SIMULATION AND COMPARISON OF VARIABLE DENSITY ROUND AND
PLANE JETS.

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ABSTRACT
Large-eddy simulations of plane and round variable density jets, as well as direct numerical simulations of plane jets were conducted using a variety of density ratios \( s = \rho_j/\rho_{co} \), which relates the jet nozzle density \( \rho_j \) to the freestream density \( \rho_{co} \). The initial momentum flux was kept constant for better comparison of the resulting data. Both simulations confirm experimental results, in that the jet half-width grows linearly with streamwise coordinate \( x \) and the lighter jets decay much faster than the heavy ones. The centerline velocity decay is however different between the plane and round geometries. Whereas the round jets exhibits a decay with \( 1/x \) for all density ratios, there seem to be two self-similar scalings in plane jets, in the limit of small and large density ratios. In the limit of small density ratios or incompressible flow, \( U_c \) scales as \( U_c \sim 1/\sqrt{\tau} \), for strongly heated jets on the other hand we find \( U_c \sim \sqrt{\rho_{co} / (\rho_j \rho_{co})} \sim 1/x \). Using nondimensional values for \( x \) and \( U_c \) (Chen and Rodi [1980]) collapses the round jet data. Furthermore, the streamwise growth in mean density or the decay of the velocity fluctuations in the self-similar region is stronger for round jets. The round jet simulation with a density ratio of \( s = 0.14 \) shows additionally a global instability, whose frequency agrees excellently with experimental data.

INTRODUCTION
Variable density flows exhibit a range of interesting physical phenomena and arise in a variety of circumstances, e.g. during the mixing process between flows of different densities, during combustion processes or plasma processing. One phenomenon extensively studied in the past experimentally is the occurrence of global instabilities in variable-density jets (e.g. Monkewitz et al. [1990], Kyle and Sreenivasan [1993], Raynal et al. [1996], Hallberg and Strykowski [2006], Lesshaft et al. [2006], Wang et al. [2008]) or DNS (Lesshaft et al. [2006], 2D-DNS). However, there is no detailed investigation comparing plane and round jets at various density ratios \( s = \rho_j/\rho_{co} \) (the subscript “\( j \)” indicates values at the virtual nozzle exit). Furthermore, a DNS database would be very valuable, since it would enable us to test LES and Reynolds averaged models and provide insight into important physical mechanisms. Another import aspect, as mentioned above, is the occurrence of global instabilities (Kyle and Sreenivasan [1993]), reported to occur if the ratio of the exiting nozzle fluid density to ambient fluid density is \( \rho_j/\rho_{co} < 0.6 \). Above that ratio, the shear-layer fluctuations evolve in a fashion similar to that observed in a constant-density jet, characterized by weak background disturbances. Below that threshold on the other hand, intense oscillatory instabilities may also arise. Since the oscillatory mode was shown to repeat itself with extreme regularity and is subject to strong vortex pairing, abnormally large velocity fluctuations were observed, requiring a large number of grid points. The overall behavior depends not only on the density ratio and the relation of the shear layer thickness to the nozzle width, but also on the Reynolds number (Hallberg and Strykowski [2006]).

DETAILS OF THE SIMULATIONS
The filtered compressible equations of motion are solved in cylindrical \((r, \theta, z)\) coordinates for the round jet using the compressible form of the dynamic Smagorinsky model (Martin et al. [2000]). Treatment of the centerline singularity at \( r = 0 \) is accomplished by using the centerline treatment according to Constantinou and Lele [2002], where a series expansion is used for the equations at the centerline. For time integration a low-dispersion-dissipation Runge-Kutta scheme of (Hu et al. [1996]) in its low storage form is used. Spatial differentiation uses optimized DRP-SBP (dispersion-relation-preserving summation by parts) explicit finite-difference operators (Johansson [2004]). The code employed for the plane jet has been derived from Stanley et al.
and is based on sixth-order compact Padé schemes for space derivatives and a Runge-Kutta fourth-order scheme for time advancement. Tables 1 and 2 give an overview over the different simulation parameters. The initial momentum thickness \( \delta_{0} \), initial momentum thickness \( m_{j} \), \( L_{j} \), normalized by \( r_{j} \), and \( D \) (slot width) for the round and plane jet, respectively. For the plane jet LES see Mellado [2004] for details.

### Table 1: Parameters of the simulations (\( Re = u_{\tau} \rho / \nu \), initial momentum thickness \( \delta_{0} \), initial momentum dimensionless flux \( m_{j} \), \( L_{j} \), normalized by \( r_{j} \), and \( D \) (slot width) for the round and plane jet, respectively).

<table>
<thead>
<tr>
<th>Case</th>
<th>( Re )</th>
<th>( D / \delta_{0} )</th>
<th>( s )</th>
<th>( m_{j} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>R014</td>
<td>7000</td>
<td>27</td>
<td>0.14</td>
<td>0.1</td>
</tr>
<tr>
<td>R100</td>
<td>21000</td>
<td>27</td>
<td>1.00</td>
<td>0.1</td>
</tr>
<tr>
<td>R152</td>
<td>32000</td>
<td>27</td>
<td>1.52</td>
<td>0.1</td>
</tr>
<tr>
<td>DNS PD050</td>
<td>1414</td>
<td>20</td>
<td>0.50</td>
<td>0.1</td>
</tr>
<tr>
<td>DNS PD100</td>
<td>2000</td>
<td>20</td>
<td>1.00</td>
<td>0.1</td>
</tr>
<tr>
<td>Plane jet LES</td>
<td>( \infty )</td>
<td>10</td>
<td>0.125</td>
<td>0.1</td>
</tr>
</tbody>
</table>

### Table 2: Parameters of the simulations, \( L_{j} \) normalized by \( r_{j} \) or \( D \) (slot width) for the round and plane jet, respectively.

<table>
<thead>
<tr>
<th>Case</th>
<th>( L_{j1} )</th>
<th>( L_{j2} )</th>
<th>( L_{j3} )</th>
<th>( m_{p} )</th>
<th>( m_{s} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>R014</td>
<td>60</td>
<td>26</td>
<td>16</td>
<td>256</td>
<td>112</td>
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<tr>
<td>R100</td>
<td>60</td>
<td>26</td>
<td>16</td>
<td>256</td>
<td>112</td>
</tr>
<tr>
<td>R152</td>
<td>60</td>
<td>26</td>
<td>16</td>
<td>256</td>
<td>112</td>
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<tr>
<td>PD050</td>
<td>22</td>
<td>22</td>
<td>13</td>
<td>768</td>
<td>512</td>
</tr>
<tr>
<td>PD100</td>
<td>22</td>
<td>22</td>
<td>13</td>
<td>768</td>
<td>512</td>
</tr>
<tr>
<td>Plane jet LES</td>
<td>26</td>
<td>16</td>
<td>12</td>
<td>192</td>
<td>160</td>
</tr>
</tbody>
</table>

## RESULTS

Figure 3 shows the mean velocity difference \( \Delta u = \bar{u} \), normalized by the centerline velocity at various streamwise positions for cases R014 and R100. All profiles for the round jet collapse nicely, indicating self-similar behavior. They furthermore agree well with an exponential of the form \( u = \exp[-(r/r_{j})^{4} \ln 2] \). For the plane jet, however, we observe a collapse of the streamwise velocity difference at various \( z \)-locations and for different density ratios (figure 4). The decay of the centerline velocity \( U_{c} \), of variable density jets has been investigated by many researchers, before. No general similarity solution may be found for plane jets, as pointed out by Mellado [2004]. For incompressible flow and low density ratios dimensional analysis and conservation of momentum suggests that \( U_{c} / U_{j} \sim \sqrt{h / \delta} \). Figure 5, which shows the centerline velocity decay for the plane jet LES simulations, confirms this relationship. We furthermore see that lighter jets decay much faster than heavy jets, owing to the conservation of streamwise momentum. For high \( s \), however, density itself scales with \( \rho_{c} / \rho_{j} \sim x / D \), leading to \( U_{c} / U_{j} \sim h / \delta \). A mixed scaling was proposed by Mellado [2004] to cover both limits, using the centerline density \( \rho_{c} \) instead of \( \rho_{j} \), giving \( U_{c} / U_{j} \sim \sqrt{h / \rho_{c} / \rho_{j}} \). This scaling collapses the profiles for the plane jet as shown in figure 6. Such a simple scaling didn’t work for the round jet case, whose centerline velocity decay is shown in figure 7. Here, the Witze scaling (Witze [1974]) is used to obtain a common potential core collapse location \( x_{c} / r_{0} = 0.77(n^{2} / s)^{-1 / 2} \) where \( n = 2.08(1 - 0.136/s)^{-0.22} \). The simulations are compared with data from (Crow and Champagne [1971], Bridges and Wernet [2003], Bodony and Lele [2005]) at \( s = 0.95 \) (Bridges and Wernet [2003], Bodony and Lele [2005]) and \( s = 1 \) (incompressible Crow and Champagne [1971]). The observed linear growth of \( 1 / U_{c} \) with \( z \) agrees with the prediction of self-similar analysis (So [1986]), although several assumptions are made, including that of a constant Chapman-Rubesin parameter. A better collapse of the data is obtained by using the analysis of Chen and Rodi [1980], expressing the velocity decay using a nondimensional downstream distance parameter \( x^{*} = s^{-1 / 2} x \) and centerline velocity \( U_{c}^{*} = s^{-1} U_{c} / U_{c, 0} \), as seen in figure 8.

The development of the centerline density is seen in figure 9, for the round jet case with \( s = 1 / 7 \) and the plane jet cases with \( s = 0.8, 1 / 8 \). Both, plane and round jets, first show only minor mixing before nonlinear growth occurs. As reported by Mellado [2004], after this nonlinear growth phase the jet density continues to increase almost linearly for case \( s = 1 / 8 \). Self-similar analysis predicts this mean density linear regime for strongly heated jets, before the core density reaches values comparable to the co-flow; beyond that point, temperature becomes a passive scalar if no buoyancy is present. The round jet centerline density grows faster, but starts to increase slightly further downstream than the plane jet simulations. This seems to be caused by the strong global instability, to be discussed below, and slightly different initial conditions. The stronger growth in simulation R014 compared to the plane jet simulations could be expected, however, due to the larger surface undergoing turbulent mixing relative to the plane jet.

Contrary to the centerline velocity decay, the jet half width (position at which the streamwise velocity dropped to half the centerline value) grows linearly in \( x \) (figure 10 and 11). The half width for plane and round jet was, irrespective of the density ratio, of similar magnitude, 0.112 vs. 0.116, respectively. The round jet data is compared with a simulation from Bodony and Lele [2005] at \( s = 0.95 \), the plane jet data with experiments from Browne et al. [1983], Ramaprian and Chandrasekhar [1985]. When shifted by the virtual origin \( x_{v} \), it can be seen that the growth rate is the same within the round or plane jet data, irrespective of the density ra-
Figure 12 shows a comparison of the evolution of the Reynolds stress components in radial direction for the round jet and cross-stream direction for the plane jet (other components show similar trends). Interestingly, the peak values are of the same order of magnitude. With increasing density ratio higher fluctuation levels are observed, consistent with results from Djeridane et al. [1996]. The initial growth for the round jet cases is different, but it is possible, by using the Witze scaling, collapse the profiles in the early development region. As observed for the mean density growth, during the self similar stage a different evolution of round and plane jets is obvious here, too. The peak is more pronounced for the round jet simulations, with a stronger decrease thereafter.

In Case R014, high amplitude oscillations were observed, especially for the streamwise velocity and density, leading to strong gradients which made the simulation unstable at first. Adding artificial dissipation (Fiorina and Lele [2007]) helped stabilizing the calculation. This strong oscillating mode together with the strong vortex pairing events as well as the occurrence of side jet phenomena points toward the existence of a global instability in this simulation. Evidence for this is shown in figure 13, plotting the power spectrum of the streamwise velocity at various streamwise locations. The power spectrum clearly shows the fundamental frequency and its sub-harmonics at various streamwise positions (shifted for better visibility). The peak is clearly visible until a sudden transition to turbulence has occurred around x/D = 7. The Strouhal number of this oscillating mode $St_0 = fD/U_j$ was calculated and compared to various experimental datasets, depicted in figure 14. Excellent agreement is seen between the simulation results and the experimental data. For the plane jet simulations no such global mode was observed, neither in the DNS nor the LES simulations. After Hallberg and Strykowski [2006], the global oscillations depend on the density ratio $s$, D/δ₀ and the Reynolds number, as long as compressibility and buoyancy effects are unimportant. Going further than Kyle and Sreenivasan [1993], Hallberg and Strykowski [2006] non-dimensionalize the frequency by the viscous time scale $D^2/ν$, thus retaining the Reynolds number in the frequency dependence. They were then able to collapse all data onto a straight line, when plotting $fD^2/ν$ over $Re\sqrt{D/δ₀}(1+\sqrt{s})$. Having similar initial momentum thickness and density ratio and higher Re than required for the occurrence of global modes at a given momentum thickness in the plane jet cases (Hallberg and Strykowski [2006]) a possible reason might be the plane jet geometry itself or the different Mach number. Simulations at various Mach numbers are performed currently, to answer this question.

CONCLUSIONS

Results of DNS simulations of plane jets and LES simulations of round and plane jets at various density ratios, ranging from $s = 0.125$ to $s = 1.52$ have been presented in the text. The streamwise linear growth of the round and plane jet half width was found to be of similar magnitude, with slightly lower growth of the round jets and independent of the density ratio, a fact reported in the literature before. The centerline velocity decay rates are strongly affected by heating and exhibit different behavior for round and plane jets. Whereas the round jets exhibits a decay with $1/x$ for all density ratios, there are two self-similar scalings found in plane jets, in the limit of small and large density ratios. In the limit of small density ratios or incompressible flow, $U_c$ scales as $U_c \sim 1/\sqrt{s}$. For strongly heated jets on the other hand we find $U_c \sim \sqrt{\rho_0}\left(\frac{x}{\theta_0}\right) \sim 1/x$, which is due to a linear growth of the centerline density. Using nondimensional values for $x$ and $U_c$ (basically scaling the quantities using the factor $s^{-1/4}$ after Chen and Rodi [1980]) collapses the round jet data. This clearly shows a difference in the development and entrainment process of round and plane jets, which should be considered when developing RANS models. Furthermore, it was seen that the streamwise growth in mean density or the decay of the velocity fluctuations in the self-similar region, after the transition region, is stronger for round jets. The core density reached co-flow values much faster for the round jet and therefore the interval of distances $x$ in which there is a strong density difference between the core and the co-flow is shortened, and it is more difficult to find an intermediate self-similar regime. We believe, that further insight might be found in analyzing the vorticity development and enstrophy budget for both geometries. This, however, requires DNS simulations of the round jet, currently performed on 1024 processors at the Julich supercomputing center, in order to resolve the small scales appropriately, necessary to reliably interpret the enstrophy budget.

The round jet data at $s = 0.14$ furthermore shows a global instability, with the fundamental frequency in excellent agreement with previous experimental data, as summarized by Hallberg and Strykowski [2006]. Such a global mode couldn’t be seen in the plane jet simulations, probably due to the different Mach numbers. This, however, is ongoing research.

References


Figure 1: Contour plot of the density for case R014.

Figure 2: Contour plot of the density for case P05. Color table similar to figure 1.
Figure 3: Normalized velocity difference at various streamwise positions for the round jet LES, normalized by the jet half width.

Figure 4: Normalized velocity difference at various streamwise positions for the DNS of the plane jet normalized by the jet half width.

Figure 5: Centerline velocity decay for the plane jet LES results. —— $s = 1$, —— $s = 0.8$, —— $s = 1/8$, ○ Browne et al. [1983] and ▲ Ramaprian and Chandrasekhara [1985].

Figure 6: Centerline velocity decay for the plane jet LES results, using the centerline density. —— $s = 1$, —— $s = 0.8$, —— $s = 1/8$.

Figure 7: Centerline streamwise velocity decay of the round jet using the Witze scaling (Witze [1974]).

Figure 8: Centerline velocity decay using $U^*_c = s^{-1/4}U_c/U_{co}$ and $x^* = s^{-1/4}(x - x_s)$. The streamwise coordinate was shifted using the virtual origin $x_s$. 

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Figure 9: Growth of centerline density \((c - \rho)/j\) for case R014, compared to the plane jet LES cases with \(s = 1/8\) and \(s = 0.8\). \(h\) is the slot width.

Figure 10: Streamwise evolution of the velocity half-width of the round-jet LES, shifted by using the virtual origin \(x_s\).

Figure 11: Streamwise evolution of the velocity half-width of the plane-jet DNS.

Figure 12: Streamwise evolution of the centerline cross-stream Reynolds stress normalized using the centerline velocity difference, taken from the LES results.

Figure 13: Frequency power spectrum of \(u_x\) at various streamwise positions (curves shifted for better visibility).

Figure 14: Strouhal number of the global instability mode, compared with experimental data.