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THE SWEEP-STICK MECHANISM OF HEAVY PARTICLE CLUSTERING IN HOMOGENEOUS AND INHOMOGENEOUS TURBULENCE

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ABSTRACT

Our work focuses on the sweep-stick mechanism of particle clustering in turbulent flows introduced in [6] for 2D inverse cascading homogeneous,isotropic turbulence (HIT), whereby heavy particles cluster in a way which mimics the clustering of zero acceleration points. We extend this phenomenology to 3D HIT, where it was previously reported that zero acceleration points were extremely rare, which allows us to discard the modified sweep-stick mechanism of [13]. Having obtained a unified mechanism we quantify the Stokes number dependency of the probability of the heavy particles to be at zero acceleration points and show that in the inertial range of Stokes numbers the sweep-stick mechanism is dominant over the conventionally proposed mechanism of heavy particles being centrifuged from high vorticity regions to high strain regions. Finally, we present preliminary work on the impact of the structure of the turbulent acceleration field on the spatial distribution of heavy particles in a turbulent channel flow. It is shown that the streamwise acceleration structures correlate strongly with the heavy particle distribution, especially as one moves closer to the wall. We also present fluid acceleration statistics sampled at heavy particle positions and validate the Maxey relation for small Stokes numbers. It is observed that near the wall heavy particles reside in low acceleration regions compared to fluid elements.

INTRODUCTION

Suspensions of dust, droplets, bubbles or other kinds of small particle in turbulent incompressible flows, are present in many stirring and mixing situations encountered in both natural and industrial situations. These include rain initiation in warm clouds, combustion processes in diesel engines, the formation of planetesimals in the early solar system, and transport of sediment and chemicals in rivers, to mention just a few.

There have been several mechanisms proposed to predict where heavy particles reside in turbulent flows, depending on the scales of interest. For particle relaxation times larger than the integral time of the turbulence, it has been shown that caustics can cause an unbounded increase in the concentration of particles in finite time [8, 26]. Meanwhile, for particle relaxation times and distances smaller than the respective Kolmogorov time and length scales, it has been proposed that the motion of inertial particles is governed by fluid strain; namely, heavy particles are centrifuged out of coherent eddies and accumulate in low vorticity and high strain rate regions [2, 9]. This centrifuging phenomenology has also been implicitly extended to particle relaxation times within the inertial range of the turbulence and is widely believed to account for clustering in this range [19, 23, 1]. Whilst it seems that these centrifuging effects are indeed predominant at sub-dissipative scales or in cases where the energy spectrum is mostly concentrated around a single length-scale, such as in low Reynolds number turbulence, at higher Reynolds numbers the clustering is not a single scale phenomemon, but rather has a multiscale nature [4]. At higher Reynolds numbers not only do the smallest eddies play a role in segregating the heavy particles, there are also larger coherent eddies which effect the process.

Work by [6] and [12] has confirmed the observation that inertial particles in stationary, homogeneous, isotropic turbulence (HIT) cluster in a multiscale structure. These works have also qualitatively shown that in 2D inverse-cascading turbulence, the clustering of inertial particles is a direct reflection of the clustering of zero-acceleration points. The mechanism behind this coincidence bewteen inertial particle and zero-acceleration points has been called the *sweep-stick* mechanism.

This mechanism is easily described qualitatively. Firstly, low acceleration points of the fluid are swept by the local fluid velocity $\mathbf{u}$, as demonstrated for 2D inverse-energy cascading HIT in [6] and [25]. [20] has shown that for small Stokes numbers, the velocity of the inertial particle $\mathbf{v_p}$ is well approximated by $\mathbf{v_p} \approx \mathbf{u}(\mathbf{x_p}, t) - \tau_p \mathbf{a}(\mathbf{x_p}, t)$, where $\mathbf{x_p}$ is the position of the particle and $\mathbf{a}$ is the fluid acceleration. As a result, particles which coincide with zero-acceleration points move together with these points with velocity $\mathbf{u}$, whereas particles coincident with non-zero acceleration points move away from these points with relative velocity $-\tau_p \mathbf{a}$.

The basic principles of the sweep-stick mehanism seem to be readily extendable to 3D and also to inhomogeneous turbulent flows in both 2D and 3D. There has already been some preliminary work with regards to the extension to 3D. [13] investigated the clustering of inertial particles and zero-acceleration points in 3D HIT and suggested a qualitative correlation between the clustering of inertial particles and surfaces where $\mathbf{e_1} \cdot \mathbf{a} = 0$ and $\lambda_1 > 0$, where $\mathbf{e_1}$ is the eigenvector corresponding to the largest positive eigenvalue λ_1 of the symmetric part of the acceleration gradient tensor $\nabla\mathbf{a}$. We investigate the validity of these sweep-stick mechanisms in 2D and 3D HIT and 3D turbulent channel flow.

NUMERICAL SIMULATIONS

The particles considered are passive, small, heavy, rigid spheres all of the same size. Their mass density ρ_p is much greater than the fluid density ρ_f i.e. $\rho_p \gg \rho_f$. The particle radius is also assumed to be smaller than the smallest length scale of the turbulence. Under these assumptions, the equation of motion for a particle, neglecting gravity, is [21]

$$\frac{d\mathbf{x_p}(t)}{dt} = \mathbf{v_p}(t) \quad (1)$$

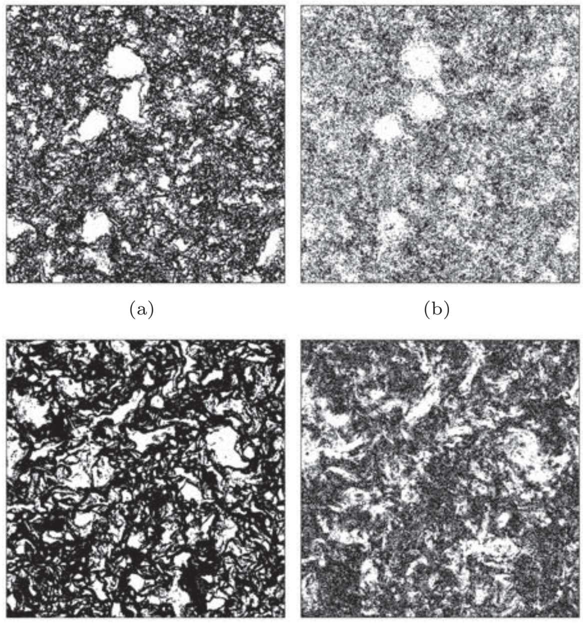
$$\frac{d\mathbf{v_p}(t)}{dt} = \frac{C_D Re_p}{24\tau_p}(\mathbf{u}(\mathbf{x_p}(t),t) - \mathbf{v_p}(t)) \quad (2)$$

where $\mathbf{v}_p(t)$ is the particle velocity at time t and $\mathbf{u}(\mathbf{x},t)$ is the fluid velocity at position $\mathbf{x}$ at time t. The equation implies that the particle velocity relaxes to that of the fluid within a timescale τ_p, where $\tau_p = 2\rho_p a^2/(9\mu)$, where μ is the fluid viscosity. A non-dimensional time scale can be obtained by normalising with a timescale of the turbulence, which is chosen to be the Kolmogorov time τ_η in the case of HIT and t^+ for channel flow . These normalized times are called the Stokes number and are defined by $S_\eta \equiv \tau_p/\tau_\eta$ and $St^+ \equiv \tau_p/\tau^+$ respectively. Although the system is much simplified, in many cases particles are so massive that the assumptions made here are justified. Equation (1) is integrated together with the appropriate solution of the Navier-Stokes equations for an incompressible fluid. The fluid velocity at the particle position $\mathbf{x}_p(t)$ is obtained using a sixth order Lagrange interpolation.

For HIT, The fluid velocity field is obtained using a pseudo-spectral Direct Numerical Simulation (DNS) of the Navier-Stokes equations. In the 2D case, the Navier-Stokes equations are integrated with an external small scale energy source and a large scale energy sink. Periodic boundary conditions are applied in two orthogonal directions. Full details of the simulation can be found in [11]. By using 2048^2 and 4096^2 grid points, there is a region between the forcing scale η and the integral length scale L, spanning approximately two decades of wavenumber space where the energy spectrum E(k) has the form $E(k) \sim \epsilon^{2/3}k^{-5/3}$.

In the 3D case a Fourier spectral method is used with a 4th order Runge-Kutta-Gill time scheme. Dealiasing is achieved using the phase-shift method. The amplitudes of Fourier components of velocity in a low-wavenumber range are kept constant in time to realize a statistically stationary state [10]. Periodic boundary conditions are applied in all three orthogonal directions.

For the channel flow, we use the code of [16] where spatial derivatives are estimated using a sixth-order compact finite-difference scheme and the Navier-Stokes equations are numerically integrated with a frational step method using a threestage third-order Runge-Kutta scheme. The fractional step method projects the velocity field to a divergence free velocity field and the Poisson pressure equation is solved in Fourier space with a staggered grid for the pressure field and an FFT for non-uniform grids. The staggered grid for the pressure was used for numerical stability purposes as was the skew-symmetric implementation of the non-linear term in the Navier-Stokes equation. The grid stretching technique maps an equally spaced co-ordinate in the computational space to a nonequally spaced co-ordinate in the physical space, in order to be able to use Fourier transforms in the inhomogeneous wall-normal direction.



(a) (b) (c) (d)

Figure 1: (a) shows the spatial distribution of heavy particles for $S_\eta = 1.6$ and (b) the position of zero acceleration points at $t \approx 4T$ in 2D HIT. The side length of the plots is approximately 800η. (c) shows the positions of particles with $S_\eta = 2$ in a thin layer of dimensions $500\eta \times 500\eta \times 5\eta$ in 3D HIT with (d) displaying the location of zero acceleration points in the same layer.

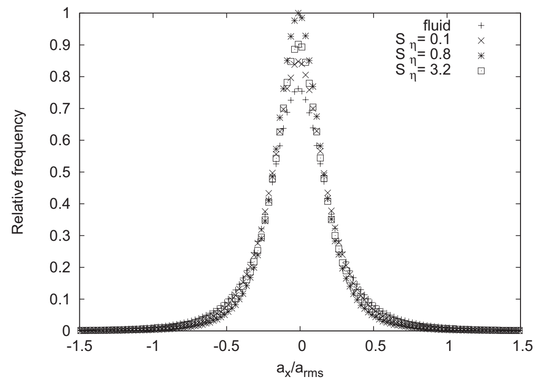
HOMOGENEOUS, ISOTROPIC TURBULENCE

We begin by summarising the results in both 2d and 3d HIT. It has recently been found [7] that the modified sweep-stick mechanism whereby heavy particles cluster in a way which mimics the clustering of $\mathbf{e}_1 \cdot \mathbf{a} = 0$ points is not necessary. The clustering of zero-acceleration points is sufficient to explain the clustering of heavy particles at inertial range lengthscales in 2d and 3d HIT Fig. 1 shows snapshots of heavy particle positions at $t \approx 4T$ in both 2D and 3D HIT at resolutions of 4096^2 and 512^3 and comparing this to the position of zero acceleration points in the fluid.

In order to quantify the tendency of heavy particles to cluster at zero-acceleration points as a function of their Stokes number, we investigate the probability distribution function (pdf) of the fluid acceleration sampled at inertial particle positions and compare it to the pdf for a uniform distribution. Figure 2 shows the relative probability distribution function (pdf) of a single component of the acceleration in 2D and 3D.

The pdfs for inertial particles are all clearly peaked around zero, with the highest peak occuring for $S_\eta = 0.8$. Even at $S_\eta = 0.1$, there is still a clear peak around $a_x = 0$ when compared to that of a homogeneous, random distribution. The pdf becomes more peaked as S_η approaches unity and then broadens again at higher Stokes number. This phenomenon can also be observed in 3D HIT, as shown in figure 2(b), which shows the pdf of a single component of the acceleration clearly peaked when sampled at the positions of particles with $S_\eta = 2.0$.

These results lead us to conclude that the sweep-stick



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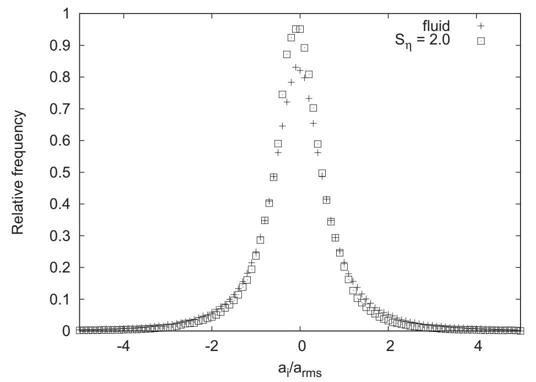


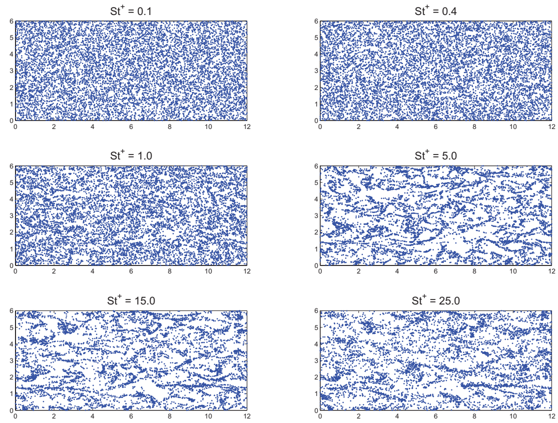
(b)

Figure 2: Pdf of a single component of acceleration for (a) 2D HIT and (b) 3D HIT, conditionally sampled at heavy particle positions and for comparison at positions homogeneously and randomly distributed throughout the fluid (labelled fluid). The pdfs for inertial particles are all clearly peaked around zero. In 2D HIT the highest peak occurs for $S_\eta = 0.8$, before dropping again for higher Stokes numbers.

mechanism proposed in [6], whereby heavy particles cluster in a way which mimics that of the clusters of zero-acceleration points of the underlying fluid over the range of lengthscales constituting the inertial range, is sufficient to explain particle clustering in both 2D and 3D HIT. There is no need for the modified sweep-stick mechanism of [13] which takes account of the compressibility of the particle velocity field.

TURBULENT CHANNEL FLOW

The great advantage of the sweep-stick mechanism over traditional phenomenologies such as the centrifuging from vortical regions is that it lends itself naturally to an unambiguous explanation of where heavy particles cluster in inhomogeneous flows. Phenomonologies which rely on centrifugal ejection of heavy particles have to introduce coarse graining of the vorticity field [12] to explain the multiscale structure of the clustering and it is not clear how this would transfer to inhomogeneous flows. There has been much recent work on clustering of inertial particles in DNS of 3D channel flow starting with [22] and continuing through to [24] and [17], as well as the experimental work of [14] and [15], but none of this work has considered the impact of the spatial structure of the acceleration field on particle clustering, which has been shown, as summarised above and discussed in [7], to be the dominant mechanism for particle



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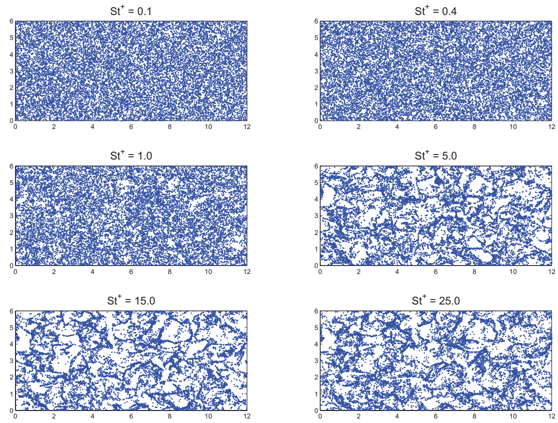


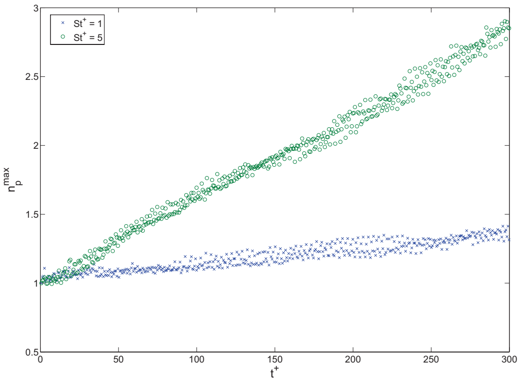
(b)

Figure 3: Spatial distributions of heavy particles of $St^+ = 0.1, 0.4, 1.0, 5.0, 15.0$ and 25.0 in a turbulent channel flow at $Re_\tau = 150$ - (a) $y^+ = 39 - 45$ and (b) $y^+ = 145 - 155$.

clustering in high Reynolds number HIT.

Figure (3) shows the positions of heavy particles of six different Stokes numbers initially seeded at the same time and positions in a turbulent channel flow with a skin friction Reynolds number of $Re_\tau = 150$ at $y^+ = 39 - 45$ and $y^+ = 145 - 155$. The most striking aspect of these pictures is initially uniformly distributed inertial particles with different values of St^+ develop holes in the same regions, and these holes become bigger with increasing St^+. This effect was noticed in [6] for the case of 2d HIT. This phenomenon implies that the clustering is clearly related to the structure of the underlying flow field and not just to the dynamics of the heavy particles. Moreover, the pictures demonstrate a clear difference in the patterns that the heavy particles form as they appraoch the wall. In the centre of the channel ($y^+ = 145 - 155$), the clustering resembles that observed in HIT, as would be expected. However as the wall is approached the particles begin to cluster in streak like structures.

To validate the accuracy of the numerical simulations, we compute the statistics of particle number density and mean particle velocities as a function of distance from the wall. These are shown in figure (4). All of these curves show good agreement with the same statistics published in [18]. Figure (4a) shows the trend of increasing maximum particle number density (n_p^{max}) as a function of time and Stokes number which has been observed for St^+ upto 25. Figure (4b) shows



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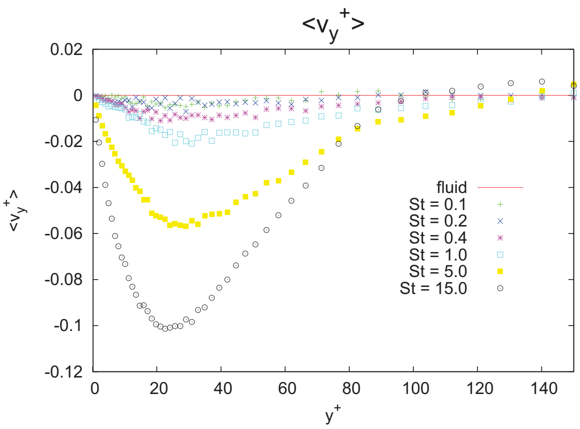
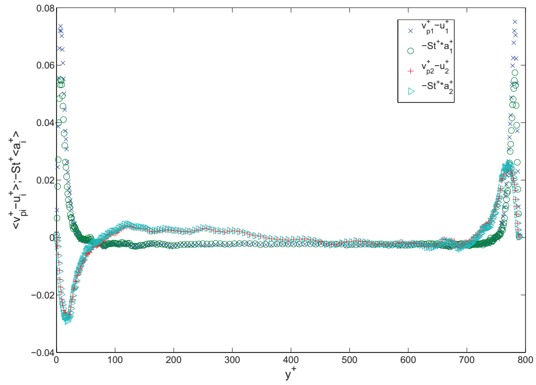


(b)

Figure 4: (a) Maximum value of particle number density n_p^{max} as a function of time for $St^+ = 1$ and 5, (b) mean wall normal particle velocities as a function of distance from the wall for $St^+ = 0.1, 0.4, 1.0, 5.0, 15.0$ and 25.0.



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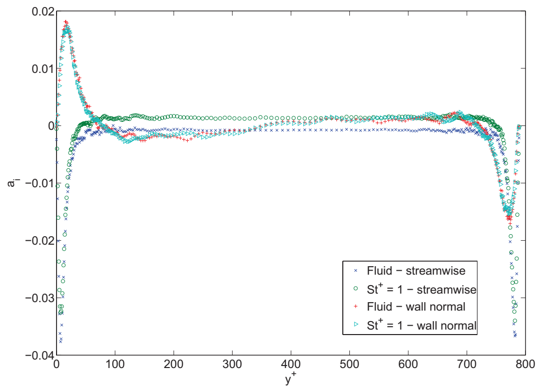


(b)

Figure 5: (a) The average difference in particle velocity and local fluid velocity and the local fluid acceleration as a function of y^+ for $Re_\tau = 395$ for $St^+ = 1$ and (b) mean acceleration in the fluid and mean acceleration sampled at $St^+ = 1$ particle positions.

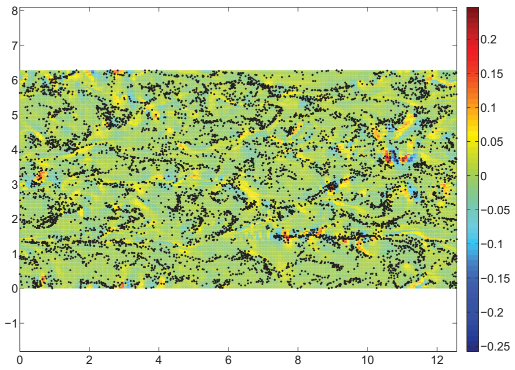
the mean wall normal particle velocity as a function of distance from the wall. The motion is characterized by a mean drift velocity to the wall which increases with Stokes number in the range of Stokes numbers investigated here.

Many authors, beginning with [19], have explained particle clustering by appealing to the leading-order asymptotic expansion of equation (1)

$$\mathbf{v}_p(t) \approx \mathbf{u}(\mathbf{x}_p(t), t) - \tau_p \mathbf{a}(\mathbf{x}_p(t), t), \tag{3}$$

with $\tau_p \ll 1$. As part of our preliminary investigation of clustering in the channel flow, figure (5a) shows numerical verification of this relationship for $St^+ = 1$ and $Re_\tau = 395$. The importance of the approximate validity of this relationship is crucial to the sweep-stick mechanism as it guarantees that the difference between the particle velocity and the fluid velocity is zero at zero acceleration points. In HIT, this fact, combined with the fact that these zero acceleration points are swept with the local fluid velocity means that inertial particles and zero acceleration points are swept together. Figure (5b) shows both the mean streamwise and wall normal accelerations as a function of distance from the wall for a uniform distribution and sampled at $St^+ = 1$ particle positions for $Re_\tau = 395$. The lines are fairly ragged due to a limited number of statistics but some trends are still clear. In the streamwise direction and away from the wall particles go to regions of higher acceleration than that of a uniform distribution as one would expect from the fact that statsitically the difference between particle velocity and local fluid velocity is negative. Close to the walls, heavy particles reside in lower magnitude negative acceleration regions than fluid tracers. The statistical uncertainity makes it difficult to draw conclusions about the wall normal direction. There appears to be no difference between the mean acceleration of the fluid at heavy particle positions and fluid positions, but it seems that heavy particles go to lower acceleration regions near to the wall than fluid elements. This undersampling of high acceleration regions by inertial particle is consistent with the fact that heavy particles have a finite relaxation time. In fact, in HIT, it has been observed that the root mean square of acceleration of heavy particles decreases with increasing Stokes number [3].

As the simulations for $Re_\tau = 395$ are still ongoing, our simulations at $Re_\tau = 150$ provide the bulk of the data. The resolution of these simulations is such that is difficult to locate many zero acceleration points. Instead, figure (6) shows the streamwise acceleration at the two different distances from the wall (corresponding to the distances in figure (3)) with the positions of $St^+ = 15$ particles superimposed. The impact of the streamwise acceleration on the particle clustering is clearly visible, especially at $y^+ = 39 - 45$. Here, the streamwise acceleration has a string like structure, and the heavy particles clearly preferentially reside on these string



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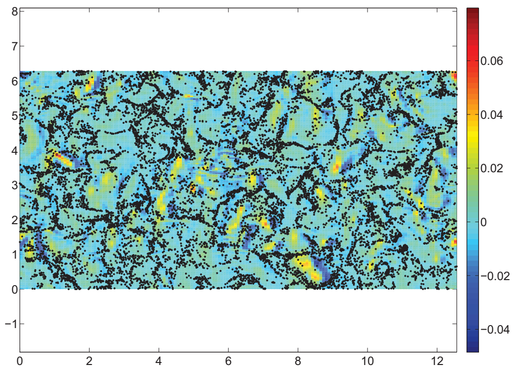

(b)

Figure 6: Spatial distributions of heavy particles of $St^+ = 15.0$ and streamwise acceleration in a turbulent channel flow at $Re_\tau = 150$ - (a) $y^+ = 39 - 45$ and (b) $y^+ = 145 - 155$.

like structures, which are regions of small positive acceleration. As we move to the centre of the channel, these acceleration structures disappear and consequently so does the streaky nature of the particle distribution. It is clear is that particles do not cluster at zero points of the total acceleration, as in HIT. This is not unexpected given the mean accelerations which are present in the turbulent channel flow and further invetigations are ongoing to investigate the effect of the acceleration field due to the fluctuating velocity on the inertial particles.

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