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EFFECTS OF NOZZLE-EXIT BOUNDARY-LAYER THICKNESS ON THE TURBULENT DEVELOPMENT OF INITIALLY LAMINAR CIRCULAR JETS

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ABSTRACT

Round jets at Mach number 0.9 and Reynolds number 10^5, displaying nozzle-exit boundary layers characterized by laminar Blasius velocity profiles, low velocity fluctuations and momentum thicknesses between $0.003r_0$ and $0.02r_0$, where r_0 is the jet radius, are computed by Large-Eddy Simulation. The effects of the outlet boundary-layer thickness on initially laminar jets are investigated. Jets with thinner boundary layers show earlier but slower turbulent development, leading to longer potential cores and lower turbulence intensities. Their flow properties compare well with corresponding experimental data for jets at high Reynolds numbers. Finally the influence of adding random noise of low magnitude in the jet nozzle is also explored.

INTRODUCTION

Since many experimental works conducted during the seventies including those by Hill *et al.* (1976), it has been well known that the development of axisymmetric jets depends strongly on the characteristics of the nozzle-exit boundary layer. Parameters such as the momentum thickness of the velocity profile, the levels of velocity fluctuations, as well as the laminar or turbulent state of the boundary layer have thus to be considered, because their variations are expected to result in noticeable modifications of jet flow features. Unfortunately they have been neither controlled nor documented in most experiments, except in some cases as in Hussain and Zedan (1978a, 1878b) and in Zaman (1985a, 1985b). In round jets of radius r_0, Zaman (1985a, 1985b) measured for instance an initial shear-layer momentum thickness of $\delta_\theta = 0.008r_0$ and negligible turbulent intensities at a diameter-based Reynolds number $\mathrm{Re}_D = 7 \times 10^4$, but $\delta_\theta = 0.004r_0$ and velocity fluctuations close to ten per cent of the jet velocity at $\mathrm{Re}_D = 2.5 \times 10^5$.

A difficulty in experiments aiming at studying the effects of initial conditions is that the inflow parameters cannot be usually changed independently, except for some very careful investigations such as those by Hussain and Zedan (1978a, 1878b). To distinguish the influence of jet inflow parameters, it then appears natural to perform numerical simulations. Works have been carried out to this end by Stanley and Sarkar (2000), Bogey and Bailly (2005), Bogey *et al.* (2008), and by Kim and Choi (2009). However, some questions remain because the computing limitations led to differences between experimental and numerical inflow conditions. Regarding the initial shear-layer momentum thickness, values of $\delta_\theta = 0.016r_0$ and $\delta_\theta = 0.011r_0$ were for instance specified in Bogey *et al.* (2008) and in Kim and Choi (2009), respectively, which is higher than the values by Zaman (1985a, 1985b) mentioned above. Consequently some discrepancies were observed with respect to experiments, in particular shorter potential core lengths.

In the present study, the effects of the nozzle-exit boundary-layer thickness in initially laminar round jets are investigated by performing Large-Eddy Simulations (LES) combining low-dissipation and low-dispersion schemes and relaxation filtering for dissipating subgrid-scale energy. The jets are at Mach number 0.9 and at Reynolds number 10^5. To get closer to experiments, a part of a pipe nozzle is included in the computational domain, and laminar boundary layers of momentum thicknesses between $\delta_\theta = 0.003r_0$ and $0.02r_0$ are imposed at the nozzle inlet. In this way the influence of the nozzle-exit δ_θ can be examined down to thickness values typically found in experiments. The results will allow us to discuss whether specifying thin boundary layers in simulations is sufficient to predict jet flow features corresponding to those encountered in practical jets at high Reynolds numbers. The issue of adding random disturbances of low amplitude in the jet nozzle, and especially its impact on the early stage of the shear-layer transition, will be also adressed.

SIMULATION PARAMETERS

Numerical procedure

The LES are performed by solving the three-dimensional cylindrical filtered compressible Navier-Stokes equations, using centered and non-centered low-dissipation and low-dispersion schemes developed by Bogey and Bailly (2004) and by Berland *et al.* (2007). Fourth-order 11-point finite differences are implemented for spatial discretization, and a 2nd-order 6-stage low-storage Runge-Kutta algorithm is applied for time integration. Grid-to-grid oscillations are removed every time step by an explicit 6th-order 11-point filtering of the flow variables, which is designed to damp only the shortest waves discretized. The filtering enables to take into account the effects of the subgrid energy-dissipating scales without affecting significantly the scales accurately resolved. More details on this LES approach based on relaxation filtering (LES-RF) can be found in Bogey and Bailly (2006a, 2006b, 2009a).

Table 1: Thickness of the inlet Blasius boundary layer δ, maximum amplitude of possible random pressure disturbances in the pipe, and line types used in the plots.

Reference	δ/r_0	Inlet noise	Line type
JetD02	0.2	-	
JetD01	0.1	-	
JetD005	0.05	-	
JetD0025	0.025	-	
JetD005p250	0.05	250 Pa	
JetD005p2000	0.05	2000 Pa	

Jet definition

Round isothermal jets at Mach number $\mathrm{M} = u_j/c_a = 0.9$ and at Reynolds number $\mathrm{Re}_D = u_j D/\nu = 10^5$, originating from a pipe nozzle of radius r_0 and length $1.1r_0$, are considered (u_j is the jet inflow velocity, c_a is the speed of sound in the ambient medium, $D = 2r_0$ is the nozzle diameter, and ν is the kinematic molecular viscosity). At the exit section of the nozzle at $z = 0$, the width of the nozzle lip is $0.053r_0$. At the pipe inlet at $z = -1.1r_0$, laminar Blasius boundary layers characterized by thicknesses $\delta = 0.025r_0$, $0.05r_0$, $0.1r_0$ and $0.2r_0$ are imposed. Four jets, referred to as JetD0025, JetD005, JetD01 and JetD02, are thus computed as described in table 1. To better match experiments, two additional jets with initial boundary layer thickness $\delta = 0.05r_0$ are simulated. In these two cases JetD005p250 and JetD005p2000, unlike the four previous jets, random pressure disturbances are introduced in the pipe, within the boundary layer between $z = -0.4r_0$ and $-0.2r_0$. They are of maximum amplitude 250 and 2000 Pa, respectively.

Computational parameters

The cylindrical grids used are specified in table 2. They contain from 22 to 48 millions of points. The numbers of time steps required to obtain simulation times T between $187.5D/u_j$ and $275D/u_j$ are also reported. The mesh spacings are minimum at the nozzle lip, with $\Delta_r = \delta/7$ at $r = r_0$, and $\Delta_z = 2\delta/7$ at $z = 0$. They are then stretched to reach maximum values $\Delta_r = \Delta_z = 0.056r_0$ so that the time frequency f of waves discretized by four grid points corresponds to Strouhal number $\mathrm{St} = fD/u_j = 10$. The flow variables are computed in this way up to $r = 8.6r_0$ in the radial direction, and up to $z = 25.5r_0$ in the axial direction.

To initially seed the jet turbulent transition, random pressure fluctuations of maximum amplitude 200 Pa are added in the shear layer between $z = 0.25r_0$ and $4r_0$ up to time $t = 18.75D/u_j$. The flow variables calculated along the centerline and the lip line are recorded from $t = 37.5D/u_j$, at a frequency allowing estimation of spectral components up to Strouhal number 10. The velocity spectra are moreover evaluated from overlapping samples of duration $10D/u_j$. Finally the flow statistics are determined from $t = 87.5D/u_j$, and results are averaged in the azimuthal direction.

Nozzle-exit conditions

The flow conditions at the nozzle exit are briefly presented here. They are first illustrated in figure 1 with the mean and root-mean-square (rms) turbulent profiles obtained for the axial velocity at $z = 0$. The profiles of $<u_z>$ agree with the Blasius boundary-layer profiles specified at the pipe inlet for the different jets, while the fluctuation intensities are of low amplitude. Therefore all the jets simulated in the present study can be regarded as initially laminar. The momentum thicknesses δ_θ determined from $< u_z >$ at the exit plane are provided in table 3. They range from $0.0025r_0$ for JetD0025 to $0.023r_0$ for JetD02. As for the rms values of u'_z, maxima around $0.003u_j$ are found near the wall for all jets except for JetD005p2000. For this jet, the level peak is about $0.02u_j$. Considering this, following Zaman (1985a), the initial boundary layers of the jets are fully laminar for the four jets without inlet noise and JetD005p250, and nominally laminar for JetD005p2000.

Table 2: Numbers of grid points (n_r, n_θ, n_z) and time steps n_t, and simulation time T. Parameters for JetD005p250 and JetD005p250 same as those for JetD005.

Reference	$n_r \times n_\theta \times n_z$	n_t	Tu_j/D
JetD02	$173 \times 256 \times 505$	$48,800$	275
JetD01	$215 \times 256 \times 543$	$46,800$	275
JetD005	$249 \times 256 \times 595$	$86,000$	250
JetD0025	$287 \times 256 \times 651$	$127,800$	187.5

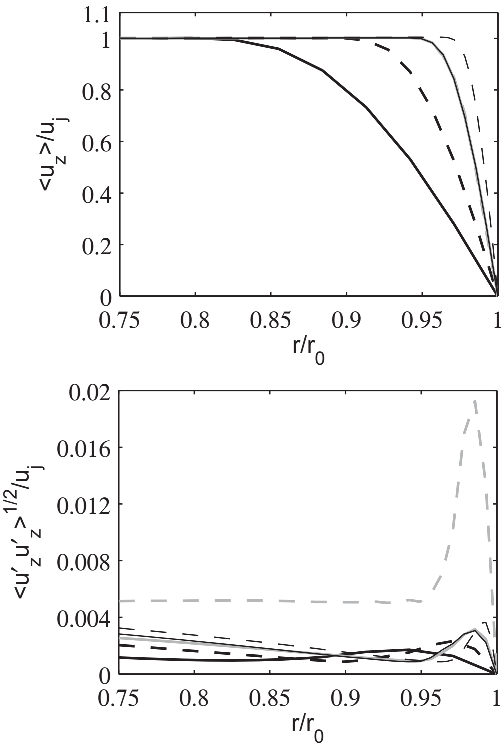

Figure 1: Profiles at $z = 0$ of mean axial velocity $< u_z >$, and of the rms values of fluctuating axial velocity u'_z. See line types in table 1.

To further characterize the initial jet shear layers, the integral length scales of axial velocity u'_z in the azimuthal direction, $L_{11}^{(\theta)}$, estimated at $r = r_0$ and $z = 0.1r_0$, are given in table 3. Significant values up to 0.59π are found, which indicates that azimuthal correlation is high in the vicinity of the nozzle exit. The addition of random pressure disturbances in the pipe however appears to lower the azimuthal correlation: from $L_{11}^{(\theta)} = 0.53\pi$ for JetD005, one gets indeed $L_{11}^{(\theta)} = 0.39\pi$ for JetD005p250, and only $L_{11}^{(\theta)} = 0.06\pi$ for JetD005p2000.

Table 3: Shear-layer momentum thickness δ_θ and rms peak value of velocity u'_z at $z = 0$, and integral length scale in the azimuthal direction $L_{11}^{(\theta)}$ calculated from u'_z at $r = r_0$ and $z = 0.1r_0$.

Reference	δ_θ/r_0	$<u_z'^2>^{1/2}/u_j$	$L_{11}^{(\theta)}/\pi$
JetD02	0.0232	0.0017	0.59
JetD01	0.0116	0.0023	0.61
JetD005	0.0056	0.0031	0.53
JetD0025	0.0025	0.0036	0.40
JetD005p250	0.0056	0.0032	0.39
JetD005p2000	0.0056	0.0192	0.06

Table 4: Jet experiments: Mach and Reynolds numbers M and Re_D, initial shear-layer momentum thickness δ_θ, intensities of fluctuating velocity u'_z, and symbols used.

Reference	M	Re_D	δ_θ/r_0	u'_z/u_j	Symbol
Husain	0.09	2.5×10^5	0.003	0.025	△
Lau	0.9	10^6	na	na	o
Arakeri	0.9	5×10^5	0.05	0.10	□
Fleury	0.9	7.7×10^5	na	na	◇

Corresponding experiments

The simulation results will be compared in the next section with measurements, provided by Husain and Hussain (1979) for an axisymmetric mixing layer, and by Lau *et al.* (1979), Arakeri *et al.* (2003) and Fleury *et al.* (2008) for round jets at Mach number 0.9 and Reynolds numbers higher than 5×10^5. Some available parameters of these experiments are documented in table 4. The initial properties of the axisymmetric mixing layer investigated by Husain and Hussain (1979) are in particular similar to those exhibited in the four jets with nozzle-exit momentum thicknesses $\delta_\theta \leq 0.0056r_0$ in table 3.

SIMULATION RESULTS

The effects of the jet initial conditions are illustrated here by vorticity snapshots, and by providing characteristics of the mean and turbulent developments of the shear layers and jet flows.

Vorticity snapshots

Snapshots of vorticity norm are displayed in figure 2 for the different jets, for $z \leq 12r_0$ to focus on the development of the shear layers. In the four jets without inlet noise, JetD02, JetD01, JetD005 and JetD0025, the mechanisms taking place downstream of the nozzle lip are the same. The turbulent transition in the initially laminar mixing layer first consists of the processes of vortex roll-up and pairing. This pattern is particularly visible in JetD02, but it is also observed in the other jets with thinner boundary-layer momentum thicknesses. After the first vortex pairing, three-dimensional turbulence appears and becomes dominant.

The variations of the initial momentum thickness affect the size of the coherent vortical structures generated by the shear-layer roll-up. As δ_θ decreases, these structures are significantly smaller, as expected. The vortex roll-up also occurs closer to the nozzle-exit section, but the shear layer, once turbulent, seems to develop more slowly. This latter

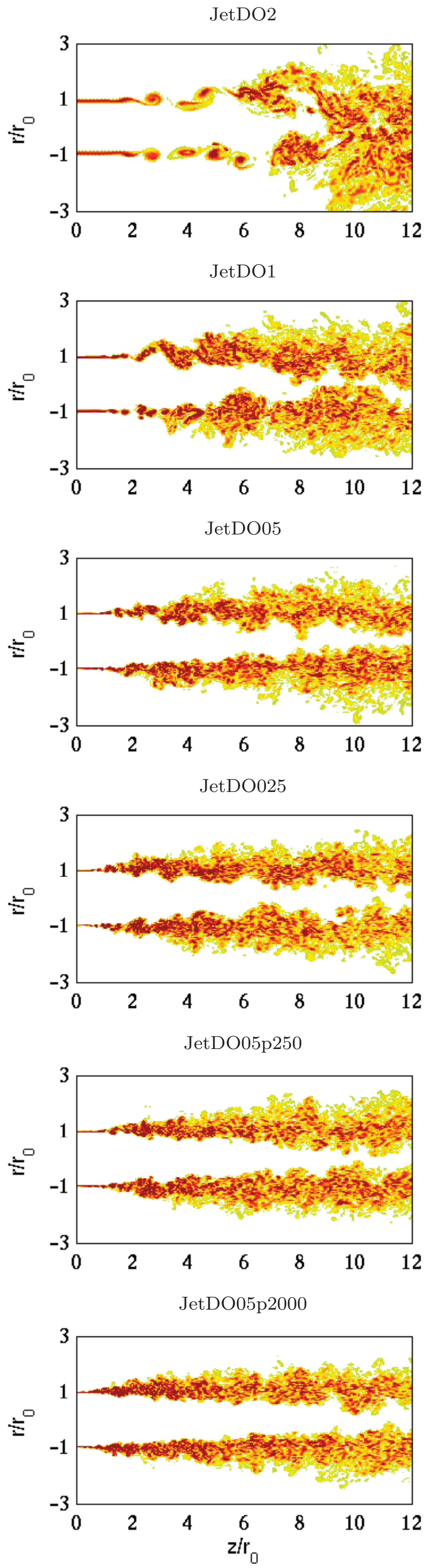

Figure 2: Snapshots in the (z, r) plane of vorticity norm. The color scale ranges up to the level of $8u_j/r_0$.

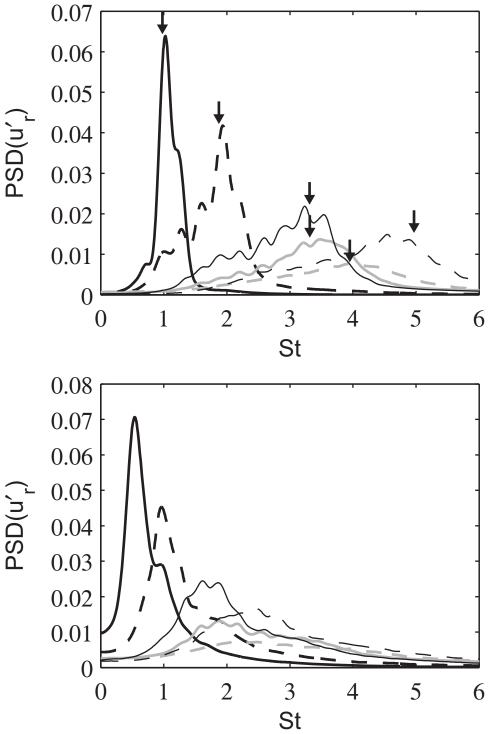

Figure 3: Spectra of velocity u'_r at $r = r_0$. Top: around vortex roll-up at $z/r_0 = 2.5$, 1.7, 1.1, 0.8, 0.95 and 0.6, bottom: downstream of pairing at $z/r_0 = 4.8$, 2.6, 1.6, 1.2, 1.3 and 0.9, respectively for JetD02, JetD01, JetD005, JetD0025, JetD005p250, and JetD005p2000. See line types in table 1.

point will be supported later by profiles of centerline velocity indicating longer potential core for thinner initial momentum thickness.

Finally the vorticity fields obtained for JetD005p250 and JetD005p2000 are fairly similar to that for JetD005, also with $\delta_\theta = 0.0056 r_0$ at $z = 0$. In the two former jets, however, the mixing layers develop earlier while exhibiting less organized vortical structures. This suggests that the addition of random noise inside the pipe inhibits the roll-up/pairing process.

Shear-layer development

To characterize the initial development of the shear layers, spectra of fluctuating radial velocity u'_r are computed along the lip line at two positions corresponding roughly to the location of the vortex roll-up and to the end of the first pairing, respectively. They are presented in figure 3. In top figure, the roll-ups appear to be dominated by frequency peaks, whose values agree well with the frequency $f\delta_\theta/u_j = 0.012$ found experimentally in initially laminar shear layers (see in Zaman (1985a, 1985b) for instance), which is indicated by the arrows for the different jets. In bottom figure, after the first pairing, the velocity spectra show peaks at the first subharmonics of the roll-up frequencies. This behaviour is observed in all jets. However it is less pronounced in jets with thinner initial shear layers, and when inlet random noise is introduced in the pipe nozzle, which implies that the roll-up/pairing process is then weaker.

The spreading of the jet shear layers is quantified in figure 4 displaying the variations of the shear-layer momentum thickness for $z \leq 8r_0$. As δ_θ becomes smaller at the nozzle exit, the shear layer develops earlier but at a slower rate, and this trend is strengthened by the addition of inlet random

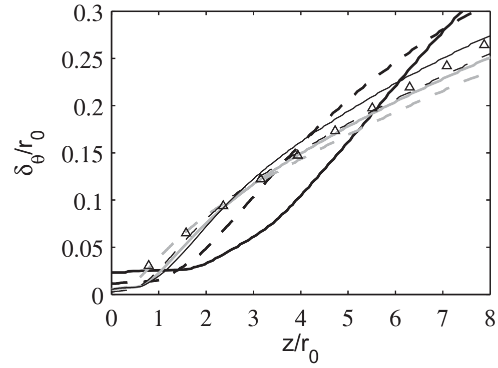

Figure 4: Variations of shear-layer momentum thickness δ_θ. See line types and symbols in tables 1 and 4.

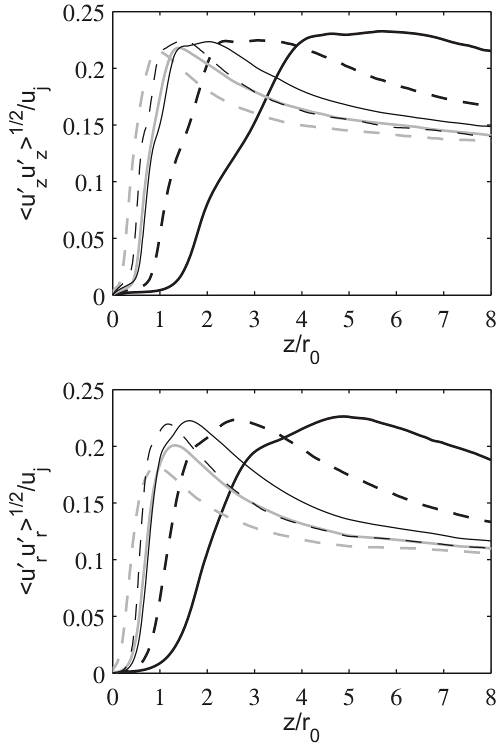

Figure 5: Variations along the lip line at $r = r_0$ of the rms values of fluctuating velocities u'_z and u'_r. See line types in table 1.

noise in the nozzle. In addition, there is a good agreement between the plots of δ_θ obtained for the jets with thin nozzle-exit boundary layer, *i.e.* with $\delta_\theta \leq 0.0056 r_0$ at $z = 0$, and the experimental results provided by Husain and Hussain (1979) for an axisymmetric initially laminar mixing layer with $\delta_\theta = 0.003 r_0$. For similar inflow conditions, simulation and experiment thus appear to provide close solutions.

The intensities of turbulence in the shear layers are shown in figure 5 with the variations of the levels of fluctuating axial and radial velocities along the lip line. In agreement with the previous results, they grow more rapidly with smaller shear-layer thickness, and with the introduction of upstream random noise. The curves obtained for the jets without inlet noise, JetD02, JetD01, JetD005 and JetD0025, display a dual-peak shape, which is typical, from Zaman and Hussain (1980), of the presence of strong vortex pairings at a fixed location. This feature is however not observed for JetD005p250 and JetD005p2000. It is also in-

Table 5: Position of the end of the potential core z_c, and rms peak values of velocity u'_r along centerline and lip line.

Reference	z_c/r_0	$<u'^2_r>^{1/2}/u_j$ max at r=0	$<u'^2_z>^{1/2}/u_j$ max at r=r_0
JetD02	8.6	0.139	0.226
JetD01	10.8	0.093	0.223
JetD005	14.1	0.071	0.222
JetD0025	16.8	0.057	0.222
JetD005p250	16.4	0.071	0.201
JetD005p2000	18.1	0.069	0.181

teresting to notice that the peak levels obtained for velocity u'_z are all around $0.23u_j$, whereas the peak levels for velocity u'_r are around $0.22u_j$ for JetD02, JetD01, JetD005 and JetD0025, but decrease down to $0.18u_j$ for JetD005p2000, as reported in table 5. These results give evidence of the significant changes that can occur in the shear-layer turbulence when transition is initially affected by random disturbances even of very low magnitude, the roll-up/pairing process being of less importance in this case. This sensitivity to inflow disturbances may have weak influence on the jet downstream development, but notable consequences on the radiated acoustic field (Bogey and Bailly, 2009).

Jet flow development

The effects of the nozzle-exit boundary-layer thickness on the mean flow field of the circular jets are shown in figure 6 with the profiles of centerline mean axial velocity u_c and of the jet half-width $\delta_{0.5}$ defined by $<u_z>(r=\delta_{0.5})=u_c/2$. When δ_θ is reduced at $z=0$, as suggested by the shear-layer properties presented previously, the jets develop on the centerline, and spread radially, at farther axial distance. This results in longer potential cores, in agreement with the simulations of Kim and Choi (2009) for a jet at $\mathrm{Re}_D=10^5$. To quantify this, the positions z_c of the end of the potential core are evaluated for the present jets using $u_c(z=z_c)=0.95u_j$, and given in table 5. For the four jets without inlet noise, they range from $z_c=8.6r_0$ for JetD02 up to $z_c=16.8r_0$ for JetD0025. The values of z_c obtained for JetD005p250 and JetD005p2000, are similar to the latter value. They also clearly increase with the magnitude of the inflow noise. The addition of random disturbances in the pipe therefore seems to slow down the jet mean flow development.

For comparison with experimental data, the centerline velocity decays and jet spreadings obtained by Lau *et al.* (1979), Arakeri *et al.* (2003) and Fleury *et al.* (2008) for Mach 0.9 jets at Reynolds numbers $\mathrm{Re}_D \geq 5\times10^5$ (see in table 4) are depicted in figure 6. Despite uncertainties and possible variations in the experimental initial conditions, the measurements are rather close. For both u_c and $\delta_{0.5}$, they fairly agree with the curves determined for the jets simulated with thin initial shear layers. They fall in particular especially well on the profiles for JetD005.

To characterize the magnitude of the jet turbulence, the levels of the fluctuating axial and radial velocities along the centerline are presented in figure 7. As the initial shear-layer thickness decreases, they reach peaks with lower amplitude, farther in the downstream direction, both for the axial velocity u'_z as in Kim and Choi (2009), and for the radial velocity u'_r. In table 5, the maxima of the centerline rms levels for u'_r are for instance reported to be $0.134u_j$ for JetD02, but only $0.057u_j$ for JetD0025. This trend could be due to the

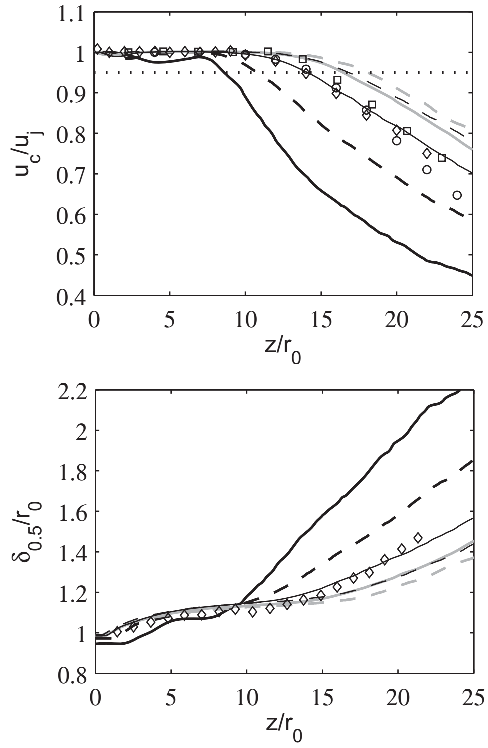

Figure 6: Variations of centerline mean axial velocity u_c and jet half-width $\delta_{0.5}$. The dotted line represents $u_c=0.95u_j$. See other line types and symbols in tables 1 and 4.

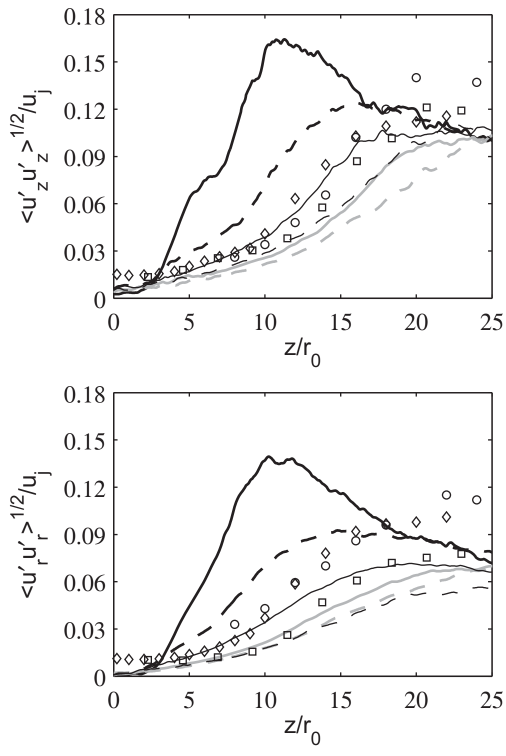

Figure 7: Variations along the centerline at $r=0$ of the rms values of fluctuating velocities u'_z and u'_r. See line types and symbols in tables 1 and 4.

combination of getting earlier shear-layer development and longer potential core as δ_θ becomes smaller at the nozzle exit. The shear-layer transition, and consequently its related high rms velocity levels around $0.2u_j$ observed in figure 5, then occur farther from the end of the potential core where the shear-layer turbulence merges on the jet axis. Regarding the influence of adding inflow noise, it appears minor because the peaks of the fluctuation levels for JetD005p250 and JetD005p2000 are similar to those for JetD005 with same initial δ_θ.

As previously for the mean flow, comparisons with measurements by Lau *et al.* (1979), Arakeri *et al.* (2003) and Fleury *et al.* (2008) are made. Albeit significantly scattered, the experimental data are somewhat closer to the results obtained for the jets with thinner nozzle-exit boundary layer. The data from Arakeri *et al.* (2003) are especially in good agreement with the rms velocity levels for JetD005.

CONCLUSION

The present LES of initially laminar subsonic jets at Reynolds number 10^5 clearly illustrate the significant effects of the thickness of the nozzle-exit boundary layer on round jet development. Decreasing the initial shear-layer momentum thickness δ_θ leads in particular to an increase of the length of the potential core, and to a reduction of the turbulence intensities on the jet axis. For both flow features, a good agreement with experimental data available for high Reynolds number jets is thus found for $\delta_\theta \simeq 0.005r_0$. Therefore it appears necessary in jet simulations to specify sufficiently thin initial boundary layers. It could still be recommended that they should be as close as possible to the experimental conditions, when they are known.

In the same way it turns out to be important to control the properties of the initial velocity disturbances. In the present LES, the jet shear-layer transition is indeed shown to be appreciably modified by the addition of small inlet noise, leading notably to a weakening of the processes of vortex roll-up and pairings. Such changes likely arise even for minute differences in the inflow conditions, as it can strikingly be seen from the results obtained for JetD005 and JetD005p250, which are two initially fully laminar jets with same nozzle-exit boundary-layer thickness and turbulent intensities.

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