langle \delta z^2 \rangle = \frac{\varepsilon_P}{N^2} \tag{5}$$

in the long time limit. The statistical mechanical argument which is the basis for the expression (5) suggests that this expression is the eddy diffusivity for any scalar which is following fluid particles. The expression (5) is equivalent to the Osborn (1980) expression for the eddy diffusivity of the buoyancy. However, Osborn's analysis leads to different expressions for the eddy diffusivity of different scalars.

REFERENCES

van Aartrijk M. , Clercx H. J. H. and Winters K. B., 2008, "Single-particle, particle-pair, and multiparticle dispersion of fluid particles in forced stably stratified turbulence", *Phys. Fluids*, Vol. 20, 025104.

Brethouwer G., Billant P., Lindborg E. and Chomaz J.-M., 2007, "Scaling analysis and simulation of strongly stratified turbulent flows", *J. Fluid Mech.*, Vol. 585, pp. 343-368.

Brethouwer G., and Lindborg, E., 2009, "Numerical study of vertical dispersion by stratified turbulence", *J. Fluid Mech.*, in press.

Kaneda, Y., and Ishida, T., 2000, "Suppression of vertical diffusion in strongly stratified turbulence". *J. Fluid Mech.*, Vol. 402, pp. 311-327.

Kimura, Y., and Herring, J. R., 1996, "Diffusion in stably stratified turbulence". *J. Fluid Mech.*, Vol. 328, pp. 253-269.

Liechtenstein, L., Godeferd, F. S., and Cambon, C., 2005, "Nonlinear formation of structures in rotating stratified turbulence". *J. Turbul.*, Vol. 6, pp. 1-18.

Liechtenstein, L., Godeferd, F. S., and Cambon, C., 2006, "The role of nonlinearity in turbulent diffusion models for stably stratified and rotating turbulence". *Int. J. Heat and Fluid Flow*, Vol. 27, pp. 644-652.

Lindborg, E., and Brethouwer, G., 2007, "Stratified turbulence forced in rotational and divergent modes". *J. Fluid Mech.*, Vol. 586, pp. 83-108.

Lindborg, E., and Brethouwer, G., 2008, "Vertical dispersion by stratified turbulence", *J. Fluid Mech.*, Vol. 614, pp. 303-314.

Nicolleau, F., and Vassilicos, J. C., 2000, "Turbulent diffusion in stably stratified non-decaying turbulence". *J. Fluid Mech.*, Vol. 410, pp. 123-146.

Nicolleau, F., Yu, G., 2007, "Turbulence with combined stratification and rotation: Limitations of Corrsin's hypothesis". *Phys. Rev. E*, Vol.76, 066302.

Nicolleau, F., Yu, G., and Vassilicos, J. C., 2008, "Kinematic simulation for stably stratified and rotating turbulence". *Fluid Dyn. Res.*, Vol. 40, pp. 68-93.

Osborn, T. R., 1980, "Estimates of the local rate of vertical diffusion from dissipation measurements". *J. Phys. Oceanogr.*, Vol. 10, pp. 83-89.

Pearson, H. J., Puttock, J. S., and Hunt, J. C. R., 1983, "A statistical model of fluid-element motions and vertical diffusion in a homogeneous stratified turbulent flow". *J. Fluid Mech.*, Vol. 129, pp. 219-249.

Venayagamoorthy, S. K., and Stretch, D. D., 2006, "Lagrangian mixing in decaying stably stratified turbulence". *J. Fluid Mech.*, Vol. 564, pp. 197-226.