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SGS MODELLING IN LES OF WALL-BOUNDED FLOWS USING TRANSPORT RANS MODELS: FROM A ZONAL TO A SEAMLESS HYBRID LES/RANS METHOD

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ABSTRACT

Employment of an advanced, Reynolds-stress anisotropy-accounting RANS (Reynolds-Averaged Navier-Stokes) model in the hybrid RANS/LES (Large Eddy Simulation) framework was investigated in the present work. The model in question is the four-equation, near-wall elliptic-relaxation eddy-viscosity model, solving the transport equations for $\zeta = \overline{v^2}/k$ (with $\overline{v^2}$ representing a scalar variable reducing to the normal-to-the-wall Reynolds stress component by approaching the solid wall) and a corresponding elliptic function f in addition to the k and ε-equations. This model, proposed by Hanjalic et al. (2004), is employed in both hybrid LES/RANS strategies tackled in the present work: a zonal and a seamless one. In the former method the k-ε-ζ-f model resolving the near-wall region is bridged at a discrete, dynamically determined interface with the conventional LES covering the outer layer (flow core). In the framework of the seamless method the k-ε-ζ-f model mimics a subgrid-scale (SGS) model in the entire flow domain in line with the PITM (Partially-Integrated Transport Model) procedure proposed by Chaouat and Schiestel (2005, 2007). The feasibility of both computational schemes is assessed in the flow over periodically arranged hills (Fröhlich et al., 2005; Breuer, 2005) featuring flow separation from a continuous curved surface. The results obtained by both hybrid LES/RANS methods exhibit reasonable agreement with reference database especially in the separated shear layer region contrary to their RANS counterpart.

INTRODUCTION

Large majority of subgrid-scale (SGS) models represents an adaptation of the eddy viscosity concept. Hereby, the most widely applied model, whose representative length ($\sim \Delta$) and velocity scales ($\sim \Delta \overline{S}$) are modelled in terms of the mesh size Δ (grid scale): $\nu_{SGS} = (C_S \Delta)^2 \overline{S}$ is proposed by Smagorinsky (1963). However, the Smagorinsky model performs poorly if an irregular, highly non-uniform grid is to be used. Such non-uniform grids with higher aspect ratio, especially in the wall vicinity, are usually encountered in everyday Computational Fluid Dynamics. A further weakness of this model is the determination of the constant C_S, which requires different values for different flows, even for different flow regions. This is especially inconvenient if new flow configurations (with no reference data existing) are to be computed. This problem was solved by introducing a second, so-called test filter in addition to the implicit grid filter in the framework of the dynamic determination (localization) of the coefficient C_S, Germano et al. (1991). Similar as the Smagorinsky model, its dynamic version also assumes the cutoff wave number corresponding to the inertial region, the condition being not fulfilled in the case of the industrially relevant coarser grids, where the position of the spectral cutoff usually coincides with the very beginning of the inertial subrange. In spite of great progress, the opinion is frequently expressed that the LES method will not reach the standard of the industrially relevant numerical tool in the near future. The most important reason for such a situation, beside the afore-mentioned issues, the uncertainties with the definition of inflow conditions and other numerically relevant problems (e.g., Moin, 2002), is the treatment of the near-wall regions. The near-wall resolution becomes progressively important with the Reynolds number in accordance to the relationship $N \sim \mathrm{Re}^{1.76}$ (compared to $N \sim \mathrm{Re}^{0.4}$ in the off-wall region), resulting in the fact, that almost *50%* of the total number of the numerical nodes should be situated in the viscous sublayer and the buffer layer, Pope (2000). All that makes a wall-resolved LES too costly. Furthermore, a high cell aspect ratio of a typical near-wall grid implies a highly anisotropic grid in this region. Similar situation is encountered also in off-wall regions, as e.g. in the regions of shear (mixing) layers and recirculation zones, both phenomena being massively encountered in separating flow configurations. Among other effects, the anisotropy of turbulence is often the consequence of such an anisotropic grid, being presented also in the flow configurations where the local isotropy assumptions are valid. Keeping in mind the fact that the anisotropies in the turbulent flow not resolved by the grid represent the anisotropies of the SGS motion, it becomes clear that Smagorynski-like models, being inherently isotropic, are regarded as inappropriate to deal with anisotropic meshes and the associated turbulence. Carati and Cabot (1996) initiated a discussion about necessity for an anisotropic eddy-viscosity model, which should be generally based on a 4-th order eddy-viscosity tensor, simplified by using tensor and flow symmetries.

The work here reported aims at formulating a hybrid LES/RANS model in both zonal and seamless frameworks using an advanced anisotropy-reflecting RANS model based on the eddy-viscosity concept. The goal is to achieve the accuracy comparable to that of a conventional LES, but at much lower computational costs.

COMPUTATIONAL MODEL

The three-dimensional, incompressible unsteady equations governing the velocity field read

$$\frac{\partial \bar{U}_i}{\partial t}+\frac{\partial\left(\bar{U}_i\bar{U}_j\right)}{\partial x_j}=-\frac{1}{\rho}\frac{\partial \bar{p}^*}{\partial x_i}+\frac{\partial}{\partial x_j}\left[(\nu+\nu_m)\left(\frac{\partial \bar{U}_i}{\partial x_j}\right)\right] \quad (1)$$

Depending on the method applied the eddy-viscosity ν_m originates from the model formulation either for the subgrid-stress tensor τ_{ij} or for the Reynolds-stress tensor $\overline{u_i u_j}$. Both tensors are expressed in terms of the mean strain tensor $\bar{S}_{ij}$ via the Boussinesq relationship (see next subsections). The rationale and the most important features of the turbulence model used in the present work are outlined in the following subsections.

k-ε-ζ-f RANS model

This is a new and more robust variant of the Durbin's $\overline{\upsilon^2}-f$ model (1991), proposed recently by Hanjalic et al. (2004). The model relies on the elliptic relaxation (ER) concept providing a continuous modification of the homogeneous pressure-strain process as the wall is approached to satisfy the wall conditions, thus avoiding the need for any wall topography parameter. This model approach represents a further contribution towards more robust use of advanced closure models. The variable ζ represents the ratio $\overline{\upsilon^2}/k$ ($\overline{\upsilon^2}$ is a scalar property which reduces to the wall-normal stress in the near-wall region) providing more convenient formulation of the equation for ζ and especially of the wall boundary conditions for the elliptic function *f*. The corresponding eddy-viscosity is defined as:

$$\nu_m=\nu_t=C_\mu \zeta k\tau \quad (2)$$

with ζ obtained from the following equation:

$$\frac{D\zeta}{Dt}=f-\frac{\zeta}{k}P_k+\frac{\partial}{\partial x_j}\left[\left(\nu+\frac{\nu_t}{\sigma_\zeta}\right)\frac{\partial \zeta}{\partial x_j}\right] \quad (3)$$

τ represents the switch between the Kolmogorov time scale and the turbulent time scale:

$$\tau=\max\left[\min\left(\frac{k}{\varepsilon},\frac{a}{\sqrt{6}C_\mu S\zeta}\right),C_\tau\left(\frac{\nu}{\varepsilon}\right)^{1/2}\right] \quad (4)$$

The coefficients appearing in Equations (3) and (4) take the values $a=0.6$, $C_\mu=0.22$ and $C_\tau=6.0$. The corresponding *k* and ε equations take the following form:

$$\frac{Dk}{Dt}=\left(P_k-\varepsilon\right)+\frac{\partial}{\partial x_j}\left[\left(\nu+\frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] \quad (5)$$

$$\frac{D\varepsilon}{Dt}=\frac{C_{\varepsilon 1}P_k-C_{\varepsilon 2}\varepsilon}{\tau}+\frac{\partial}{\partial x_j}\left[\left(\nu+\frac{\nu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_j}\right]. \quad (6)$$

with $P_k=-\overline{u_i u_j}\,\partial U_i/\partial x_j$. These equations are solved in conjunction with an equation governing the elliptic relaxation function f which is formulated by utilizing the pressure-strain model of Speziale et al. (SSG; 1991):

$$L^2\nabla^2 f-f=\frac{1}{\tau}\left(C_1+C_2'\frac{P_k}{\varepsilon}\right)\left(\zeta-\frac{2}{3}\right)-\left(\frac{C_4}{3}-C_5\right)\frac{P_k}{k} \quad (7)$$

where f takes zero value at the wall $f_{wall}=\lim_{y\to 0}(-2\nu\zeta/y^2)$. The appropriate length scale L is obtained from:

$$L=C_L\max\left(\min\left(\frac{k^{3/2}}{\varepsilon},\frac{k^{1/2}}{\sqrt{6}C_\mu S\zeta}\right),C_\eta\left(\frac{\nu^3}{\varepsilon}\right)^{1/4}\right) \quad (8)$$

Hanjalic et al. (2004) neglected the last term in Eq. (7) due to small values of $(C_4/3-C_5)\approx 0.008$ and decreased the C_2' coefficient from its original SSG value 0.9 to 0.65 in order to take into account the discrepancy in the definition of ε in the log-law region. The readers interesting in more details are referred to the original publication of Hanjalic et al. (2004).

Zonal hybrid LES/RANS (HLR) k-ε-ζ-f model

In this method the *k-ε-ζ-f* RANS model (used in its original form) covers the near-wall region and the LES using the Standard Smagorinsky SGS model the remainder of the flow domain. The equations of motion (1) operate as the Reynolds-averaged Navier-Stokes equations in the near-wall layer ($\bar{U}_i$ represents the ensemble-averaged velocity field) or as the filtered Navier-Stokes equations in the outer layer ($\bar{U}_i$ represents the spatially filtered velocity field). Both sub-regions share the same temporal resolution. The coupling of the instantaneous LES field and the ensemble-averaged RANS field at the interface is realised via the turbulent viscosity, which makes it possible to obtain solutions using one system of equations. This means practically that Eqs. (1) are solved in the entire solution domain irrespective of the flow sub-region (LES or RANS). Depending on the flow zone, the hybrid method implies the determination of the turbulent viscosity either from the RANS model (Eq. 2) or from the LES formulation:

$$\nu_m = \nu_{SGS} = (C_S\Delta)^2 \left|\overline{S}\right| \tag{9}$$

The Smagorinsky constant C_S takes the value of 0.1. $\Delta = (\Delta x \times \Delta y \times \Delta z)^{1/3}$ represents the filter width and $\left|\overline{S}\right| = \left(\overline{S}_{ij}\overline{S}_{ij}\right)^{1/2}$ the strain rate modulus. Because k and ε (as well as f and ζ, i.e. $\overline{v^2}$) are not provided within the LES sub-domain, it is to estimate their SGS values at the interface (to be used as the boundary conditions for the RANS sub-domain) using the proposal of Mason and Callen (1986):

$$k_{SGS} = (C_S\Delta)^2 \left|\overline{S}\right|^2 \Big/ 0.3 \qquad \varepsilon_{SGS} = (C_S\Delta)^2 \left|\overline{S}\right|^3 \tag{10}$$

Different strategies with respect to the determination of f and ζ in the LES sub-regions were tested. Finally, both equations were solved in the entire flow domain independent of the computational region, i.e. interface. The RANS equations k and ε are solved in the entire flow field, but with the discretization coefficients taking zero values in the LES sub-region. By manipulating appropriately the source terms (in line with the well-known procedure for setting the value of a variable at a computational node), the numerical solution of these equations in the framework of the finite volume method provides the interface values of the k_{RANS} and ε_{RANS} being equal to the corresponding SGS values. By doing so, the boundary condition at the LES/RANS interface (*ifce*) implying the equality of the modelled turbulent viscosities (by assuming the continuity of their resolved contributions across the interface, Temmerman et al., 2005) at both sides of the interface:

$$\nu_{t,ifce}\,|_{RANS-side} = \nu_{SGS,ifce}\,|_{LES-side} \tag{11}$$

is implicitly imposed without any further adjustment, see Fig. 1 for illustration. In such a way a smooth transition of the turbulent viscosity is ensured.

One of the advantages of a zonal approach is the possibility to predefine the LES-RANS interface. However, in unknown flow configurations, this could be a difficult issue. Therefore, a certain criteria expressed in terms of a control parameter should be introduced. Presently, following control parameter

$$k^* = \left\langle \frac{k_{\text{mod}}}{k_{\text{mod}} + k_{res}} \right\rangle \tag{12}$$

is adopted, representing the ratio of the modelled (SGS) to the total turbulent kinetic energy in the LES region, averaged over all grid cells bordering the interface on the LES side. Thus, the interface will be positioned in accordance with the requirement for the k^*-parameter, i.e. for the modelled turbulent kinetic energy, to stay under a certain limit which is to be prescribed. A typical value of 20% was adopted in the present work, corresponding approximately to the reference value an LES resolution should comply with (Pope, 2000). As soon as the k^*-value exceeds 20 % (implying a coarser grid at this position, being inadequate for LES to be applied), the interface is moved farther from the wall (the RANS sub-region will be enlarged) and in the opposite direction when the value goes below 20 % (denoting a finer grid at the respective position being suitable for LES to be appropriately applied). This additionally ensures that in the limit of a very fine grid (very low level of the residual turbulence) LES is performed in the most of the solution domain. Contrary, in the case of a coarse grid, RANS prevails. As the interface separates the near wall region from the reminder of the flow, it would be suitable to choose a wall-defined parameter for denoting the interface location. In the present study, the dimensionless wall distance y^+ was adopted. It is noted, that the interface y^+ is not active in the computational procedure. It only denotes the computational nodes, at which the prescribed value of k^* was obtained. Fig. 2 displays the time evolution of the interface position in the periodic flow over a 2-D hill, averaged over all cells at the interface. The final position of the interface obtained at the end of this computation corresponds to y^+=*60*.

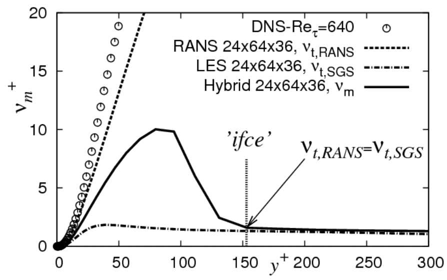

Figure 1: Variation of the modelled turbulent viscosity across the interface ('*ifce*') in a fully-developed channel flow (DNS: Abe et al., 2004)

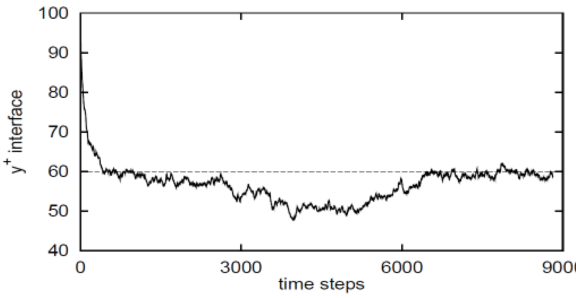

Figure 2: Temporal variation of the interface position in terms of dimensional wall distance y^+

More details about zonal hybrid LES/RANS procedure including numerical realizations of individual steps can be found in Jakirlic et al. (2009).

Seamless hybrid LES/RANS (PITM) k-ε-ζ-f model

In this method the k-ε-ζ-f RANS model has been employed in the entire flow domain modelling the unresolved subgrid-scale viscosity, whereas the large grid-scale motion is fully resolved in accordance with the appropriately adopted time and space resolution as in the conventional LES framework. Such a model is expected to overcome deficiencies of the inherently isotropic, Smagorinsky related subgrid-scale models. The model should be capable of dealing with mesh and turbulence anisotropies present in complex flows when employing coarser space (where an extremely anisotropic grid is typically used for its discretization) and time resolutions. Starting from the original k-ε-ζ-f model scheme, modifications of the relevant terms in the transport equations for corresponding scale-supplying equation were made in line with the PITM (Partially Integrated Transport Model) method proposed by Chaouat and Schiestel (2005, 2007). The coefficients in the destruction term of the dissipation rate equation were made grid- (filter width) dependent, as they were made dependent on the location of the spectral cutoff, by applying a multiscale modelling procedure originating from spectral splitting of filtered turbulence. Here, more general expression for model coefficients based on analytically formulated energy spectrum $E(\kappa)$, which is valid for both large and small eddy ranges (Chaouat and Schiestel, 2007; TSFP5), was adopted. The form of the functional dependency in the model coefficient multiplying the destruction term in the scale-supplying equation is of decisive importance: $C_{\varepsilon_2,SGS} = C_{\varepsilon_1} + \left(C_{\varepsilon_2} - C_{\varepsilon_1}\right) \Big/ \left(1 + \beta_\eta \eta_C^3\right)^{2/9}$, with $C_{\varepsilon_1} = 1.44$, $C_{\varepsilon_2} = 1.92$ and $\beta_\eta = \left(2C_K/3\right)^{9/2}$, $C_K = 0.36$ (see Fig. 5 for its graphical illustration). It should be noted that the value $C_K = 1$ was used by Chaouat and Schiestel in their PITM model employing a near-wall Second-Moment Closure model. The dimensionless parameter $\eta_C = \kappa_C L_e$ ($\kappa_C = \pi/\Delta$; $\Delta = \left(\Delta x \times \Delta y \times \Delta z\right)^{1/3}$) involves the turbulent length scale $L_e = k_{tot}^{3/2}/\varepsilon_{tot}$ built by means of total turbulent kinetic energy and corresponding total dissipation rate including its resolved part but also the SGS fraction. Such a model coefficient form provides a dissipation rate level (the value $C_{\varepsilon_2} = 1.92$ prevailing in the near-wall region reduces towards the value $C_{\varepsilon_1} = 1.44$ in the core region) which suppresses the turbulence intensity towards the subgrid (i.e. subscale) level in the region where large coherent structures with a broader spectrum dominate the flow, allowing in such a way evolution of structural features of the associated turbulence. Herewith, a seamless coupling (with no discrete interface), i.e. a smooth transition from LES (generally covering the core flow) to RANS (capturing in general the wall vicinity) and opposite is enabled. The model proposed functions as a subgrid-scale model in the LES region and as a RANS model in the RANS region.

RESULTS AND DISCUSSION

Some selected results (Figures 3-10) obtained by computing the periodic flow over a 2D hill (Re_h=10595; LES: Breuer, 2005 – 13 million) exhibiting a number of features typically associated with the highly-unsteady shear layer that separates the main stream from the recirculation (Fig. 3), demonstrate that both hybrid LES/RANS models presented (denoted by PITM- k-ε-ζ-f and HLR- k-ε-ζ-f) is capable of representing the subgrid-stress transport in the framework of LES in the flows dominated by the organized, large-scale coherent structures, influencing to a large extent the overall flow behaviour. In addition to hybrid models, the results of the 3-D computations using RANS- k-ε-ζ-f models are also displayed for the sake of their comparative assessment. The computational grid used (Fig. 4) comprises only 250.000 (80x100x32) computational cells, which is about eighteen times coarser than the grid used for the reference LES (Breuer, 2005). Fig. 6 compares the wall shear stress evolution, from which the separation ($(x_S/h)_{LES}=0.22$) and reattachment points ($(x_R/h)_{LES}=4.72$) locations can be determined. Both hybrid schemes resulted in the reattachment lengths being fairly close to the reference one unlike the pure RANS model overpredicting it ($(x_R/h)_{LES}=5.2$). The results shown in Figs. 7-10 document the capability of reproducing the vortex structure by both hybrid models in the separated shear layer (being beyond the reach of any RANS model), Fig. 8, which consequently led to a proper shape and magnitude of the turbulence kinetic energy and shear stress profiles (Figs. 10) and correct velocity profile evolution (Fig. 7). Low intensity of the modelled turbulence (Fig. 10) corresponding to the subgrid-scale level was of crucial importance in capturing the structural flow features.

CONCLUSIONS

Zonal and seamless hybrid LES/RANS schemes utilizing the same near-wall RANS model (representing the Hanjalic's et al. four-equation, elliptic-relaxation eddy-viscosity model) were comparatively analysed in computing the flow over a periodic hill at moderate Reynolds number. Promising results obtained by both schemes with respect to the structural characteristics of the instantaneous flow field, the mean velocity field and associated integral parameters (wall shear stress) as well as the turbulence quantities demonstrate their feasibility. The results obtained on the substantially coarser grid than that of the fine LES follow closely the reference LES results.

REFERENCES

Breuer, M., 2005, "New Reference Data for the Hill Flow Test Case", http://www.hy.bv.tum.de/DFG-CNRS/

Chaouat, B. and Schiestel, R., 2005, "A new partially integrated transport model for subgrid-scale stresses and dissipation rate for turbulent developing flows", *Phys. Fluids*, Vol. 17 (065106), pp. 1-19

Fröhlich, J., Mellen, C.P., Rodi, W., Temmerman, L., and Leschziner, M.A., 2005, "Highly resolved large-eddy simulation of separated flow in a channel with streamwise periodic constrictions", *J. Fluid Mech.*, Vol. 526, pp. 19-66

Hanjalic, K., Popovac, M. and Hadziabdic, M., 2004, "A robust near-wall elliptic-relaxation eddy-viscosity turbulence model for CFD", *Int. J. of Heat and Fluid Flow*, Vol. 25, pp. 1047-1051

Jakirlić, S., Kniesner, B., Kadavelil, G., Gnirß, M. and Tropea, C., 2009, "Experimental and computational investigations of flow in a single-annular combustor configuration”, *Flow, Turbulence and Combustion* (under consideration for publication)

Moin, P., 2002, “Advances for Large-Eddy Simulation methodology for complex flows”, *Int. J. Heat and Fluid Flow*, Vol. 23(5), pp. 710-720

Pope, S., 2000, “Turbulent flows”, Cambridge University Press

Temmerman, L., Hadžiabdić, M., Leschziner, M.A., and Hanjalić, K., 2005, “A hybrid two-layer URANS-LES approach for large eddy simulation at high Reynolds numbers”, *Int. J. Heat and Fluid Flow*, Vol. 26, pp. 173-190

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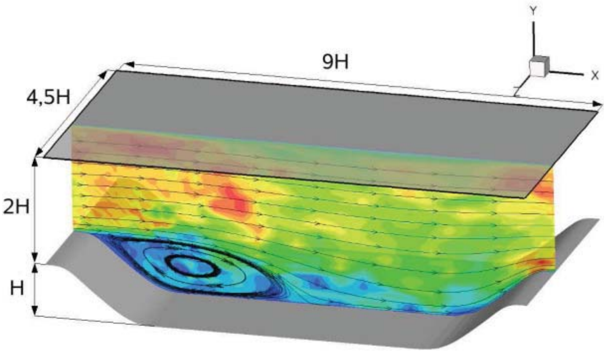

Figure 3: Flow domain and topology

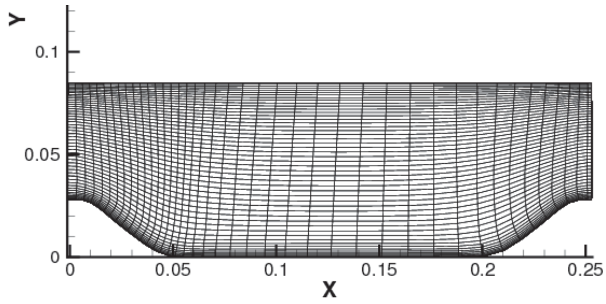

Figure 4: Computational grid used

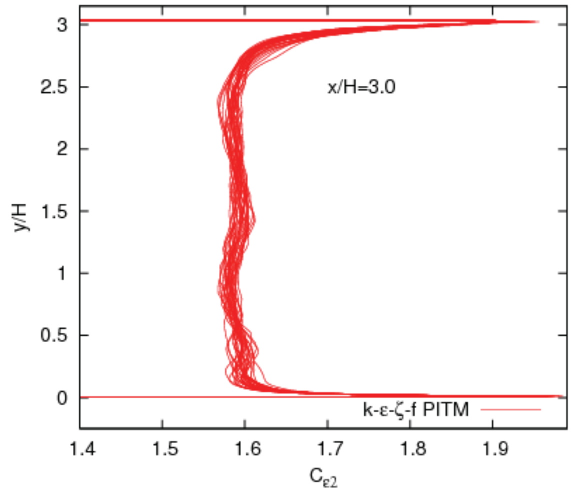

Figure 5: Time-dependent profiles of the $C_{\varepsilon 2,SGS}$-coefficient at a location crossing the separation bubble

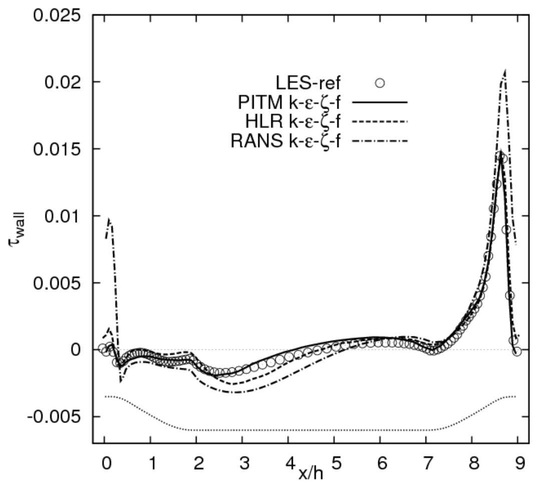

Figure 6: Wall shear stress evolution along the lower channel wall

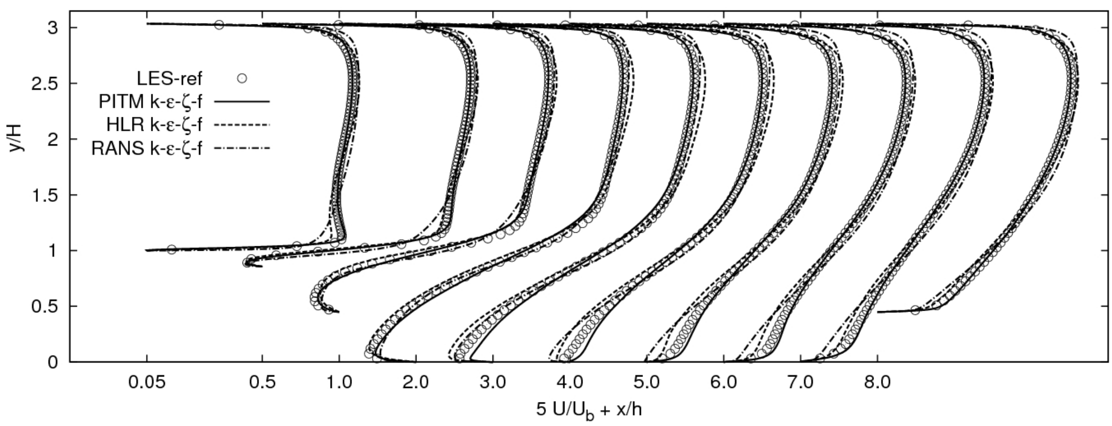

Figure 7: Evolution of the mean axial velocity profiles in the periodic flow over a 2-D hill

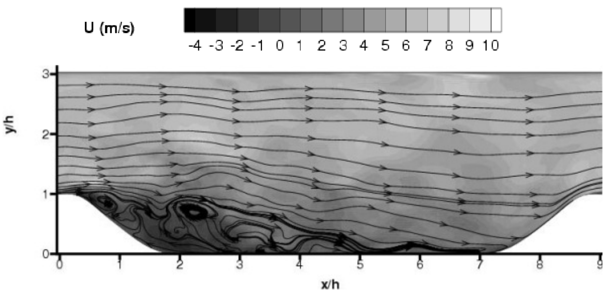

Figure 8: Instantaneous velocity field and associated streamlines obtained by the PITM-k-ε-ζ-f model

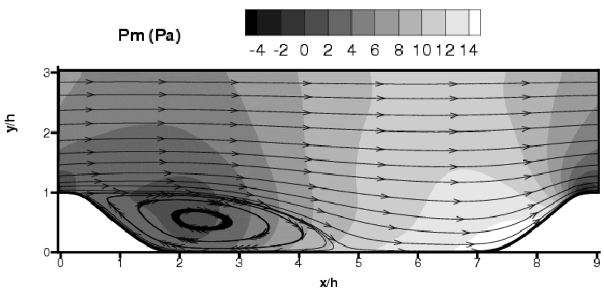

Figure 9: Mean pressure field and associated time-averaged streamlines obtained by the HLR-k-ε-ζ-f model ($(x_R/h)=4.71$)

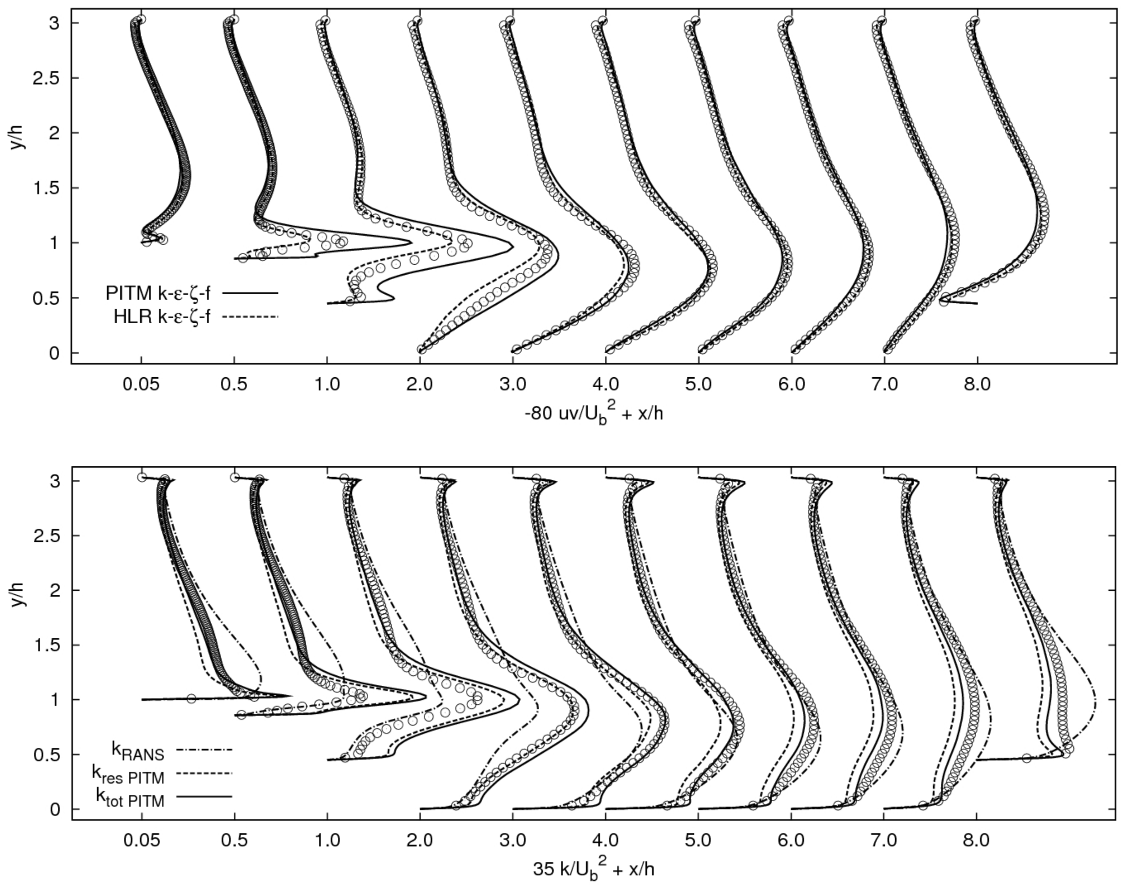

Figure 10: Evolution of the shear stress and kinetic energy profiles in the periodic flow over a 2-D hill